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OPTIMAL CONTROL ANALYSIS OF AN AGE-STRUCTURED MODEL OF HIV/AIDS TRANSMISSION DYNAMICS

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ABSTRACT

Optimal control analysis is a mathematical approach used to determine the best combination, timing, and intensity of interventions given to curb the spread of a disease over time, while considering both the disease transmission and the costs of interventions. In this study, an age-structured mathematical model for HIV/AIDS transmission dynamics in Nigeria has been formulated and analyzed. The model incorporates two distinct age classes: immature susceptibles (children under 15 years) and mature susceptibles (adults aged 15-64 years), giving rise to a system of twelve ordinary differential equations. The basic reproduction number R_0 was derived using the next-generation matrix method and computed as $R_0>1$, indicating persistent disease transmission without interventions. Optimal control characterization through Pontryagin's maximum principle and numerical simulations over a 20-year horizon demonstrated that aggressive combined controls reduce the effective reproduction number to 0.41 and decrease peak infections by over 93%. Cost-effectiveness analysis revealed that adult prevention contributes approximately 32% of total infection reduction. The findings provide evidence-based guidance for public health policy, recommending prioritized allocation of resources toward adult prevention interventions and the implementation of combination strategies integrating prevention, screening, and treatment. This research contributes to the advancement of age-structured epidemiological modeling and supports the global initiative to end the AIDS epidemic by 2030.

Keywords: HIV/AIDS, Age structured, Optimal control; Transmission dynamics

1 INTRODUCTION

The WHO HIV/AIDS Guidelines define HIV as a virus that targets white blood cells and weakens a person´s immunity. According to the United Nations programme on AIDS **(**UNAIDS), HIV is a retrovirus that infects the cellular immune system (CD4-positive T-cells and macrophages) [15]. Centers for Disease Control and Prevention CDC [16] describes HIV as a specific viral pathogen that attacks the body's white blood cells. The Human Immunodeficiency Virus (HIV) and Acquired Immune Deficiency Syndrome (AIDS) continue to pose a significant global health challenge. HIV is a chronic viral infection where the virus attacks CD4 white blood cells, progressively destroying the immune system. AIDS on the other hand is described as the collection of symptoms and infections associated with acquired deficiency immune system [17]. Infections with HIV has been established as the underlying cause of AIDS. The level of immunodeficiency or the appearance of certain infections are used as indicators that HIV infection has progressed to AIDS**.** If not treated HIV can advance to Acquired Immune deficiency Syndrome (AIDS), leaving the body defenseless against severe, life-threatening opportunistic infections and cancers. The virus targets CD4+ T-lymphocytes, inserting its genetic material and hijacking cellular machinery to replicate, which kills the host cell. The first stage of HIV/AIDS infection is the acute stage [1] which occurs after weeks of exposure. At this stage, infected Individuals may experience flu-like symptoms (fever, rash, swollen lymph nodes) while viral load peaks. The second stage is the chronic/latent stage. Individuals at this stage remain active at lower levels of the virus with T-cell counts over years gradually depleting without causing obvious symptoms. The last stage is the AIDS stage which is diagnosed when CD4 counts drop below 200 cells/mm^3 (normal is 500–1,600) or when specific opportunistic illnesses manifest, such as Pneumocystis pneumonia and Kaposi's sarcoma [14] . HIV/AIDS can be transmitted specifically through bodily fluids—blood, semen, vaginal and rectal secretions, breast milk and primarily through unprotected sex or shared injection equipment. The virus cannot be transmitted through casual contact, kissing, hugging, or sharing food. It can be prevented through the use of barrier methods (condoms), Pre-exposure prophylaxis (PrEP), and Post-exposure prophylaxis (PEP). Daily combination medications eg. Antiretroviral Therapy (ART) suppress viral replication, preserving immune function [4].

Optimal control analysis of HIV/AIDS combines mathematical modeling with public health strategies to minimize new infections, reduce disease progression, and lower costs using interventions like antiretroviral therapy (ART), screening, PrEP and PEP (Edward et al [5] . Mathematical modeling has become an indispensable tool for understanding disease transmission dynamics and evaluating intervention strategies. Age-structured models are particularly important for HIV/AIDS, as transmission rates, risk behaviors, and the effectiveness of interventions vary substantially across age groups. While previous models have provided valuable insights, they often do not explicitly incorporate the distinct roles of children and adults in the transmission dynamics [3].

Linlin et al [7] studied and formulated a work on "Dynamical analysis and optimal control for an age-structured HIV transmission model". They investigated the transmission dynamics and optimal control for HIV/AIDS mathematical model with age-structure. Local asymptotic stability of disease-free equilibrium was proven by applying operator theory and spectral boundary principle. The existence of endemic equilibrium was obtained by using fixed-point theorem and monotone iterative procedure. The optimal control of the model was conducted using prevention, education and anti-HIV treatment as control strategies. Numerical simulations with appropriate parameters were conducted to show that comprehensive publicity and education on prevention and control for susceptible individuals and prompt treatment of HIV-infected individuals can help to reduce HIV impact.

A HIV/AIDS study carried out by Agha et al. in [20] estimated parameters and performed a sensitivity analysis for an age-structured HIV/AIDS transmission model in Nigeria. The model considered two age groups: individuals aged 15–60 years and those under 15. Results showed that the population of unaware infectives consistently exceeded that of aware infectives, indicating a significant hidden reservoir that sustains ongoing transmission. This finding highlights the critical need to reduce transmission rates and enhance case detection through targeted awareness campaigns. Urgent prevention and control measures are therefore essential to curb the spread of the infection.

Shewafera et al [10] carried out a work titled "Analyses of an age-structured HIV/AIDS compart-mental model with optimal control theory". They observed that HIV/AIDS is among the major viral infectious diseases that are a major threat with high human basic reproduction number (R_0). Specifically, they demonstrated that when the basic reproduction number is less than 1, a globally stable disease-free steady state exists; this implies that the disease will reduce. In scenarios where disease-induced death is absent, the endemic steady state becomes the sole existing steady state and exhibits local asymptotic stability for the basic reproduction number is more than 1, thereby perpetuating the disease. To further enhance the applicability of their model, they incorporated control functions such as physical distancing, educational campaigns, and treatment and then formulated an optimal control problem and rigorously prove the existence and uniqueness of optimal control solution. Their approach provided a robust theoretical foundation for designing effective intervention strategies. Numerical simulations conducted to illustrate the core findings of their study indicated the critical role of age variability in the dissemination dynamics of HIV/AIDS. Notably, their results suggested that targeting educational interventions to achieve self-protection awareness toward younger populations yields

significantly greater effectiveness compared to implementing uniform strategies across all age groups. Their insight underscored the importance of age-specific approaches in optimizing resource allocation and maximizing public health impact.

This paper extends existing frameworks of Lin-Xue et al in [18] by explicitly incorporating two distinct age classes: children under 15 years (immature susceptibles) and adults aged 15-64 years (mature susceptibles). The objectives of this research are to: (1) formulate an age –structured mathematical model incorporating two age classes; (2) analyze the model´s behaviour, including equilibrium points and the basic reproduction number; (3) examine the impact of age-specific factors on HIV transmission; (4) evaluate the effectiveness of different control strategies; (5) determine optimal control strategies to mitigate the spread of infection.

2 THE MODEL

2.1 Model Formulation

The total population is partitioned into two age groups: Children, C (aged 0-14 years) and Adults, A (aged 15-64 years). Each group is further divided into six epidemiological compartments; susceptibles (S), exposed (E), aware infected (I_1), unaware infected (I_2), treated (T) and AIDS (A). The forces of infection for children and adults, respectively are given by:

$$\lambda_C(t)=\frac{\beta_C(I_{1C}+I_{2C}+\varepsilon EC)}{N_C}, \quad (1)$$

$$\lambda_A(t)=\frac{\beta_A(I_{1A}+I_{2A}+\varepsilon EA)}{N_A}, \quad (2)$$

where N_C=S_C+E_C+I_{1C}+1_{2C}+T_C+A_C and N_A=S_A+E_A+I_{1A}+1_{2A}+T_A+A_A.

The parameter β is the effective contact rate and ε accounts for the reduced infectivity of the latent individuals.

The schematic diagram (figure 1) of the age-structured HIV/AIDS model is as shown and governed by the following system of ordinary differentiate equations:

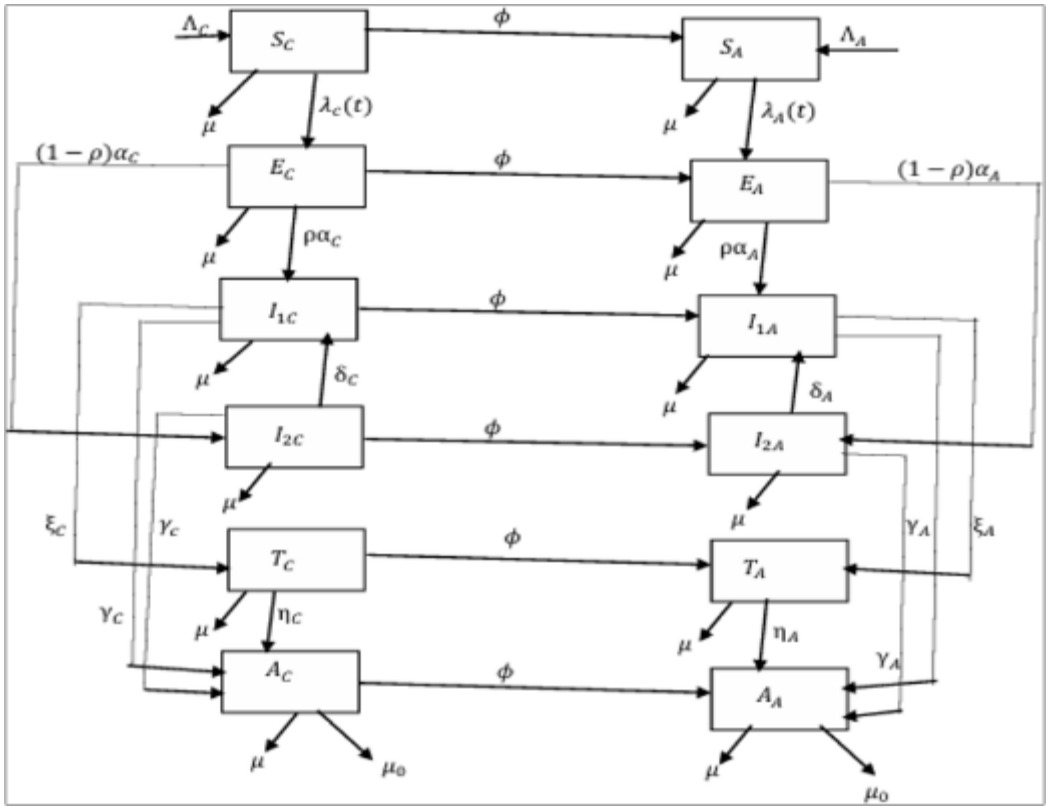

Figure 1: Schematic Diagram of HIV/AIDS dynamics without control

Equations of the model without control

$$
\begin{aligned}
\frac{dS_C}{dt} &= \Lambda_C - \phi S_C - \lambda_C S_C - \mu S_C \\
\frac{dS_A}{dt} &= \Lambda_A + \phi S_C - \lambda_A S_A - \mu S_A \\
\frac{dE_C}{dt} &= \lambda_C S_C - (\alpha_C + \phi + \mu) E_C \\
\frac{dE_A}{dt} &= \lambda_C S_A + \phi E_C - (\alpha_A + \mu) E_A \\
\frac{dI_{1C}}{dt} &= \rho \alpha_C E_C + \delta_C I_{2C} - (\phi + \xi_C + \gamma_C + \mu) I_{1C} \\
\frac{dI_{1A}}{dt} &= \rho \alpha_A E_A + \phi I_{1C} + \delta_A I_{2A} - (\xi_A + \gamma_A + \mu) I_{1A} \\
\frac{dI_{2C}}{dt} &= (1-\rho)\alpha_C E_C - (\delta_C + \phi + \gamma_C + \mu) I_{2C} \\
\frac{dI_{2A}}{dt} &= \phi I_{2C} + (1-\rho)\alpha_A E_A - (\delta_C + \gamma_A + \mu) I_{2A} \\
\frac{dT_C}{dt} &= \xi_C I_{1C} \Lambda_C - (\phi + \eta_C + \mu) T_C \\
\frac{dT_A}{dt} &= \xi_A I_{1A} + \phi T_C - (\eta_A + \mu) T_A \\
\frac{dA_C}{dt} &= \gamma_C (I_{1C} + I_{2C}) + \eta_C T_C - (\phi + \mu) A_C \\
\frac{dA_A}{dt} &= \gamma_A (I_{1A} + I_{2A}) + \eta_A T_A + (\phi A_C - (\mu + \mu_0) A_A
\end{aligned}
\qquad (3)
$$

All parameters are positive and their descriptions are provided in Table 1. Recruitment into S_C occurs only by birth at rate Λ_C, while recruitment into S_A occurs only by immigration at rate Λ_A. Children mature into the adults' class at rate ϕ.

Table 1: Description of State Variables and Parameters for the age –structured HIV/AIDS model

Variable/Parameter	Description
S_C, S_A	Susceptible children and adults
E_C, E_A	Exposed children and adults
I_{1C}, I_{1A}	Aware infected children and adults
I_{2C}, I_{2A}	Unaware infected children and adults
T_C, T_A	Treated children and adults
A_C, A_A	AIDS children and adults
Λ_C, Λ_A	Recruitment rates of children and adults
β_C, β_A	Effective contact rates for HIV transmission
Φ	Rate at which children mature to adults
Ω_C	Rate at which exposed children test negative and return to SC
Ω_A	Rate at which exposed adults test negative and return to SA
α_C, α_A	Progression rates from latent to infected
P	Fraction progressing to aware class (I_1)
$1 - \rho$	Fraction progressing to unaware class (I_2)
E	Reduced infectivity of latent individuals
δ_C, δ_A	Rates at which unaware infected become aware
ξ_C, ξ_A	Rates at which aware infected receive treatment

γ_C, γ_A	Progression rates to AIDS
η_C, η_A	Treatment failure rates
μ	Natural death rate
μ_0	AIDS-related mortality rate

2.2 Basic Reproduction Number (R_0)

The basic reproduction number, R_0 is computed using the next-generation matrix method [13]. The infected compartments are X= (E_C, E_A, I_{1C} , I_{1A}, I_{2C}, I_{2A})^T. The model is decomposed into two transition matrices F and V.The matrices F (inflow) and V (outflow) are derived from the linearized system at the disease-free equilibrium (DFE), where $S^0C = \Lambda_C/(\phi+\mu)$ and $S^0A = (\Lambda_A + \phi S^0C)/\mu$. The Jacobian matrices F and V are computed, and R_0 is the spectral radius of the next-generation matrix FV^{-1}:

$R_0 = R_{0C} + R_{0A}$

where

$$R_{0C}=\beta_C\left(\frac{\varepsilon}{k_1}+\frac{(1-\rho)\alpha_C}{k_1k_3}+\frac{(1-\rho)\alpha_C\delta_C}{k_1k_3k_5}\right),$$

$$R_{0A}=\beta_A\left(\frac{\varepsilon}{k_2}+\frac{(1-\rho)\alpha_A}{k_2k_6}+\frac{(1-\rho)\alpha_A\delta_A}{k_2k_4k_6}\right),$$

and the constants are defined as $k_1=\Omega_C+\alpha_C+\phi+\mu$, $k_2=\alpha_A+\mu$, $k_3=\phi+\xi_C+\gamma_C+\mu$, $k_4=\xi_A+\gamma_A+\mu$, $k_5=\delta_C+\phi+\gamma_C+\mu$, $k_6=\delta_A+\gamma_A+\mu$.

2.3 Optimal Control Problem

To identify the most effective and cost-efficient intervention strategies, we extend the age- structured model (equation 3) by incorporating six time-dependent control functions. These controls represent public health interventions implemented over a finite time horizon [0, T]. Each control is bounded between 0 and 1, where 0 represents no effort and 1 represents maximum effort.

2.3.1 Control Variables

- We define the following controls:
- $U_1(t)$: Pre-exposure prophylaxis (PrEP)for children, reducing their susceptibility to infection.
- $U_2(t)$: Screening for children, increasing the rate at which exposed children are detected and become aware of their infection.
- $U_3(t)$: Antiretroviral therapy (ART)for children, increasing the treatment rate for aware infected children.
- $U_4(t)$: Screening for adults, increasing the detection rate for exposed adults.
- $U_5(t)$: Educational campaigns and behavioral interventions for adults, reducing their effective contact rate.
- $U_6(t)$: Antiretroviral therapy (ART) for adults, increasing the treatment rate for aware infected adults.

2.3.2 Controlled state schematic diagram and equations of HIV/AIDS dynamics

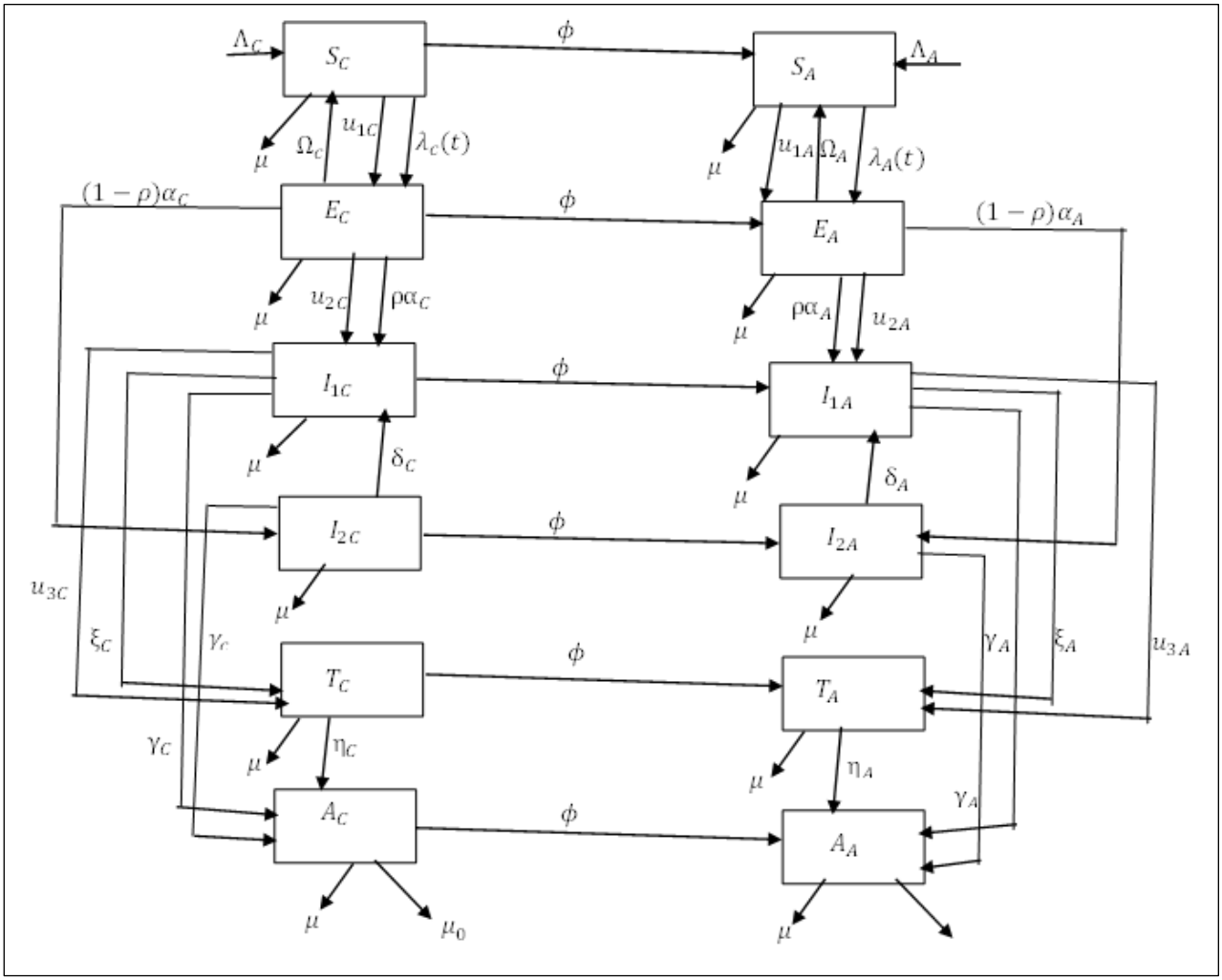

Figure 2: Schematic Diagram of HIV/AIDS dynamics with control

Incorporating these controls modifies equations 3, hence the controlled system is as follows:

$$
\begin{aligned}
\frac{dS_C}{dt} &= \Lambda_C + \Omega_C E_C - \varphi S_C - u_1 S_C - (1-u_1)\lambda_C S_C - \mu S_C \\
\frac{dS_A}{dt} &= \Lambda_A + \varphi S_C + \Omega_A E_A - u_{1A} - u_5 S_A - (1-u_5)\lambda_A S_A - \mu S_A \\
\frac{dE_C}{dt} &= u_1 S_C + (1-u_1)\lambda_C S_C - \Omega_C E_C - u_2 E_C - \rho\alpha_C E_C - (1-\rho)\alpha_C E_C - \varphi E_C - \mu E_C \\
\frac{dE_A}{dt} &= u_5 S_A + (1-u_5)\lambda_A S_A + \varphi E_C - \Omega_A E_A - u_4 E_A - \rho\alpha_A E_A - (1-\rho)\alpha_A E_A - \mu E_A \\
\frac{dI_{1C}}{dt} &= u_2 E_C + \rho\alpha_C E_C + \delta_C I_{2C} + u_3 T_C - \varphi I_{1C} - \xi_C I_{1C} - \gamma_C I_{1C} - \mu I_{1C} \\
\frac{dI_{1A}}{dt} &= u_4 E_A + \rho\alpha_A E_A + \varphi I_{1C} + \delta_A I_{2A} + u_6 T_A - \xi_A I_{1A} - \gamma_A I_{1A} - \mu I_{1A} \\
\frac{dI_{2C}}{dt} &= (1-\rho)\alpha_C E_C - \delta_C I_{2C} - \varphi I_{2C} - \gamma_C I_{2C} - \mu I_{2C} \\
\frac{dI_{2A}}{dt} &= \varphi I_{2C} + (1-\rho)\alpha_A E_A - \delta_A I_{2A} - \gamma_A I_{2A} - \mu I_{2A} \\
\frac{dT_C}{dt} &= \xi_C I_{1C} + u_3 I_{1C} - u_3 T_C - \varphi T_C - \eta_C T_C - \mu T_C \\
\frac{dT_A}{dt} &= \xi_A I_{1A} + u_6 I_{1A} + \varphi T_C - u_{6T_A} - \eta_A T_A - \mu T_A \\
\frac{dA_C}{dt} &= \gamma_C I_{1C} + \gamma_C I_{2C} + \eta_C T_C - \varphi A_C - \mu A_C \\
\frac{dA_A}{dt} &= \gamma_A I_{1A} + \gamma_A I_{2A} + \eta_A T_A + \varphi A_C - \mu A_A - \mu_{iA} A_A
\end{aligned}
\qquad (4)
$$

where the forces of infection, λ_C and λ_A, are defined as in Equations (1) and (2).

2.3.3 Objective Functional

The goal is to minimize the number of individuals in the AIDS stages (A_C and A_A) and the costs associated with implementing the controls. We assume a quadratic cost for the controls, reflecting the increasing marginal cost of interventions. The objective functional to be minimized is:

$$J(u)=\int_0^T \left[A_C(t)+(A_A(t)+\sum_{i=1}^{6}\frac{W_i}{2}u_i(t)^2\right]dt, \qquad (5)$$

where $W_i>0$ are positive weight constants. We seek an optimal control set $u^*=(u_1^*,....,u_6^*)$

such that

$$J(u^*)=\min_{u\epsilon U} J(u),$$

where the control set is defined as $U=\{u(t)\epsilon R^6 / 0\leq u_i(t)\leq 1, i=1,...,6\}$.

2.3.4 Hamiltonian and Optimality Conditions

The necessary conditions are derived using Pontryagin´s Maximum Principle [9]. We define the Hamiltonian H as:

$$H=A_C+A_A+\sum_{i=1}^{6}\frac{W_i}{2}u_i^2+\lambda^T.f(X,u)$$

where $X=(S_C,S_A,E_C,E_A,I_{1C},I_{1A},1_{2C},I_{2A},T_C,T_A,A_C,A_A)^2$ is the state vector, f represents the right-hand of equation 4 and $\lambda=(\lambda S_C,\lambda S_A,\lambda E_C,\lambda E_A,\lambda I_{1C},\lambda I_{1A},\lambda 1_{2C},\lambda I_{2A},\lambda T_C,\lambda T_A,\lambda A_C,\lambda A_A)$

The costate equations are given by $\dot{\lambda}_j=-\frac{\partial H}{\partial X_j}$, *with transversality conditions* $\lambda_j(T)=0$. The optimal controls are characterized by $\frac{\partial H}{\partial u_j}=0$ on the interior of the control set.

2.3.5 Characterization of the Optimal Controls

Applying the optimal condition yields explicit expressions by employing the method of Emmanuel et al in [19]:

$$u_1^\star = min\left(1, max\left(0, \frac{(\lambda S_C - \lambda E_C)(1-\lambda_C)S_C}{W_1}\right)\right). \quad (6)$$

$$u_2^\star = min\left(1, max\left(0, \frac{(\lambda E_C - \lambda I_{1C})E_C}{W_2}\right)\right). \quad (7)$$

$$u_3^\star = min\left(1, max\left(0, \frac{(\lambda I_{1C} - \lambda T_C)(I_{1C} + T_C)}{W_3}\right)\right). \quad (8)$$

$$u_4^\star = min\left(1, max\left(0, \frac{(\lambda E_A - \lambda I)(I_{1C} + T_C)}{W_3}\right)\right). \quad (9)$$

$$u_5^\star = min\left(1, max\left(0, \frac{(\lambda S_A - \lambda E_A)(1-\lambda_A)S_A}{W_5}\right)\right). \quad (10)$$

$$u_6^\star = min\left(1, max\left(0, \frac{(\lambda I_{1A} - \lambda T_A)(I_{1A} + T_A}{W_6}\right)\right). \quad (11)$$

The optimal system consists of the 12 state equations (Equation 4), the 12 costate equations, and the 6 control characterizations. This two-point boundary value problem is solved numerically using the forward sweep method with fourth-Order Runge- Kutta integration [5]

3 RESULTS

3.1 Parameter Values and R_0 Computation

Baseline parameter values for Nigeria were obtained from the Nigerian Centre for Disease Control (NCDC), UNAIDS), and relevant literatures (Table 2). Using these values, the basic reproduction number was computed as $R_0 \approx 5.15$. The adult population contributes the vast majority (R_{0A}=4.95, 96%) to this value, indicating that interventions targeting adults will have the greatest epidemiological impact.

Table 2: Baseline parameter values for the age -structured HIV/AIDS model

Parameter	Value(year^{-1})	Parameter	Value(year^{-1})
Λ_C	3.2×10^6	δ_C	0.40
Λ_A	8.5×10^5	δ_A	0.55
β_C	0.85	ξ_C	0.35
β_A	1.20	ξ_A	0.45
Φ	0.0667	γ_C	0.10
Ω_C	2.50	γ_A	0.12
Ω_A	0.30	η_C	0.05
α_C	0.15	η_C	0.06
α_A	0.12	μ	0.014
P	0.65	μ_0	0.33
E	0.30		

3.2 Dynamics without controls and optimal control strategies

Simulations over a 20- year horizon without controls show rapid epidemic growth with the total infected population approaching 25 million and stabilizing at an endemic equilibrium (Figure 4).

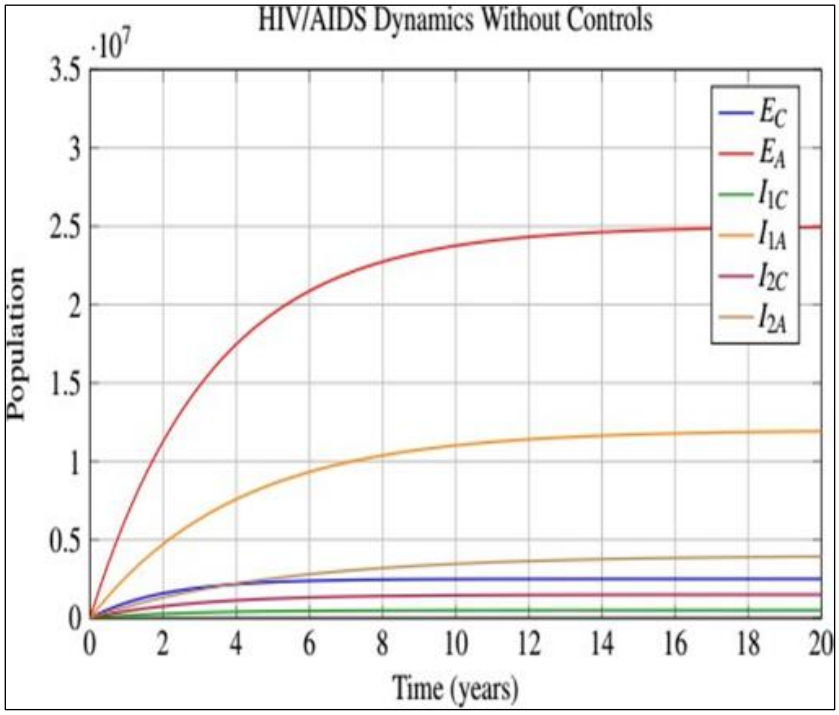

Figure 4: Time series of infected compartments without control interventions.

The red curve represents exposed adult (E_A) and orange curve represents aware adult (I_{1A}). They are the largest, reaching approximately 25 million and 12 million, respectively, within 10 years. This reflects the epidemiological reality that adults are the primary drivers of HIV transmission due to higher rates of sexual activity and intravenous drug use. The exposed adult population (E_A) grows fastest and reaches equilibrium at about 2.5×10^7, indicating that a massive latent reservoir exists in the adult population. Children compartments remain smaller: The blue curve represents the exposed children (E_C) and purple curve represent unaware children (I_{2C}) which saturates at 2.5×10^6 and 1.5×10^6, respectively about one order of magnitude (ten times) lower than their adult counterparts. This is consistent with vertical transmission (mother-to-child) and lower contact rates among children. The green curve represents the aware children (I_{1C}) and brown curve represents the unaware adults (I_{2A}) and both show intermediate values (0.5million and 4 million respectively). The brown curve intersects with the blue curve at time T=4 implying that both unaware adults and exposed children have the same population (about 2 million) within a space of about 4 years. In summary, Figure 4 clearly demonstrates that without intervention, HIV/AIDS reaches catastrophic levels within one generation, with adults bearing the heaviest burden. The saturating dynamics reveal a stable but devastating endemic equilibrium, underscoring the urgent need for sustained control strategies.

Table 3: Effective reproduction number under different control scenarios

Scenario	R_e	Reduction (%)
Baseline (no control)	5.15	0%
Adults only (prevention+ treatment)	1.85	64.1%
Three controls combined (aggressive)	0.85	83.5%
Full optimal (six controls)	0.41	93.0%

The epidemic grows rapidly, with the adult population being the primary driver. The full optimal control strategy (all six controls applied optimally) reduces the total infected population by over 93% (Figure 5) and reduces the effective reproduction number to Re =0.41 (Table 3). The optimal controls profiles (Figure 6) shows that control efforts should be highest in the initial years and can gradually decrease as the disease burden declines.

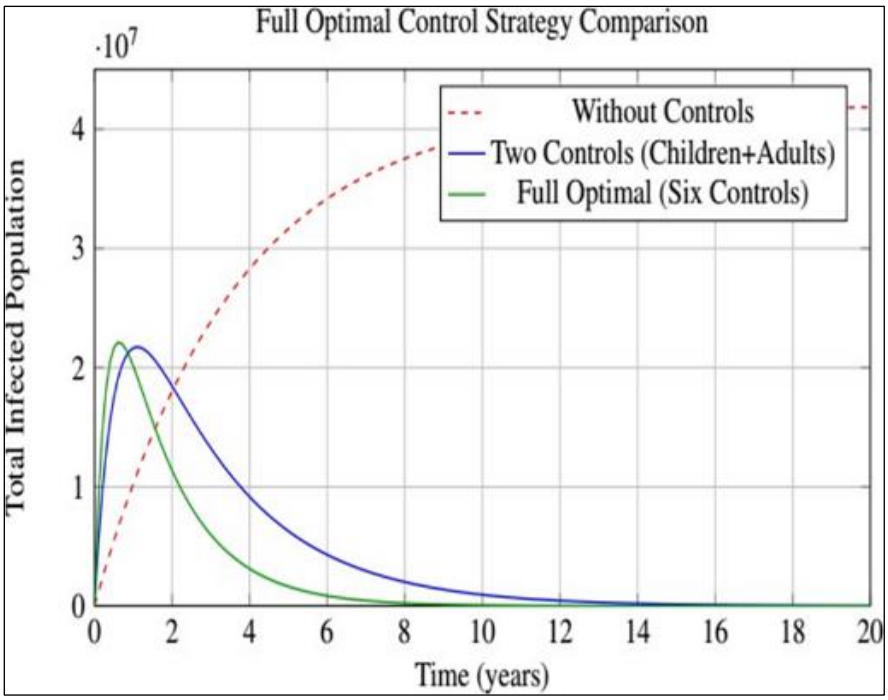

Figure 5: Comparison of total infected population without controls versus with the full control

The full optimal control strategy (green curve) dramatically outperforms both the no-control scenario (red dashed) and the two-control combination (blue curve). While the no-control scenario sees the total infected population grow monotonically, and the two-control strategy reduces the endemic equilibrium to about 12 million, the full six-control strategy reduces the peak the infection. This remarkable decline is achieved through the synergistic effect of simultaneously implementing PrEP, screening, education, and ART across both age groups, consistent with the findings researchers in [6], [4] and [5] who demonstrated that comprehensive multi-intervention approaches are necessary to approach disease elimination. The green curve's steep initial rise (years 0–3) followed by a sharp decline reflects the time needed for interventions to reach full coverage and for the combined effects of prevention and treatment to overcome the epidemic's momentum.

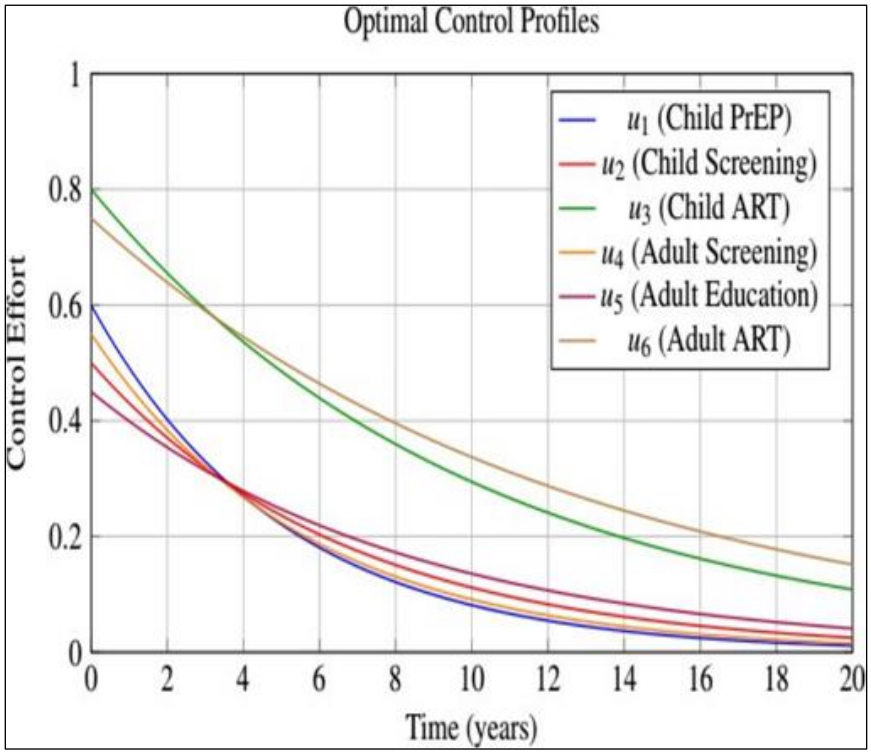

Figure 6: Time-dependent optimal control strategies over the 20 years horizon.

Figure 6 displays the time-dependent optimal control efforts for all six interventions over the 20-year horizon. All control profiles follow a decaying exponential pattern, with the highest efforts concentrated in the first 5–8 years, after which they gradually taper off as the disease burden diminishes. Adult ART (u_6, brown curve) requires the highest initial effort at approximately 0.75, followed closely by child ART (u_3, green) at 0.8 reflecting the central role of antiretroviral therapy in both reducing transmission and preventing disease progression, as emphasized by Nsuami et al., 2018 in [7], and [8]. Child PrEP (u_1, blue) and adult screening (u_4, orange) require moderate initial efforts (0.55–0.6), while adult education (u_5, purple) requires the lowest effort (0.45), consistent with Shaibu et al., 2018 in [9] and Okolo et al., 2023 in [10], who found that behavioral interventions alone are less resource-intensive but also less impactful. The intersection between the brown curve and the green curve indicates that at time T= 3 years the control measures of u_3 (Child ART) and u_6 (Adults ART) have the same control effort. (about 0.58). Similarly, the other four control measures (Child Prep, Child Screening, Adult Screening and Adult Education) have the same control effort (about 0.3) at time T=3.5 years. The gradual decline in all controls after year 8 aligns with the optimal control theory principle that efforts should be front-loaded to rapidly reduce the susceptible-infected pool, after which maintenance levels suffice a strategy advocated by Rana et al., 2023 in [11] and Shewafera and Iannelli, 2025 in [5] for cost-effective epidemic management. Notably, child ART (u_3) and adult ART (u_6) maintain relatively higher efforts throughout the horizon (decay rates of 0.1 and 0.08, respectively), underscoring the sustained need for treatment to prevent resurgence a finding that supports the UNAIDS 90-90-90 targets and the goal of ending the AIDS epidemic by 2030, as discussed by Tibebu and Temesgen, 2020 in [12].

4 DISCUSSION

The computed basic reproduction number $R_0 = 5.15$ is consistent with estimates for high prevalence settings in sub-Saharan Africa [11]. The finding that adults contribute 96% of R_0 confirms that HIV transmission in Nigeria is predominantly driven by the adult population, and control strategies must prioritize this group to achieve epidemic control [18].

5 CONCLUSION

This study developed and analyzed a comprehensive age-structured mathematical model for HIV/AIDS transmission in Nigeria. The key findings are: (1) the basic reproduction number is $R_0 = 5.15$, with adults driving the epidemic; (2) optimal control strategies can reduce the effective reproduction number to 0.41 and decrease peak infections by over 93%; and (3) combination prevention packages are essential for achieving elimination.

In conclusion, HIV elimination in Nigeria is mathematically feasible with the implementation of optimal control interventions. The study recommends: (1) prioritize adult-focused prevention interventions; (2) implement combination prevention packages; (3) aggressively scale up ART coverage; and (4) maintain child-focused interventions for individual health benefits. Future research should incorporate spatial structure, behavioral dynamics, and comprehensive health economic evaluation.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

I declared that there is no conflict of interests exist.

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