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DECOMPRESSION BASED ASSESSMENT OF ELASTIC AND CRACKED PRESTRESSED CONCRETE SECTIONS

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ABSTRACT

The decompression threshold provides a useful reference for relating prestress, tensile stress and cracking in concrete sections. This study develops a reproducible assessment framework that combines classical kern theory with a normalized residual moment and compatible section analysis. Three modelling levels are compared: a gross elastic section with constant prestressing force, an uncracked transformed section with bonded reinforcement, and a locally cracked section with zero tensile concrete stress. A deterministic campaign considers rectangular, T-shaped and I-shaped sections with varying eccentricity, prestress, reinforcement ratios and concrete strength. Of 729 candidate configurations, 621 satisfy the initial tendon-stress filter, yielding 40,188 accepted loading states. Gross-section decompression moments differ from compatible elastic predictions by -2.80% to $+2.05\%$, supporting their usefulness for preliminary threshold assessment. At a normalized loading coordinate of 0.50, gross-section curvature errors have a median of 3.19% and a maximum of 12.95%. At the first sampled cracked state, uncracked extrapolation underestimates local curvature by 21.32–85.92%. The normalized coordinate consistently aligns decompression and theoretical cracking while revealing the influence of geometry and reinforcement on subsequent response. These findings provide a practical basis for selecting the appropriate modelling level and separating threshold assessment from deformation prediction. The residual moment remains a loading indicator; cracked-section response requires the coupled contributions of concrete, passive reinforcement and tendons. The results describe local cracked sections and establish a foundation for future member-level assessment with tension stiffening and experimental validation.

Keywords: Prestressed Concrete; Decompression; Kern Theory; Partial Prestressing; Cracked Section; Serviceability.

1 INTRODUCTION

Prestressing enables concrete members to sustain substantial external bending before tensile stress develops at a selected fibre. Its effectiveness results from the combined action of uniform compression and an eccentric bending moment, as established in classical prestressed concrete analysis [1,2]. A convenient interpretation should retain both effects while remaining compatible with the changing response of concrete and reinforcement.

Partial prestressing provides a well-established connection between reinforced and fully prestressed concrete. Its development demonstrates how controlled tension and cracking can be balanced against deformation, durability and strength requirements [3–5]. This perspective is particularly useful for preliminary design, where a transparent loading indicator can help identify when a more detailed section analysis becomes necessary.

The decompression threshold offers such an indicator. Through kern theory, it identifies the external moment at which the compression at a selected fibre becomes zero. Magnel diagrams place this condition within the wider feasible domain defined by transfer and service stresses [6]. Load balancing addresses the complementary structural problem of representing tendon actions through equivalent loads [7]. These methods provide the classical foundation for the present framework.

Compatible analysis of cracked prestressed sections is also well established. Nilson [8], Tadros [9] and Mast [10] developed methods for evaluating stresses after cracking, including the joint participation of prestressing and passive steel. Dilger and Suri [11] used decompression as a reference for steel-stress increments. Serviceability-based design and partial prestressing research further demonstrate the value of passive reinforcement for controlling response [12–14]. Accordingly, the present contribution lies in a reproducible comparison of modelling levels, rather than in claiming a new decompression formula or a new constitutive law.

The distinction between section response and member deformation is central to this comparison. Effective-inertia methods account for the influence of uncracked regions on deflection [15], while progressive-cracking analyses address stress transfer and tensile concrete behaviour [16,17]. Nonlinear and staged approaches extend this treatment to broader loading histories [18,19]. More recent studies demonstrate additional requirements for unbonded tendons [20] and deteriorated members [21]. The fib serviceability bulletins provide a wider framework for relating these analyses to structural verification [22,23].

This study evaluates how accurately a residual-moment representation supports threshold and response assessment. Its objectives are to establish consistent signs and definitions, quantify the difference between gross and transformed elastic sections, and assess uncracked extrapolation against a compatible local cracked-section model. A normalized coordinate places decompression at zero and theoretical first cracking at unity, enabling direct comparisons between geometries. The resulting numerical benchmark supports practical model selection and identifies the physical mechanisms required for subsequent member-level development.

2 MATERIALS AND METHODS

2.1 Mechanical assumptions and scope

The analysis considers uniaxial bending about a principal centroidal axis, small strains and plane sections remaining plane. The coordinate y is positive downward from the gross-section centroid; tensile stress and an external moment producing bottom-fibre tension are positive. The tendon eccentricity e is positive below the centroid. The strain field is $\varepsilon(y) = \varepsilon_0 + \kappa y$, and the positive bottom-fibre section modulus is $W_t = I_g/y_t$, where $y_t > 0$. Gross concrete area and second moment of area are denoted by A_g and I_g .

The reference state represents a defined service stage after prior prestress losses. Subsequent loading is monotonic; creep, shrinkage, relaxation and friction are not evolved during this increment. Tendons and passive steel are perfectly bonded and remain elastic within the adopted stress limits. Reinforcement areas are concentrated at their centroidal levels, with displaced concrete subtracted at those levels. The cracked model represents a section at a crack, with linear compressed concrete and no tensile concrete stress. It therefore provides local curvature and steel stresses rather than member-average curvature or crack width.

The term *surface-fibre decompression threshold* denotes zero normal stress at the selected outer fibre. Code decompression requirements may instead concern a compressed region around a tendon or duct under a prescribed load combination [6,24]. EN 1992-1-1:2023 supplies the contemporary normative context [25], whereas the benchmark material relationships are explicitly those of EN 1992-1-1:2004+A1:2014 [24]. National implementation remains relevant when selecting the applicable edition and National Annex [26]. The historical BPEL classes I, II and III provide context for no-tension, limited-tension and cracked verification [27]; they are not used as Eurocode categories or universal moment intervals. Serviceability assessment and ultimate resistance remain separate checks.

2.2 Decompression and residual moment

For a positive effective prestressing force P , the actions on the concrete are $N_p = -P$ and $M_p = -Pe$. The bottom-fibre stress in the gross elastic model is

$$\sigma_t = -\frac{P}{A_g} + \frac{M - Pe}{W_t}. \quad (1)$$

The kern distance associated with that fibre is $k_t = W_t/A_g = I_g/(A_g y_t)$. Setting equation (1) to zero gives

$$M_{\text{dec},g} = P(e + k_t), \quad M_{r,g} = M - M_{\text{dec},g}, \quad \sigma_t = \frac{M_{r,g}}{W_t}. \quad (2)$$

Equation (2) is exact within the adopted gross, uncracked elastic model. It follows directly from classical kern and pressure-line theory [1,2]. The compression resultant lies at $y_c = e - M/P$ and reaches $-k_t$ at bottom-fibre decompression. Thus Pe is a mechanical moment, while the additional term Pk_t accounts for the uniform compression that must also be overcome. For a rectangle, $k_t = h/6$ and $M_{\text{dec},g} = P(e + h/6)$. The expression $h + 6e$, obtained by rearrangement, is not a physical equivalent depth.

A specified tensile strength $f_{ct,\text{crit}}$ defines the theoretical first-cracking moment and normalized residual coordinate:

$$M_{\text{cr},g} = M_{\text{dec},g} + W_t f_{ct,\text{crit}}, \quad \lambda_{r,g} = \frac{M - M_{\text{dec},g}}{M_{\text{cr},g} - M_{\text{dec},g}}. \quad (3)$$

The coordinate is zero at decompression, unity at theoretical cracking and negative while the selected fibre remains compressed. Unlike M/M_{dec} , it directly scales the decompression-to-cracking interval. The tensile-strength convention must match the analysis objective; mean axial strength, effective age-dependent strength and flexural strength are not interchangeable.

For a transformed uncracked section with a fixed initial state, the fibre stress remains affine in external moment, $\sigma_{t,U} = a_t + b_t M$. Hence

$$M_{\text{dec},U} = -\frac{a_t}{b_t}, \quad M_{\text{cr},U} = M_{\text{dec},U} + \frac{f_{ctm}}{b_t}, \quad \lambda_{r,U} = \frac{M - M_{\text{dec},U}}{M_{\text{cr},U} - M_{\text{dec},U}}. \quad (4)$$

The slope b_t is positive for the retained configurations. Its inverse acts as an effective stress-to-moment modulus for this response. The numerical campaign uses $\lambda_{r,U}$, ensuring that unity coincides with the compatible model's cracking threshold. Gross and transformed normalizations are distinguished throughout.

For statically indeterminate structures, the global analysis must first supply secondary prestress effects. With M_{sec} positive in the external-moment direction, equation (1) becomes $\sigma_t = -P/A_g + (M + M_{\text{sec}} - Pe)/W_t$, giving $M_{\text{dec},g} = P(e + k_t) - M_{\text{sec}}$. This local expression assumes known resultants at the selected stage; effects already included in a global moment must not be counted twice [6,18]. For top-fibre assessment, the coordinate direction and signs of moment and eccentricity must be redefined consistently.

2.3 Geometry and parameter space

All sections have a depth of 600 mm. Table 1 specifies the rectangular (R), T-shaped (T) and I-shaped (I) geometries; flange and web widths are centred about the vertical axis. The passive reinforcement depth is $d_s = 0.90h$ from the top. The equivalent tendon depth is the gross-centroid depth plus e . Figure 1 illustrates these arrangements.

Table 1: Cross-section geometry and calculated gross properties.

Section	Component dimensions in mm	A_g in mm^2	Centroid depth in mm	k_t in mm
R	Rectangle 300 × 600	180,000	300.00	100.00
T	Top flange 800 × 120; web 180 × 480	182,400	202.11	80.84
I	Top flange 600 × 100; web 180 × 400; bottom flange 400 × 100	172,000	270.93	126.29

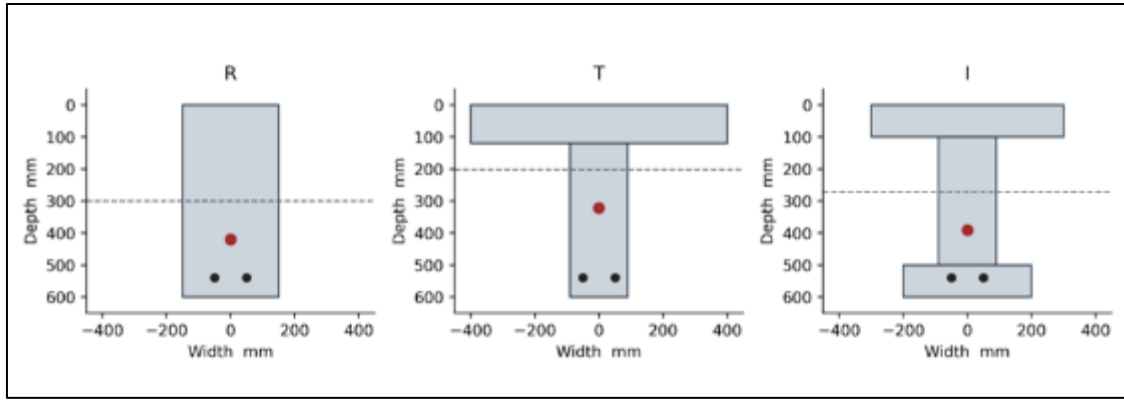


Figure 1. Cross-sections and reinforcement locations. Dashed lines mark gross centroids; red points show tendon locations for $e/h = 0.20$. Black points schematically represent the equivalent passive reinforcement layer.

Table 2 defines six three-level factors, giving 729 candidate combinations. Steel ratios are relative to gross area, $\rho_s = A_s/A_g$ and $\rho_p = A_p/A_g$. Detailed bar arrangements and duct geometry are outside the present section benchmark.

Table 2: Deterministic factorial parameter space.

Parameter	Definition	Values
Section shape	Gross geometry	R, T, I
Relative eccentricity	e/h	0.10, 0.20, 0.30
Initial precompression	$P_0/(A_g f_{ck})$	0.03, 0.06, 0.09
Passive steel ratio	A_s/A_g	0.002, 0.006, 0.010
Prestressing steel ratio	A_p/A_g	0.002, 0.004, 0.006
Concrete strength	f_{ck} in MPa	30, 40, 50

The elastic moduli are $E_s = 200,000$ MPa and $E_p = 195,000$ MPa. Mean axial tensile strength serves as a deterministic cracking threshold, with the first-generation Eurocode relationships [24]:

$$f_{ctm} = 0.30 f_{ck}^{2/3}, \quad E_c = 22,000 \left(\frac{f_{ck} + 8}{10} \right)^{0.3}. \quad (5)$$

All strengths and moduli in equation (5) are in MPa. The resulting tensile strengths are 2.90, 3.51 and 4.07 MPa; elastic moduli are approximately 32.84, 35.22 and 37.28 GPa. These values define reproducible inputs rather than experimental distributions. No aggregate-specific modulus adjustment is introduced.

2.4 Compatible section models and initialization

Model G uses gross-section rigidity and constant force P_0 , giving $\kappa_G = (M - P_0 e)/(E_c I_g)$. Model U includes compatible bonded steel and uncracked concrete. Model C uses the same geometry and initial prestress as U, but removes tensile concrete at the cracked section. The G–U comparison isolates gross-section and constant-force approximations; the U–C comparison quantifies uncracked extrapolation after cracking.

For reproducibility, introduce $\mathbf{q} = (\varepsilon_0, \kappa h)^T$ and $\mathbf{v}(y) = (1, y/h)^T$, so that $\varepsilon(y) = \mathbf{v}^T \mathbf{q}$. Subscripts s and p denote the passive and prestressing steel levels. The initial moment is $M_0 = 0.5 P_0 (e + k_t)$, and the prescribed tendon stress is $\sigma_{p0} = P_0/A_p$. The initial stiffness and strain state are

$$\mathbf{K}_0 = E_c \text{diag}(A_g, I_g/h^2) + (E_s - E_c) A_s \mathbf{v}_s \mathbf{v}_s^T - E_c A_p \mathbf{v}_p \mathbf{v}_p^T, \quad (6)$$

$$\mathbf{q}_0 = \mathbf{K}_0^{-1} \begin{pmatrix} -P_0 \\ (M_0 - P_0 e)/h \end{pmatrix}, \quad c_p = \frac{P_0}{A_p} - E_p \mathbf{v}_p^T \mathbf{q}_0. \quad (7)$$

The passive steel stress is compatible with the initial concrete strain and is not reset to zero. The tendon offset c_p ensures the prescribed initial force is recovered. At subsequent stages, $\sigma_s = E_s \mathbf{v}_s^T \mathbf{q}$ and $\sigma_p = E_p \mathbf{v}_p^T \mathbf{q} + c_p$. The uncracked solution is

$$\mathbf{K}_U = \mathbf{K}_0 + E_p A_p \mathbf{v}_p \mathbf{v}_p^T, \quad \mathbf{q}_U(M) = \mathbf{K}_U^{-1} \left[\begin{pmatrix} 0 \\ M/h \end{pmatrix} - A_p c_p \mathbf{v}_p \right]. \quad (8)$$

The tendon is an internal constituent of the composite section, so no additional external force equal to its prestress is applied to models U or C. Evaluating $E_c \mathbf{v}_t^T \mathbf{q}_U$ gives the affine coefficients in equation (4).

For C, the concrete law is $\sigma_c = E_c \min(\mathbf{v}^T \mathbf{q}, 0)$. With zero external axial force, the two equilibrium equations are written compactly as

$$\mathbf{R}(\mathbf{q}) = \int_{A_g} \sigma_c \mathbf{v} dA - \sum_{j=s,p} A_j \sigma_c(y_j) \mathbf{v}_j + A_s \sigma_s \mathbf{v}_s + A_p \sigma_p \mathbf{v}_p - \begin{pmatrix} 0 \\ M/h \end{pmatrix} = \mathbf{0}. \quad (9)$$

Each section consists of constant-width rectangular bands. Integration is exact over the compressed portion of each band, using the primitives

$$E_c b \left(\varepsilon_0 y + \frac{\kappa y^2}{2} \right), \quad \frac{E_c b}{h} \left(\frac{\varepsilon_0 y^2}{2} + \frac{\kappa y^3}{3} \right). \quad (10)$$

The integration limits are the intersection of each band with the negative-strain region. A hybrid root solver resolves the two unknowns with a step tolerance of 10^{-10} . An equilibrium solution is accepted when $\|\mathbf{R}\|_\infty / (A_g f_{ck}) < 10^{-7}$. Analytical integration avoids fibre-discretization sensitivity for these geometries.

2.5 Loading protocol and comparison indicators

An initial filter imposes $\sigma_{p0} \leq 1,200$ MPa, excluding 108 configurations. The remaining 621 are initially uncracked. For each, 81 values of $\lambda_{r,U}$ are sampled from 0 to 4 in increments of 0.05. The reference follows U up to unity and C thereafter. The first cracked solution starts from the U solution at cracking; subsequent solutions use the preceding cracked state. This branch change is an explicit local idealization rather than a progressive cracking law.

The loading sequence stops at the first exceedance of 400 MPa in absolute passive-steel stress, 1,500 MPa in tendon stress, or $0.60 f_{ck}$ in maximum concrete compression. These limits define the explored domain, not a complete regulatory verification. Of 50,301 potential sampled states after initial filtering, 40,188 are retained. All 621 configurations reach $\lambda_{r,U} = 1.05$, while 174 reach 4.00. Statistics at higher loads therefore describe a progressively selected subset.

The post-cracking error indicators compare U with C:

$$\delta_\kappa = 100 \frac{|\kappa_U - \kappa_C|}{\max(|\kappa_C|, 10^{-12} \text{ mm}^{-1})}, \quad \delta_s = 100 \frac{|\sigma_{s,U} - \sigma_{s,C}|}{\max(|\sigma_{s,C}|, 1 \text{ MPa})}, \quad (11)$$

$$\Delta \sigma_p = \sigma_p - \sigma_{p0}, \quad \delta_p = 100 \frac{|\Delta \sigma_{p,U} - \Delta \sigma_{p,C}|}{\max(|\Delta \sigma_{p,C}|, 1 \text{ MPa})}. \quad (12)$$

The denominators prevent instability near zero. Tendon-stress increment is preferred to total stress because a high initial prestress can obscure a substantial error in the incremental response. The maximum of the three errors defines classes of at most 5%, above 5% to 10%, and above 10%. These are analytical accuracy criteria. Agreement before cracking is built into the common reference branch and is not an independent validation. For G–U curvature comparison, equation (11) is used with G in place of U and U as reference.

2.6 Verification of the implementation

A gross-section illustration uses a 300×600 mm rectangle, $P = 480$ kN and $e = 150$ mm. Its kern distance is 100 mm, giving $Pe = 72$ kN·m and $M_{\text{dec},g} = 120$ kN·m. At $M = 138$ kN·m, the residual moment is 18 kN·m and the bottom-fibre stress is 1.00 MPa. This example checks signs, units and equation (2).

An independent cracked-section calculation provides a stronger numerical verification. The central rectangular configuration has $e/h = 0.20$, initial precompression ratio 0.06, $\rho_s = 0.006$, $\rho_p = 0.004$ and $f_{ck} = 40$ MPa. Thus $P_0 = 432$ kN, $A_s = 1,080$ mm², $A_p = 720$ mm² and $\sigma_{p0} = 600$ MPa. When both steel layers lie below the neutral axis, axial equilibrium gives

$$\kappa = \frac{A_p c_p}{E_c b x^2 / 2 - E_s A_s (d_s - x) - E_p A_p (d_p - x)}, \quad (13)$$

where x is neutral-axis depth and d_s, d_p are steel depths measured from the top. Moment equilibrium about the top face then gives

$$M = -\frac{E_c b \kappa x^3}{6} + E_s A_s \kappa (d_s - x) d_s + A_p [E_p \kappa (d_p - x) + c_p] d_p. \quad (14)$$

Equations (13)–(14) are solved as one bracketed scalar equation using Brent's method, independently of the two-variable solver. At normalized loads 1.05, 1.50 and 2.00, neutral-axis depths are 307.571, 264.652 and 237.634 mm. The two calculations agree in curvature to better than 10^{-10} relative difference. The maximum normalized equilibrium residual across the campaign is below 6×10^{-14} . These checks establish consistency of the numerical implementation; experimental validation remains a subsequent step.

3 RESULTS AND DISCUSSION

3.1 Accurate preliminary identification of decompression

Across the 621 configurations, the signed difference $(M_{\text{dec},g} - M_{\text{dec},U})/M_{\text{dec},U}$ ranges from -2.80% to $+2.05\%$. Shape-specific ranges are -2.80% to $+1.89\%$ for R, -2.47% to $+2.05\%$ for T, and -2.26% to $+1.71\%$ for I. This close agreement supports the practical usefulness of the classical gross-section expression as an initial threshold estimate within the investigated domain.

The deformation comparison adds a valuable qualification. At $\lambda_{r,U} = 0.50$, gross-section curvature errors have a median of 3.19%, a 90th percentile of 6.55% and a maximum of 12.95%. Including steel rigidity, displaced concrete and the compatible tendon-force increment can therefore improve serviceability predictions before cracking. The results distinguish two useful design tasks: locating a threshold and predicting response to a specified accuracy. This extends the classical stress-based interpretation [1,2,6] with a quantitative assessment of model selection.

3.2 Coupled response of the central configuration

For the central rectangle, the gross and compatible decompression moments are 95.04 and 95.26 kN·m, respectively; compatible cracking occurs at 164.17 kN·m. Table 3 reports the reference response. At decompression, the passive reinforcement remains slightly compressed while the tendon has already gained stress relative to its initial state. Zero stress at the bottom concrete fibre therefore does not imply zero stress or zero increment in every constituent.

Table 3: Central rectangular configuration and uncracked curvature error.

$\lambda_{r,U}$	M in kN·m	κ in 10^{-3} m^{-1}	σ_s in MPa	σ_p in MPa	δ_κ in %
0.00	95.26	0.228	-2.74	605.15	0.00
1.00	164.17	0.570	13.08	612.58	0.00
1.05	167.61	1.018	47.31	635.47	42.30
1.50	198.62	1.529	84.20	659.48	51.53
2.00	233.08	2.128	128.71	688.85	57.15

The reference is uncracked through $\lambda_{r,U} = 1.00$ and locally cracked thereafter. Figure 2 shows how the model change affects curvature and steel response. At 2.00, the tendon-stress increment reaches 88.85 MPa; uncracked extrapolation underestimates this increment by 77.48% and passive-steel stress by 77.54%. A comparison using total tendon stress would conceal much of this incremental discrepancy. The calculation demonstrates why compatible tendon response remains relevant after decompression, consistent with established partial-prestressing analyses [9,11].

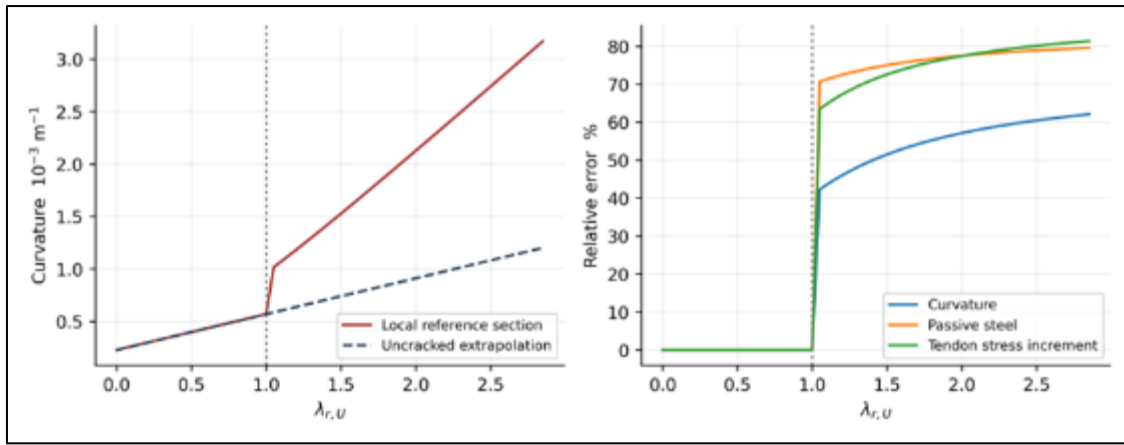


Figure 2: Central rectangular configuration. Reference curvature and uncracked extrapolation are compared with errors in curvature, passive-steel stress and tendon-stress increment. The abrupt change reflects the adopted branch switch, not a measured cracking trajectory.

3.3 Geometry and reinforcement effects after cracking

At the first sampled cracked state, $\lambda_{r,U} = 1.05$, uncracked extrapolation underestimates curvature by 21.32–85.92%. Median errors are 45.60%, 51.30% and 58.94% for R, T and I, respectively. These values reveal the effect of the selected geometries and reinforcement locations; they are not an intrinsic ranking of all rectangular, T-shaped and I-shaped members.

Table 4 and Figure 3 show the distribution of the three error indicators. Curvature and steel-stress discrepancies persist over the explored cracked range. The reduced sample size at higher loads reflects the stress-based stopping criteria and should be considered when interpreting trends.

Table 4: Median U-C errors and retained sample sizes.

$\lambda_{r,U}$	Cases	δ_κ in %	δ_s in %	δ_p in %	Curvature P10–P90 in %
1.05	621	51.69	76.11	74.88	33.05–74.35
1.50	618	58.80	78.27	80.33	42.52–78.48
2.00	593	62.37	79.42	82.88	48.11–78.93
3.00	376	63.10	78.39	83.14	53.18–74.59
4.00	174	63.88	77.17	82.18	54.80–70.71

P10 and P90 are descriptive percentiles of the retained calculations, not experimental confidence bounds.

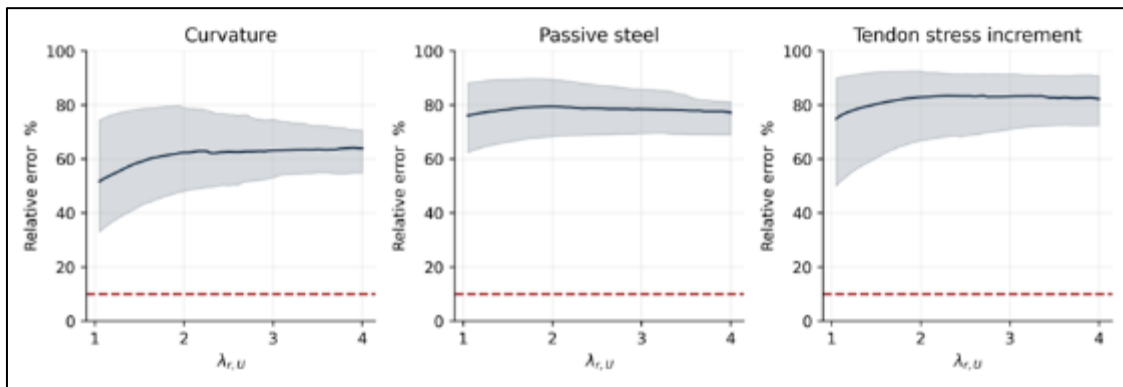


Figure 3: Median and 10th–90th percentile envelope of local post-cracking errors. The dashed horizontal line marks the 10% analysis criterion. The population retained at each load is governed by the stress limits.

Figure 4 groups the first cracked-state curvature errors by steel ratios. Greater reinforcement stiffness generally reduces the discrepancy between uncracked and cracked predictions. The amount and distribution of concrete removed from tensile equilibrium remain influential, explaining why the normalized curves do not collapse to one universal response. The normalization successfully aligns physical reference events while preserving meaningful differences between sections.

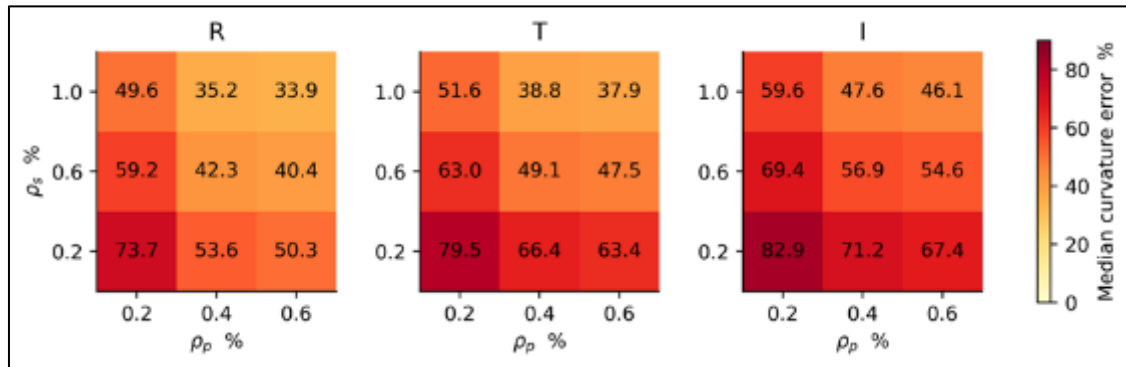


Figure 4: Median curvature errors at $\lambda_{r,U} = 1.05$ by section shape and reinforcement ratios. Cell values are percentages aggregated over admissible eccentricity, precompression and concrete-strength combinations.

All 621 configurations exceed the 10% maximum-error criterion at the first sampled cracked state; curvature error alone exceeds 10% in every case. This provides a clear model-selection result: for the local cracked idealization, compatible cracked analysis should replace uncracked extrapolation when post-cracking response is required. The sampled boundary lies between 1.00 and 1.05 because the reference changes branch in that interval. It is not a universal physical limit at 1.05. Resolving a continuous transition in average curvature requires tension stiffening and a corresponding member-level validation, as suggested by effective-inertia and progressive-cracking approaches [15–17].

3.4 Interpretation and practical design use

The residual moment measures the distance from a loading threshold. Material moment contributions have a different meaning: they are obtained by integrating constituent stresses about the same reference axis. After solving compatibility and equilibrium,

$$M = M_c + M_s + M_p, \quad \Delta M = \Delta M_c + \Delta M_s + \Delta M_p. \quad (15)$$

Each contribution depends on the common axial strain and curvature. Its value also depends on the chosen moment origin when constituent axial forces are nonzero. Consequently, $M - M_{dec}$ cannot be assigned a priori to passive reinforcement alone. A reinforced-concrete section under combined axial force and bending can be a useful representation if tendon action and its increment are treated consistently. Established cracked-section methods provide that compatibility [8–11].

For preliminary gross-section design without unaccounted secondary effects, useful rearrangements are

$$P_{req} = \frac{M}{e + k_t}, \quad e_{req} = \frac{M}{P} - k_t. \quad (16)$$

These relations assume a defined stage and a positive denominator. If a tensile stress $\sigma_{t,lim}$ is permitted while the section remains uncracked, the numerator of the first expression becomes $M - W_t \sigma_{t,lim}$. The opposite fibre, construction stages and tendon geometry must also be checked.

The findings support a practical sequence. First, obtain global resultants and effective prestress for the selected stage. Next, use gross-section stresses to locate decompression and assess proximity to cracking. Adopt a transformed uncracked section when steel ratios or the required deformation accuracy warrant it. Once cracked response governs, solve concrete and both steel systems together and select a deformation model appropriate to the scale of verification. Transfer stresses, code decompression, crack width, deflection, fatigue, shear, torsion, anchorage and ultimate resistance remain part of the complete design process [6,22–25].

3.5 Scope and opportunities for further development

The benchmark isolates the effects of section geometry, eccentricity, prestress and reinforcement through a fully specified deterministic calculation. Its value is the transparent connection between classical thresholds and measurable modelling errors. It covers three geometries at one depth, one equivalent passive layer and one equivalent tendon. Filtered and truncated factorial results are descriptive of this corpus, rather than statistical estimates for a population of structures.

Compressed concrete remains linear up to the adopted ceiling, which can introduce constitutive error near that ceiling. The model does not calculate tensile softening, crack spacing or width, long-term effects, cyclic response or size effects. Removing tensile concrete is appropriate for the selected local comparison, but member deflection requires the contribution of uncracked regions and concrete between cracks. Experimental bending data with steel-strain measurements would provide a productive next validation step. Extensions to unbonded tendons require member-level tendon deformation [20], while deterioration requires additional bond and section-loss variables [21]. These developments can build directly on the present initialization, normalization and equilibrium framework.

4 CONCLUSION

Classical kern theory provides a sound basis for interpreting decompression and organizing the response of prestressed concrete sections. The residual moment combines uniform precompression and eccentricity in an exact gross-elastic stress identity, while the normalized coordinate places decompression and theoretical cracking on a common scale.

The campaign of 621 configurations demonstrates the usefulness of gross-section decompression for preliminary assessment: deviations from compatible elastic values remain between -2.80% and $+2.05\%$ in the studied domain. Curvature requires a more discriminating choice of model; at normalized load 0.50, gross-section errors reach 12.95% . After the imposed transition to a local cracked section, the first sampled curvature errors range from 21.32% to 85.92% , clearly identifying the benefit of compatible cracked analysis.

The principal contribution is a reproducible framework that separates threshold estimation, uncracked deformation prediction and local cracked response. It supports efficient preliminary design while retaining the coupled participation of concrete, passive reinforcement and tendons. Incorporating tension stiffening and validating against member tests are the next steps toward applying the normalized framework to average deformation and crack control.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The authors declare no conflict of interest.

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