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EFFECT OF THE RAYLEIGH NUMBER ON THE MEAN NUSSELT NUMBER AND THE MEAN TEMPERATURE IN A NATURAL CONVECTION FLOW WITH AN ELLIPTICAL-CYLINDRICAL GEOMETRY

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ABSTRACT

This study presents the results of a numerical simulation of fluid flow within a chamber bounded by a horizontal, flat wall heated at a constant flow rate and an upper wall with an elliptical outline. The results of this simulation enable the implementation of a cooling system utilising natural convection. The key parameters in this study are dimensionless numbers such as the Rayleigh number and the Nusselt number, which characterise the magnitude of heat transfer by convection. To determine the thermal parameters of the problem, we shall use the heat equation and the equation of motion in our modelling. To obtain results such as the average temperature characterising heat transfer between the fluid and the wall, and the Nusselt number indicating the efficiency of convective cooling, we will solve the non-linear equations using the finite volume method and the Gauss-Siedal iterative method.

Keywords: Convection; Cooling; Nusselt Number; Rayleigh Number; Average Temperature; Convection; Finite Volume

1 INTRODUCTION

The study of natural convection within enclosed chambers or complex annular configurations is a major pillar of contemporary research in fluid mechanics and heat transfer. This mode of heat transfer, governed by buoyancy forces induced by density gradients in the presence of a gravitational field, lies at the heart of numerous cutting-edge industrial applications. These include the cooling of the new generation of miniaturised electronic components, the optimisation of compact heat exchangers, thermal energy storage systems using phase-change materials, and the passive safety of nuclear reactors. Within this vast field of research, the elliptical-cylindrical geometry—defined by the annular space confined between two elliptical cylinders that are either co-focal or eccentric—is attracting growing interest within the scientific community. Unlike conventional circular cylindrical configurations, the use of elliptical cross-sections offers crucial geometric degrees of freedom, such as the aspect ratio (eccentricity) and the angle of inclination of the walls. Recent studies [1, 2] confirm that modulating these geometric parameters allows for the passive control and optimisation of flow structures to maximise the overall heat transfer rate or, conversely, to minimise energy losses.. Wang L and Bejan A [3]. The hydrodynamic and thermal behaviour of these systems is fundamentally governed by the Rayleigh number, which quantifies the ratio between buoyancy forces and viscous and thermal dissipation forces. The overall efficiency of the convective process is, for its part, assessed by the average Nusselt number Num, which represents [4] the amplification of heat transfer relative to the pure conduction regime. As the Rayleigh number increases (generally beyond a critical threshold Rac), the flow transitions from a pure conductive regime to complex cellular structures (convective rolls) [5]. This change in regime profoundly alters the spatial distribution of the mean temperature within the fluid, creating phenomena of intense thermal stratification or localised zones of superheating at the interfaces. Although the academic literature offers numerous empirical correlations linking the Nu number and the Ra number for simple rectangular or circular cavities, the combined analysis of the effect of Ra on the mean Nusselt number and the mean temperature within a strictly ellipsoidal-cylindrical space still raises questions. Thermoconvective instabilities are particularly sensitive to wall eccentricity in such cases, rendering conventional models obsolete. Furthermore, research in recent years has increasingly focused on the use of complex fluids or hybrid nanofluids, making it essential to first establish a rigorous numerical database using standard Newtonian fluids such as air or water. The aim of this article is to make a theoretical and numerical contribution to the precise characterisation of the influence of the Rayleigh number on the mean Nusselt number and the mean temperature within a natural convection flow in an elliptical-cylindrical geometry. Using a two-dimensional numerical approach based on an elliptical coordinate formulation, we shall explore a wide range of Rayleigh numbers in order to establish new laws governing thermal behaviour. To address this gap, this study proposes a two-dimensional, steady-state numerical simulation of natural convection within an elliptical-cylindrical chamber. The main objective is to quantitatively characterise the behaviour of the mean Nusselt number and the mean fluid temperature over a wide range of Rayleigh numbers ($10^3 \leq Ra \leq 10^6$). Through this analysis, we will highlight the transitions between flow regimes and propose empirical correlations linking Num to Ra, tailored to the specific characteristics of the elliptical geometry under study [6]. At low Ra values, the flow regime is purely conductive and the mean Nusselt number tends towards a stable minimum value. As soon as the Rayleigh number crosses a critical threshold, thermal equilibrium is disrupted, triggering the formation of organised convection cells. A continuous increase in Ra intensifies the fluid mixing rate, which thins the thermal boundary layers along the walls of the elliptical cylinder and significantly increases the mean Nusselt number, often characterised by a power law of the form Nu and Ra^n. **[7]**

2 MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Description of the physical system and phenomena

We propose to study heat transfer within a fluid in a closed, impermeable and non-catalytic chamber in order to simulate the cooling of a heated plate at constant flow rate. This chamber is bounded by a half-cylinder and a horizontal flat plate of length (2.L), aligned with the major axis of the cylinder, and depth (2.l). A section of this plate, of length (2.l0), is heated at a constant heat flux. (Figure 1).

Initially, the chamber is filled with a fluid of density ρ0 in thermodynamic equilibrium at temperature T0 and thermally isolated from the external environment. The curvilinear boundary of the chamber is maintained at temperature T0

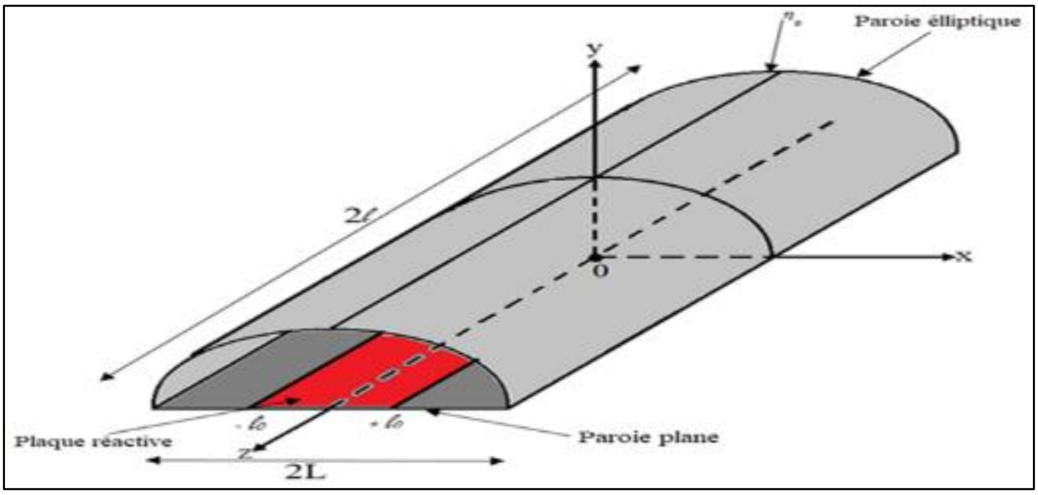


Figure 1: A schematic representation of the physical domain

From a certain point onwards, the heat flux brings the wall of thickness 2lo to Tp > T0. If the difference between the temperature of the fluid and that of the heated flat plate exceeds a certain threshold, the fluid then begins to move under the combined action of the non-uniformity in density caused by the temperature and mass fraction gradients within the region (D). In other words, this motion occurs if the driving force (buoyancy or Archimedes' thrust) is sufficient to overcome the stabilising effects, namely viscous dissipation, thermal conductivity and the diffusion mass flow. The light fluid near the hot wall will rise and then flow back down along the cold elliptical wall. We are therefore dealing with natural convection.

Based on this heat transfer by convection, we will study the fluid's movement through the chamber walls in order to simulate the cooling of the heating plate caused by the fluid's movement. The walls in contact with the moving fluid are the heated plate of thickness (2.l0) (interpole section); the extrapole plate (the unheated part of the plate); and the curvilinear section. This simulation will be carried out by taking into account the Nusselt number and the average temperature.

2.2 Dimensionless equations

Although it is possible to use the modelling equations in a Cartesian coordinate system (x, y), it is more appropriate to find a coordinate system in which the boundaries of our domain are lines of constant coordinates. We have chosen the elliptical coordinates (η,θ), which naturally seem best suited to the geometry in question. Thus, our curvilinear enclosure is transformed into a rectangular domain bounded by the coordinate lines

$\eta = 0 \ et \ \eta = \eta_p \ ; \ \theta = 0 \ et \ \theta = \pi \ ; \ z = -l \ et \ z = +l$

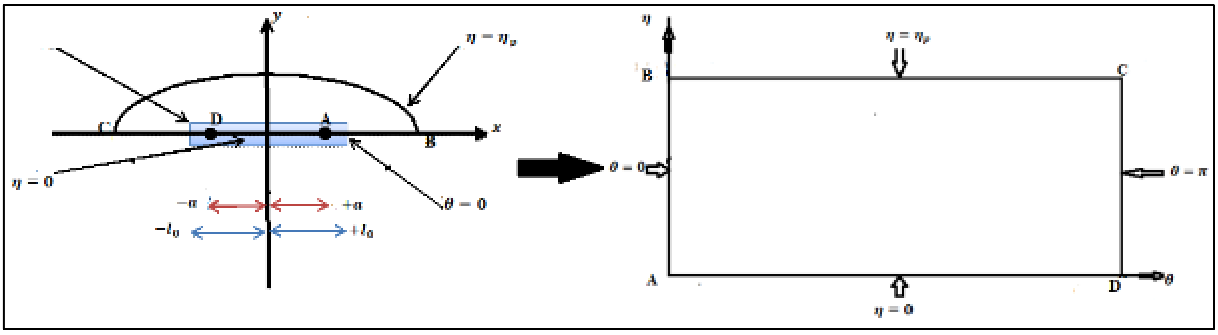


Figure 2: Physical domain in the elliptical-cylindrical coordinate system.

The dimensionless equations for modelling the problem will be written in terms of dimensionless parameters and curvilinear coordinates (η, θ)

2.2.1 Equation for the current function

$$\vec{\nabla}^*.\left(-\vec{\nabla}^*\psi^*\right) = \omega^* \qquad (1)$$

2.2.2 Equation of motion

$$\frac{\partial \omega^*}{\partial t^*} + \vec{\nabla}^*.\left(\vec{v}.\omega^* - Pr.\vec{\nabla}^*\omega^*\right) = \frac{1}{h^{*2}}\left\{Bo_T.\left(\frac{\partial T^*}{\partial \eta}.sh\eta.cos\theta - \frac{\partial T^*}{\partial \theta}.ch\eta.sin\theta\right) + Bo_m.\left(\frac{\partial C}{\partial \eta}.sh\eta.cos\theta - \frac{\partial C}{\partial \theta}.ch\eta.sin\theta\right)\right\}$$
(2)

Solving the current and motion equations [8] enables us to determine the isotherms, the average temperature and the Nusselt number via the heat equation

2.2.3 Heat Equation

$$\frac{\partial T^*}{\partial t^*} + \vec{\nabla}^*.\left(\vec{v}.T^* - \vec{\nabla}^*T^*\right) = 0 \qquad (3)$$

We complete this system with the following thermal initial and boundary conditions

2.2.4 Initial conditions

The system is in thermodynamic equilibrium, so we have

$$T^*(\eta, \theta\,) = 0 \qquad (4)$$

On the elliptical wall $\eta = \eta_p \quad ; \forall\theta \in \ ]0\ ;\ \pi[$

$$T^*\left(\eta_p, \theta\,; t^*\right) = 0 \quad ou \ \frac{\partial T}{\partial \eta} = 0 \qquad (5)$$

On the intra-pole section $\eta = 0 \quad ; \forall\theta \in \ ]0\ ;\ \pi[$

$$T^*(0,\theta\,;t^*) = 1 \quad (6)$$

On the extrapolated section $\theta = 0\,;\ \forall\eta \in \left]0\,;\eta_p\right[$

$$T^*(\eta,0;t^*) = 1 \quad ou \quad \frac{\partial T^*}{\partial\theta} = 0 \quad (7)$$

On the extrapolated section $\theta = \pi\ ;\ \forall\eta \in \left]-\eta_p\,;0\right[$

$$T^*(\eta,\pi\,;t^*) = 1 \quad ou \quad \frac{\partial T^*}{\partial\theta} = 0 \quad (8)$$

2.3 Nusselt Number

To estimate the intensity of heat transfer at the walls and determine the convective heat transfer coefficients, the Nusselt number is used; this is defined as the ratio of the convective heat flux qc to a reference conductive heat flux qref. The instantaneous local Nusselt number is therefore

$$Nu = \frac{q_c}{q_{réf}} \quad (9)$$

Where

$$q_c = -\lambda.\vec{n}_{\mathrm{p}}.\vec{\nabla}T \quad (10)$$

Where $\vec{n}_{\mathrm{p}}$ the unit normal vector to the wall. When the temperature is specified

$$q_{réf} = \frac{\lambda.(T_p - T_{rf})}{L_r} \quad (11)$$

T_{rf} is a characteristic temperature of the fluid away from the wall. Thus, we have

$$Nu = -\frac{\vec{n}_{\mathrm{p}}.\vec{\nabla}T}{T_p - T_{rf}}.L_r \quad (12)$$

We take it to be equal to the instantaneous average temperature of the fluid, given by

$$T_{rf} = \frac{1}{S}\int_{\theta=0}^{\theta=\pi}\int_{\eta=0}^{\eta=\eta_p} T(\eta,\theta\,;\,t\,).h^2.d\eta.d\theta \quad (13)$$

We take it to be equal to the instantaneous average temperature of the fluid given by

$$S = \int_{\theta=0}^{\theta=\pi}\int_{\eta=0}^{\eta=\eta_p} h^2.d\eta.d\theta \quad (14)$$

On the intra pole section $\eta = 0$

$$Nu = -\frac{1}{1-T_{rf}^*}.\frac{1}{h^*}.\frac{\partial T}{\partial\eta} \quad (15)$$

On the extra pole section $\theta = 0 \quad ou\ \theta = \pi$

$$Nu = -\frac{1}{1-T_{rf}^*}.\frac{1}{h^*}.\frac{\partial T}{\partial\theta} \quad (16)$$

On the elliptical section $\eta = \eta_p$

$$Nu = \frac{1}{T_{rf}^*}.\frac{1}{h^*}.\frac{\partial T}{\partial\eta} \quad (17)$$

3 RESULTS AND DISCUSSION

The results of the numerical simulation of natural convection in a binary fluid are presented for different Rayleigh numbers (Ra). The parameters studied include:

- Nusselt number (Nu): this characterises heat transfer by convection between the fluid and the walls of the system
- Average temperatures: these are used to characterise the overall thermal state of the system.

These results were obtained using FORTRAN and PYTHON software. They show how the average temperature and the Nusselt number vary as a function of different Rayleigh numbers: Ra = 50,000; Ra = 100,000 and Ra = 1,000,000

Le nombre de Nusselt moyen

The main observation is that an increase in the Rayleigh number (Ra) increases the average Nusselt number, indicating an intensification of convective heat transfer as seen in figure 3

Ra = 50 000 (Figures a and c) : The initial value of **Numoyen** is high and then falls rapidly before stabilising at around 20–30 for the 'curvilinear' curve (red). Slight oscillations are observed, suggesting a stable or slightly unstable laminar flow regime. The behaviour is very similar to that shown in graph 'a'. The curves stabilise at similar values. At this Ra level, convection is present but relatively moderate, leading to moderate and stable heat transfer over time.

Ra = 100 000 (Figure B) : The initial **Numoyen** is higher than for Ra=50,000. It falls rapidly and then stabilises sooner than in cases 'a' or 'c'. The equilibrium value is also higher, at around 40–50 for the red curve. The intensity of convection is greater than at Ra = 50,000, allowing for more efficient heat transfer, which reaches a steady state more quickly.

Ra = 1 000 000 (Figure d) ; This case exhibits the most dynamic behaviour. The initial peak of **Numoyen** is the highest of all the cases. The system stabilises very quickly at a high value, around 40–50 for the red curve as well, perhaps slightly higher than for Ra = 100,000. At such a high Ra, buoyancy forces dominate, leading to vigorous and potentially turbulent flow. Heat transfer is maximised and reaches its maximum efficiency very quickly.

Table 1: Determination of the average equilibrium Nusselt number and convection intensity as a function of the Rayleigh number

Ra	Initial Peak Average	Average Unweighted Equilibrium Value	Speed of establishment	Convection Intensity
50000	Moderate	20-30	slow	Moderate
10000	High	40-50	fast	High
100000	Very high	40-50	Very fast	Very high

In conclusion, the higher the Rayleigh number, the more efficient the heat transfer (increase in the Nusselt number) and the more quickly the system reaches its steady state.

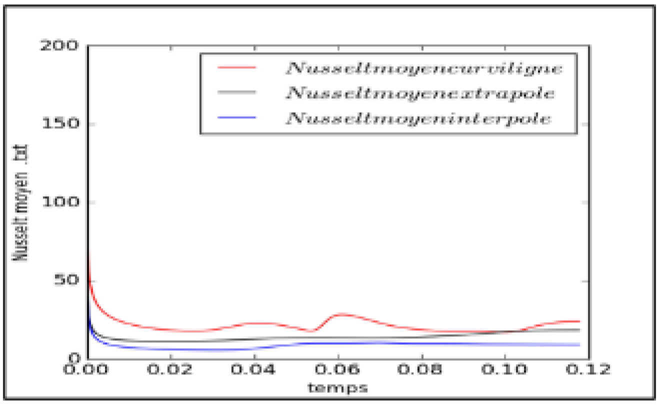
Average temperature

The graphs in figure 4 illustrate how the intensity of natural convection, determined by the Rayleigh number, affects the rate at which the system reaches thermal equilibrium and the average temperature at that equilibrium.

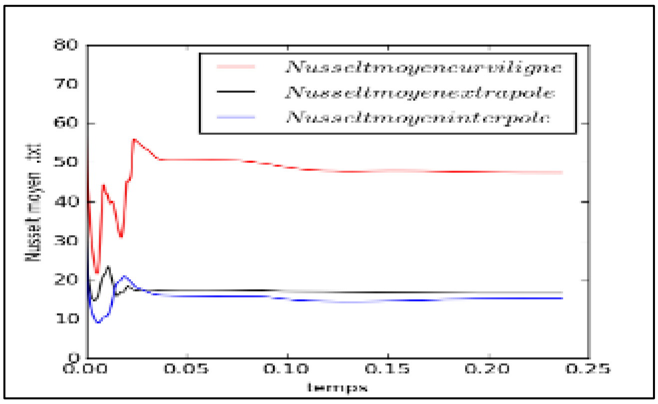
Ra = 50 000 (Figures a et c) : These curves show a gradual and slow rise in temperature compared with the other cases. The system takes the longest to stabilise. The curve showed a stepwise evolution, suggesting changes in the flow structure (formation or rearrangement of convection cells) over time. Curve c reaches a plateau more quickly than curve 'a', at around t = 0.03 s, but with a very similar equilibrium value (approximately 0.345). At this low Ra, buoyancy forces are relatively weak, the flow is less vigorous, and heat transfer is less efficient. The system is probably in laminar flow.

Ra = 100 000 (Figure b) : The temperature rise is significantly faster than at Ra = 50,000. The system reaches a quasi-steady state at around t = 0.04 s. The average equilibrium temperature is very similar to that at Ra = 50,000 (approximately 0.335–0.34), but it is reached earlier. The increase in Ra intensifies the convective flow, accelerating heat diffusion within the domain.

Ra = 1 000 000 (Figure d) : This case shows the fastest rise in average temperature. Equilibrium is practically 6reached at t ≈ 0.02 s. Slight oscillations or steps are observed during the stabilisation phase, which could indicate a transition to a more complex, potentially turbulent regime, or the rapid establishment of complex flow structures. A very high Ra indicates dominant buoyancy forces and a highly dynamic flow that is efficient at heat transfer, leading to rapid thermalisation of the system.



a



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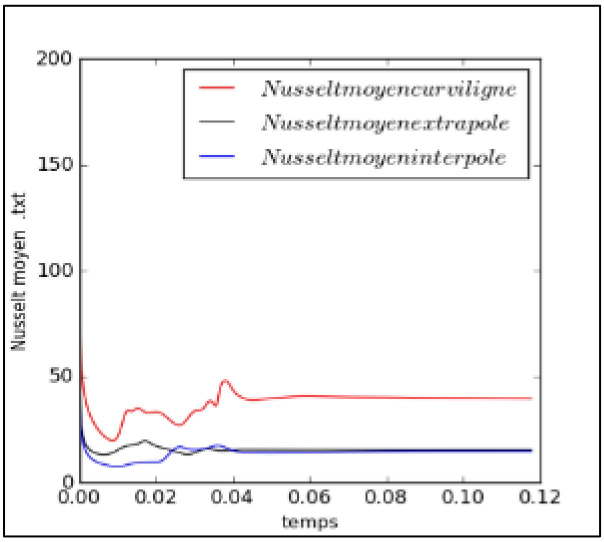
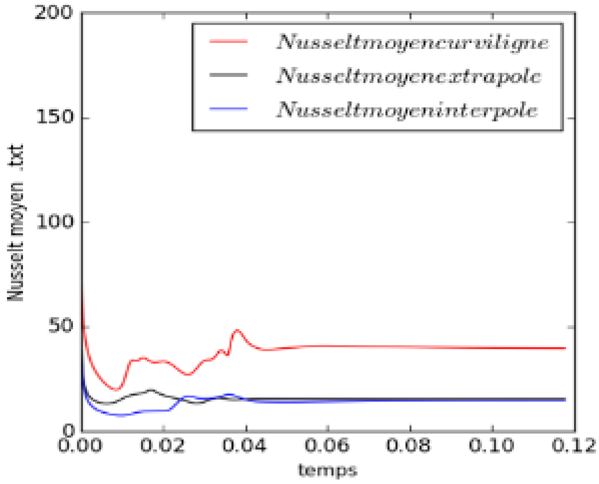


c



d

Figure 3: Nusselt number as a function of time for different values of the Rayleigh number

Table 2: Determination of the equilibrium temperature and convection intensity as a function of the Rayleigh number

Ra	Speed of establishment	Equilibrium temperature	Probable flow regime
50000 (a,b)	slow	0,34	Laminar
100000 (c)	average	0,34	Laminar/transition
1000000 (d)	Very fast	0,34	Turbulent naissant/transition

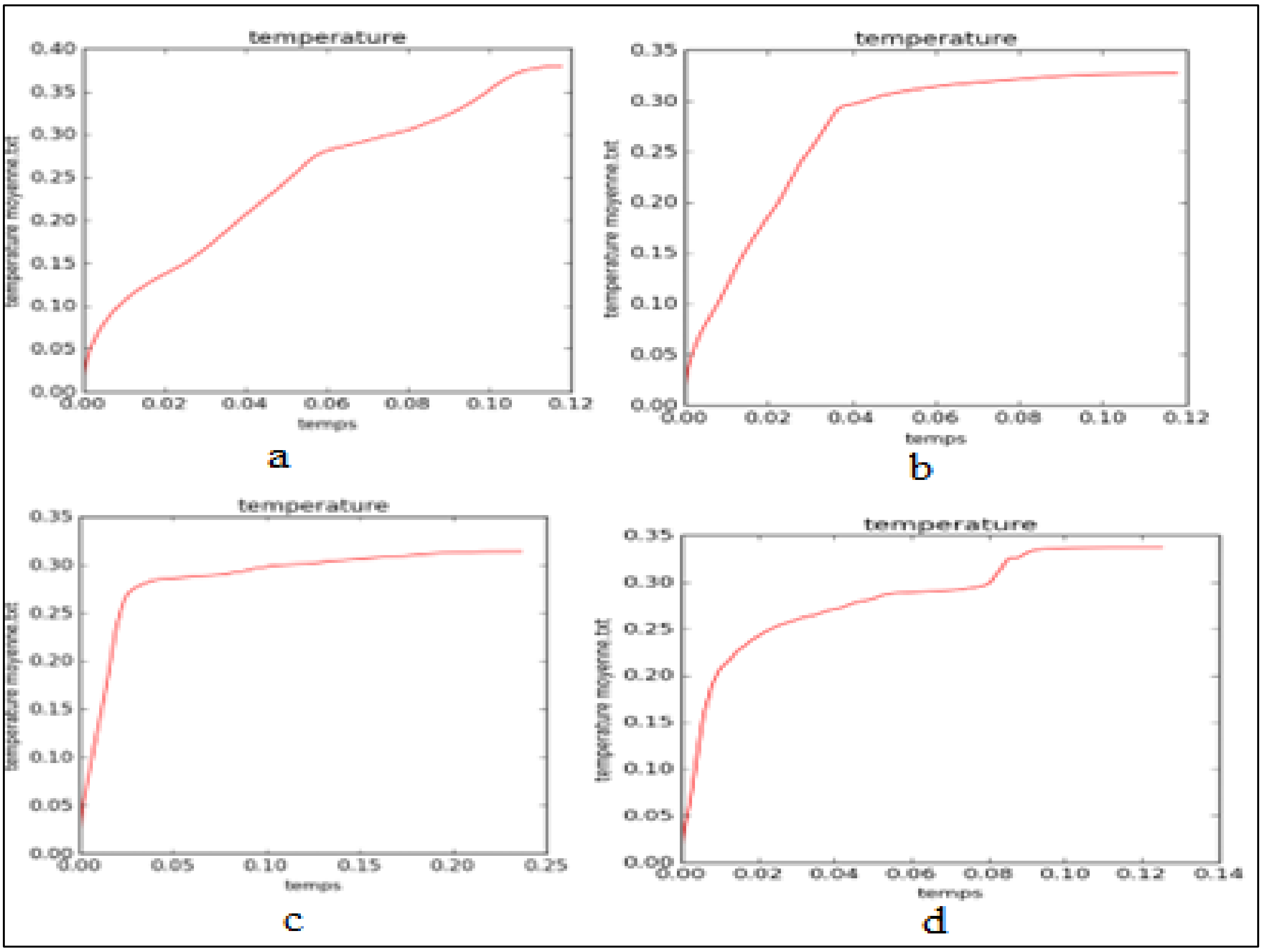


Figure 4: Average temperature as a function of time for different values of the Rayleigh number

The main conclusion is that an increase in the Rayleigh number significantly accelerates the dynamics of heat transfer, but has only a very slight influence on the final value of the average equilibrium temperature within the range studied.

4 CONCLUSION

The results presented in this study examine heat transfer by convection with a view to implementing cooling in constant-flow heating systems. They demonstrate the relevance of using the elliptical-cylindrical curvature model and of optimising the thermal boundary layers to achieve better heat transfer efficiency.

In our study, we used the Navier–Stokes equations coupled with the heat equation to determine the evolution of the flow's hydrodynamic and thermal parameters. This enabled us to deduce the mean temperature and the Nusselt number, which are important factors in convection. The results obtained by solving the non-linear equations using the finite volume method enabled us to determine the evolution of the mean temperature and the Nusselt number for different Rayleigh numbers: Ra = 50,000; Ra = 100,000 and Ra = 1,000,000

Indeed, for Ra = 1,000,000, we observe a rapid establishment of the dimensionless mean temperature at a value of 0.34, below which heat transfer occurs, and of the mean Nusselt number, with a maximum value lying between 40 and 50.

Finally, greater cooling efficiency is observed for high Rayleigh numbers, where flow structures establish themselves rapidly, promoting better convection.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

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