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DEVELOPMENT OF LEARNING TRAJECTORY-BASED INSTRUCTIONAL MATERIALS ON MAGIC POLYGON TO ENHANCE VOCATIONAL HIGH SCHOOL STUDENTS

Muhammad Zainur Rofiqi [1,*], Arika Indah Kristiana [2], Ridho Alfarisi [1], Slamin [3] and Antonius Cahya Prihandoko [1]

[1] Department of Postgraduate Mathematics Education, Faculty of Teacher Training and Education, University of Jember, Indonesia.
[2] Department of Postgraduate Mathematics Education, PUI-PT Combinatorics and Graphs, CGANT, Faculty of Teacher Training and Education, University of Jember, Indonesia.
[3] Department of informatics, Faculty of Computer Science, University of Jember, Indonesia.
* Corresponding Author

ORCID Details
Slamin: https://orcid.org/0000-0002-1280-8037
Antonius Cahya Prihandoko: https://orcid.org/0000-0001-7112-5458

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ABSTRACT

Low mathematical logical reasoning and conceptual understanding among Vocational High School (SMK) students remain a major challenge in mathematics education, largely due to procedural instruction and limited exploration of discrete mathematical structures. This study aims to develop and evaluate the validity, practicality, and effectiveness of mathematics learning materials based on the Magic Polygon Learning Trajectory to enhance students' mathematical reasoning and problem-solving abilities. This Research and Development (R&D) study adapted the 4D model (Define, Design, Develop, Disseminate). The developed instructional materials consisted of Teaching Modules, Student Worksheets (LKM), and Student Progress Tests (TPM), structured across four progressive cognitive activity levels: informal, pre-formal, formal, and mastery. The subjects of the field trial were 31 Grade X students of SMKN 1 Seririt. Data were gathered using expert validation sheets, implementation observation sheets, student response questionnaires, and mathematical progress tests (pretest and posttest). The results demonstrated that: the learning materials were highly valid, achieving an average validation score of 3.754 (93%); the materials were highly practical, evidenced by an optimal instructional execution rate of 88% and a 93% positive student response rate; and the materials were effective in significantly enhancing students' achievement, as indicated by an increase in mean test scores from 54.00 to 69.84 ($p < 0.05$). Qualitative analysis confirmed that the Magic Polygon Learning Trajectory successfully transitioned students' reasoning from random trial-and-error to structured algebraic reasoning across decomposition, pattern recognition, abstraction, and algorithm design stages.

Keywords: Learning Trajectory; Magic Polygon; Instructional Materials; 4d Model; Vocational High School

1 INTRODUCTION

The dynamic evolution of education in the era of the Fourth Industrial Revolution and Society 5.0 necessitates a paradigm shift in mathematics instruction, transitioning from the rote execution of mechanical algorithms toward high-level logical reasoning. Within modern curriculum frameworks, developing structured mathematical reasoning has become a foundational pillar in equipping learners to solve complex problems.

Despite the strategic emphasis placed on high-level mathematical reasoning within the Merdeka Curriculum, empirical realities in classrooms reveal a concerning disparity. Results from PISA 2022 indicate that Indonesian students' average mathematics score remains low at 366 (substantially below the OECD average of 472), with approximately 82% of students failing to achieve Level 2 basic proficiency. This phenomenon demonstrates that the majority of students can only respond to direct procedural instructions and struggle when confronted with open-ended tasks requiring structured logical reasoning. At the Vocational High School (SMK) level, this deficiency stems from instructional approaches that heavily emphasize mechanical formula memorization without providing adequate opportunities to explore underlying logical structures.

A novel topic in discrete mathematics rich in logical structure, yet rarely integrated into high school curricula, is graph labeling, specifically the Magic Polygon (Cai & Howson, 2013). A Magic Polygon is a planar graph composed of a set of vertices (v) and edges (e), where placing positive integers on each vertex yields a constant sum or edge weight along every side (Wallis, 2001). The puzzle-like visual characteristics of magic polygons naturally stimulate students' analytical abilities by encouraging them to decompose graph components, identify regularities in numerical relations, filter out irrelevant visual information, and structure systematic labeling steps. However, the Magic Polygon involves a high level of abstraction. Presenting formal mathematical rules directly without accommodating students' cognitive readiness frequently induces learning obstacles. To bridge this cognitive gap, an instructional trajectory commonly known as a Learning Trajectory is required. A Learning Trajectory functions as an instructional roadmap that anticipates the progression of students' mathematical reasoning from informal contextual understanding to formal structured comprehension through progressive mathematization (Simon, 1995; Gravemeijer, 2004).

Prior empirical research underscores the vital role of Learning Trajectories and structured instructional design in overcoming cognitive hurdles across various mathematical domains. Indriani (2020) demonstrated that structured learning trajectories significantly enhance students' fundamental conceptual understanding by providing scaffolding activities tailored to their cognitive levels. Wulansari et al. (2021) established the effectiveness of Hypothetical Learning Trajectories (HLT) on geometry topics in vocational high schools, proving that structured learning paths successfully reduce cognitive obstacles. Furthermore, Surya (2023) highlighted that implementing learning trajectory designs enables teachers to map potential learning obstacles early and adjust instructional scaffolding accordingly. More recently, Pratama (2024) revealed that learning trajectory-based instructional designs combined with contextual problems significantly accelerate conceptual understanding and mitigate cognitive barriers in abstract mathematical topics.

Although research on Learning Trajectories has expanded rapidly, its application within discrete mathematics topics such as the Magic Polygon at the SMK level remains sparse. Existing literature predominantly focuses on elementary arithmetic or conventional geometry. Therefore, this study aims to analyze the development process and evaluate the validity, practicality, and effectiveness of Learning Trajectory-based instructional materials on the Magic Polygon for vocational high school students.

2 MATERIAL AND METHODS

2.1 Material and Development Framework

The core output of this Research and Development (R&D) study is a comprehensive, validated instructional package consisting of three main developed products. Teaching Modules: Designed under the Merdeka Curriculum framework for Grade X Vocational High School (SMK) mathematics. The module integrates a Learning Trajectory (LT) roadmap specifically tailored for discrete mathematics topics, outlining explicit instructional objectives, teacher-facilitator guidelines, time allocations, and scaffolding strategies across four sequential cognitive phases (informal, pre-formal, formal, and mastery). Student Worksheets (LKM): Developed as interactive, problem-based work guides structured around the Magic Polygon topic. LKM 1 focuses on informal graph concepts (modeling daily routes), while LKM 2 focuses on pre-formal and formal Magic Polygon labeling tasks. The LKMs incorporate visual prompts, structural hints, and guided questions designed to directly trigger logical reasoning and systematic problem solving. Student Progress Tests (TPM): Constructed as diagnostic and evaluation instruments (pretest and posttest) consisting of essay problems involving Magic Polygons (e.g., triangle and pentagon magic graphs). Each test item is paired with a specific scoring rubric to measure students' cognitive shifts before and after using the developed materials.

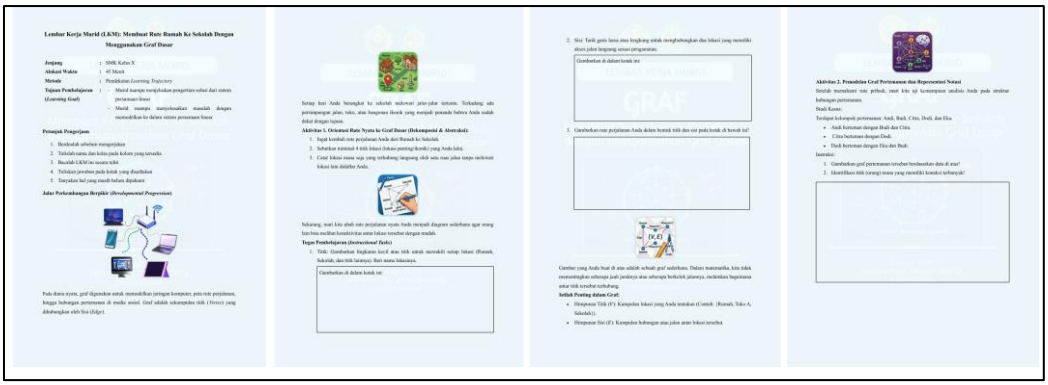

Figure 1: Design of LKM

2.2 Research Design and Development Procedure

This study adapted the four-stage 4D development model (Define, Design, Develop, Disseminate) proposed by Thiagarajan, Semmel, & Semmel (1974) to systematically construct, refine, and evaluate the instructional materials. Define Phase: Conducted five foundational analyses: Front-end analysis identified that mathematics instruction in vocational schools remains procedural; Learner analysis established that Grade X students require visual representations and structured scaffolding to overcome cognitive leaps; Concept analysis structured the Magic Polygon topic (triangle and pentagon orders) with linear equation systems; Task analysis broke down required student tasks into structured sub-tasks; and Specifying learning objectives formulated target competencies.

Design Phase: Focused on drafting the initial prototype (Draft 1) of the instructional package. Activities included constructing criterion-referenced test items (pretest and posttest), selecting worksheet media layouts, determining document formats adhering to Merdeka Curriculum standards, and embedding the three core components of a Learning Trajectory (learning goals, instructional tasks, and hypothetical learning processes). Develop Phase: Aimed to produce a final, high-quality instructional package (Draft 2) through expert validation and field trials. Two expert validators evaluated the materials. Following revisions based on expert feedback, a field trial was conducted in a vocational classroom setting to evaluate product practicality and effectiveness. Disseminate Phase: Handed over the finalized, validated instructional package to mathematics teachers at SMKN 1 Seririt and shared the development framework with partner schools.

2.3 Research Subjects and Data Collection Instruments

The field trial was conducted with 31 Grade X Accounting 1 students at SMKN 1 Seririt, Buleleng, Bali, during the odd semester of the 2026/2027 academic year. Data collection utilized four validated instruments. Material Validation Sheets: Completed by two academic experts (mathematics education specialists) to evaluate the formatting, content accuracy, language clarity, and pedagogical alignment of the Lesson Plans, LKM, TPM, observation sheets, and student questionnaires. Instructional Implementation Observation Sheet: Used by classroom observers during instructional sessions to assess teacher execution, student engagement, and the practical usability of the developed materials in real time. Student Response Questionnaire: Administered hardcopy to students post-instruction to gather user feedback regarding worksheet usability, visual layout, instructions clarity, and interest. Student Progress Tests (TPM Pretest & Posttest): Used to measure the cognitive effectiveness of the developed materials in improving mathematical performance.

2.4 Data Analysis Techniques

The quality of the developed instructional materials was evaluated based on Nieveen's (1999) product evaluation framework. Validation data from expert review sheets were analyzed by calculating the average score and percentage across all evaluated dimensions.

$$\text{Percentage } (P) = \frac{\sum \text{Obtained Score}}{\sum \text{Maximum Score}} \times 100\%$$

Product validity criteria were categorized as: $80\% < P \leq 100\%$ (Highly Valid), $60\% < P \leq 80\%$ (Valid), $40\% < P \leq 60\%$ (Fairly Valid), and $P \leq 40\%$ (Invalid). Evaluated from two data sources: (a) observer ratings on implementation sheets to calculate the percentage of instructional execution, and (b) student response percentages regarding worksheet usability. Materials were categorized as highly practical if both metrics exceeded 80%. Analyzed using student score progress from TPM pretest and posttest. Data normality was verified via the Shapiro-Wilk test. Inferential statistical

analysis was conducted using a Paired Sample t-Test ($\alpha = 0.05$) via SPSS to verify whether using the developed materials led to a statistically significant gain in students' mathematical mastery.

3 RESULTS

3.1 Results of the Define and Design Phases

The Define phase, five detailed analyses were executed. Front-End Analysis (Needs Analysis): Classroom observations at SMKN 1 Seririt revealed that mathematics instruction was predominantly procedural, leaving students struggling with open-ended problem solving and discrete mathematical structures. The Magic Polygon topic was selected as an innovative context to bridge abstract concepts and structured reasoning. Learner Analysis: Grade X Accounting students required step-by-step visual prompts and instructional scaffolding to transition smoothly from concrete ideas to abstract mathematical models without experiencing cognitive overload. Curriculum & Concept Analysis: Analyzed learning standards within the Merdeka Curriculum and structured the Magic Polygon topic (triangle and pentagon orders) to seamlessly connect graph labeling with Systems of Linear Equations (SLE). Task Analysis: Identified and organized student tasks sequentially into worksheet activities, ranging from real-world route mapping to systematic algebraic derivations. Specifying Learning Objectives: Formulated concrete instructional benchmarks focusing on graph modeling, pattern identification, linear equation formulation, and algorithmic solving steps.

The Design phase successfully produced Draft 1 of the product package: Structured for two meetings (2×2 hours). The module lays out an explicit Learning Trajectory where the teacher acts as a facilitator, guiding students from informal graph representations to formal System of Linear Equations (SLE) solutions on Magic Polygons. Produced two distinct worksheets. LKM 1 ("Basic Graph & Route Modeling") guides students to map real-world home-to-school routes into vertices (v) and edges (e). LKM 2 ("Magic Polygon Number Mystery") introduces triangle and pentagon magic graphs, systematically guiding students through node labeling, vertex counting, and formula derivation. Produced two distinct worksheets. LKM 1 ("Basic Graph & Route Modeling") guides students to map real-world home-to-school routes into vertices (v) and edges (e). LKM 2 ("Magic Polygon Number Mystery") introduces triangle and pentagon magic graphs, systematically guiding students through node labeling, vertex counting, and formula derivation.

3.2 Expert Validation Results

To establish structural and theoretical validity, Draft 1 was submitted to expert validators. The expert review recapitulation across all developed components is detailed in Table 1.

Table 1: Recapitulation Results of Expert Validation for Developed Materials

Evaluated Material Component	Average Score	Validation
Teaching Module / Lesson Plan	3,84	95,92%
Student Worksheet (LKM)	3,71	92,75%
Student Progress Test (TPM)	3,71	92,75%
Implementation Observation Sheet	3,78	94,42%
Student Response Questionnaire	3,73	93,33%
Overall Material Average	3,754	93%

The validation results demonstrate that the developed instructional materials reached an overall validity score of 3.754 (93%), qualifying as *Highly Valid*. Expert feedback focused primarily on minor formatting refinements, font adjustments, and simplifying technical graph terms to suit vocational students' readability.

3.3 Practicality Evaluation of the Materials

The practicality of the developed materials during field testing was measured via instructional implementation observations and student user feedback. Observer evaluation of classroom execution using the developed modules recorded an overall execution rate of 88% (average score 3.53), categorized as *Highly Practical*. Detailed implementation scores across phases: Student Activity (75%), Teacher Facilitation (90%), and Learning Environment (100%). Student User Feedback: Questionnaire responses from 31 students confirmed high usability and user satisfaction with the developed worksheets. The results are summarized in Table 2.

Table 2: Summary of Student Responses to the Developed Instructional Materials

Assessed Aspect of Developed Materials	Positive Response %
Enjoyability of instructional components and activities	96%
Perception that worksheets helped train computational thinking	95%
Novelty of the Magic Polygon topic and worksheet approach	91%
Clarity of language and instructions in LKMs	94%
Comprehensibility of task prompts in LKMs and TPMs	82%
Visual appeal and layout design of LKMs and test covers	97%
Student interest in participating in future LT-based lessons	97%
Overall Material Average	93%

An overall student response rate of 93% demonstrates that the developed LKMs and learning modules were user-friendly, visually engaging, and clear, fully satisfying the practicality criteria.

3.4 Quantitative Analysis of Student Progress

The cognitive impact of the developed instructional materials was evaluated by comparing TPM pretest and posttest results. Data normality was verified using the Shapiro-Wilk test ($p > 0.05$). Inferential statistical analysis via Paired Sample t-Test confirmed a significant increase in student performance ($t = -8.697$, $p = 0.000 < 0.05$).

Table 3: Paired Sample Statistics for Test Performance

Test Pair	Assessment	Mean Score	N
Pair 1	Pretest TPM	54.00	31
	Posttest TPM	69,84	31

The mean score improved significantly by 15.84 points, rising from a pretest average of 54.00 to a posttest average of 69.84. ($r = 0.719, p < 0.05$)

3.5 Qualitative Contribution of Worksheet Design across Trajectory Stages

Qualitative analysis of completed student LKMs revealed how specific design elements supported student growth across the four Learning Trajectory cognitive stages. Informal Stage (Real-World Experience): Prompts in LKM 1 guided students to deconstruct complex real-world travel routes into discrete vertex sets (v) and edge sets (e). Students isolated corner nodes from side nodes rather than treating network figures as unstructured lines. Pre-Formal Stage (Pattern Exploration): Scaffolding in LKM 2 led students to observe that numbers placed at corner vertices were counted twice across intersecting sides. Students recognized that placing larger numbers on corner vertices systematically increased the total edge sum (magic constant). Formal Stage (System of Linear Equations): Worksheet activities helped students filter out visual graph representations and focus on core algebraic relationships. Students converted Magic Polygon figures into formal Systems of Linear Equations (SLE), representing node labels as algebraic variables. Mastery Stage (Algorithmic Solving Procedures): The final section of LKM 2 prompted students to write down sequential, step-by-step procedures for assigning vertex values. Rather than relying on trial-and-error guessing, students successfully executed structured algorithms to solve new Magic Polygon orders.

4 DISCUSSION

The overall findings demonstrate that applying the 4D development model to build Learning Trajectory-based materials effectively addresses the shortage of discrete mathematics learning tools in vocational schools. Validation of Practicality and Effectiveness: High product validity (93%) and strong practicality (88% implementation rate; 93% user satisfaction) confirm that structuring LKMs around progressive mathematization creates an effective learning bridge. In terms of effectiveness, the significant increase in mean test scores from 54.00 to 69.84 ($p = 0.000 < 0.05$) proves that the instructional materials successfully upgraded students' mathematical reasoning and problem-solving execution.

Throughout the development process, time constraints during classroom trial sessions posed a notable challenge, as students required extended guidance during the transition from pre-formal pattern searching to formal algebraic modeling. Additionally, the trial was limited to a single field group of 31 Grade X vocational students in one school.

Future research should expand trials across diverse vocational majors and investigate the integration of digital manipulative tools.

The empirical results align closely with the constructivist learning trajectory theories proposed by Simon (1995) and Gravemeijer (2004), confirming that progressive mathematization successfully bridges informal intuitions and formal mathematics. The findings resonate with Indriani (2020) and Wulansari et al. (2021), who demonstrated that structured trajectories reduce cognitive obstacles in abstract topics. Furthermore, the results reinforce the findings of Surya (2023) and Pratama (2024), proving that trajectory-based instructional materials effectively eliminate learning gaps, transition students away from unguided trial-and-error, and foster robust mathematical reasoning.

5 CONCLUSION

Based on the research findings, the following conclusions are drawn. Learning Trajectory-based instructional materials on the Magic Polygon were successfully developed using the 4D model (Define, Design, Develop, Disseminate). The resulting package comprising Teaching Modules, Student Worksheets (LKM), and Student Progress Tests (TPM) effectively guides students through four progressive cognitive levels: informal, pre-formal, formal, and mastery. The developed instructional materials met high-quality standards, achieving an expert validation score of 3.754 (93%, Highly Valid), an implementation rate of 88%, and a positive student response rate of 93% (Highly Practical). The materials proved highly effective in boosting vocational students' mathematical achievement. This is evidenced by the rise in mean test scores from 54.00 to 69.84, and a paired sample t-test result of $t = -8.697, p = 0.000 < 0.05$. The Learning Trajectory framework successfully shifted student reasoning from random trial-and-error to structured algebraic problem solving.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

Statement of ethical approval

This research was approved by the Research Ethics Committee, University of Jember.

Statement of informed consent

All participants gave their informed consent before participating in this study.

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