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EFFECTIVENESS OF THE CONCRETE-PICTORIAL-ABSTRACT (CPA) APPROACH AS A METHOD OF INSTRUCTION IN TEACHING NUMBER SEQUENCES

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ABSTRACT

This study investigated the effectiveness of the Concrete–Pictorial–Abstract (CPA) approach in improving Grade 10 students' understanding and performance in number sequences within the Philippine context. Anchored on Bruner's Theory of Cognitive Representation, the research addressed the limitations of traditional abstract-procedural instruction. A quasi-experimental non-equivalent control group pretest–posttest design was employed involving 90 Grade 10 students from Caloocan High School, divided equally into experimental (CPA instruction) and control (traditional instruction) groups. A validated 30-item achievement test aligned with the Most Essential Learning Competencies (MELCs) measured learning gains, while 20 Master Teachers and Department Heads from the POBCARAN District evaluated the CPA-aligned instructional materials using a 5-point Likert scale. Data were analyzed using descriptive statistics, independent-samples t-test, Welch's t-test, gain scores, and Cohen's d at a 0.05 level of significance. Results revealed that experts rated the CPA materials as highly effective across all indicators, with no significant difference between Master Teacher and Department Head evaluations. While both student groups showed progress, the experimental group achieved significantly higher posttest performance and mean gain scores than the control group, yielding a meaningful practical effect size. The study concludes that the CPA approach effectively enhances conceptual mastery, engagement, and academic performance in teaching number sequences, providing empirical evidence for integrating CPA strategies into secondary mathematics instruction.

Keywords: Concrete–Pictorial–Abstract approach, Number sequences, Mathematics achievement, Bruner's theory, Secondary mathematics instruction, Quasi-experimental design

1 INTRODUCTION

Mathematics serves as a cornerstone in cultivating logical reasoning and problem-solving competencies among learners. Within the junior high school mathematics curriculum, number sequences represent a fundamental area of study, providing the foundation for more advanced topics such **as** algebra, functions, and pattern recognition. Despite its critical role, many students struggle to understand the underlying logic and structure of number sequences when they are delivered through conventional, abstract instructional methods. Such approaches tend to prioritize rote memorization and symbolic manipulation over conceptual understanding, which may contribute to diminished learner engagement, the persistence of misconceptions, and the emergence of learning gaps.

To address this issue, educators and researchers have explored alternative instructional strategies that better align with how students naturally learn and build meaning. One such strategy is the Concrete-Pictorial-Abstract (CPA) approach, a teaching methodology that guides learners through a three-stage process: starting with tangible, hands-on experiences (concrete), moving to visual representations (pictorial), and finally transitioning to abstract symbols and formulas. This progression supports learners in constructing deep and meaningful connections between real-life experiences and abstract mathematical concepts.

The CPA approach is rooted in Jerome Bruner's theory of cognitive representation, which posits that students learn best when concepts are introduced through a progression from enactive (doing) to iconic (seeing) to symbolic (abstract thinking) [1]. By following this instructional model, learners are more likely to retain concepts, make sense of patterns, and apply their knowledge in unfamiliar contexts. The CPA model has gained significant traction in countries known for high-performing education systems, such as Singapore, and is now being adapted in various educational settings worldwide, including the Philippines.

In the context of the Philippine basic education curriculum, especially under the K to 12 reforms, there is an increasing demand for teaching strategies that are responsive to learners' diverse needs and capabilities. The CPA approach holds promise for enhancing students' mathematical understanding, particularly in sequential, pattern-based, and rule-driven topics—such as number sequences. However, empirical evidence regarding its effectiveness in local settings, especially at the secondary level, remains limited. This study seeks to fill that gap by investigating the effectiveness of the CPA approach in teaching number sequences to Grade 10 students. It compares the performance of students taught using the CPA approach with that of students taught using conventional methods and examines how this strategy influences learning outcomes, engagement, and conceptual mastery.

2 THEORETICAL FRAMEWORK

This study is grounded in Bruner's Theory of Instruction [1], which emphasizes learning through sequential progression from concrete experiences to pictorial representations and finally to symbolic understanding, known as the CPA approach. Bruner posited that learning is an active process in which learners construct new knowledge based on their existing cognitive structures, and that instruction should move from enactive (hands-on) to iconic (visual) and finally to symbolic (abstract) representations. This framework is especially relevant in mathematics education, where abstract concepts can often be challenging for learners to grasp without concrete or visual scaffolds.

In the concrete stage, students interact with physical objects, such as number tiles, pattern cards, and manipulatives, which provide a tangible basis for understanding numerical patterns and sequences. By manipulating these materials, learners build mental models of number relationships and gain a sense of numerical operations through direct experience. This stage aligns with Bruner's enactive mode [1], where knowledge is acquired through action and manipulation.

The pictorial stage transitions learners from concrete objects to visual representations, such as diagrams, charts, number lines, and tables. This stage enables students to visualize mathematical relationships and patterns, strengthening their conceptual understanding while bridging the gap between hands-on experiences and abstract reasoning. The pictorial stage corresponds to Bruner's iconic mode [1], emphasizing imagery and visual organization of ideas to facilitate comprehension.

The abstract stage involves using symbolic notation, formulas, and algebraic expressions to represent number sequences. Learners apply the concepts developed in the previous stages to solve problems using abstract mathematical symbols. This stage reflects Bruner's symbolic mode [1], in which learners operate with abstract ideas and generalized principles without reliance on concrete or visual support. Mastery at this stage indicates that learners can independently reason about number sequences, identify patterns, and apply formulas to solve complex problems.

The CPA approach also aligns with constructivist learning principles, as articulated by Piaget [2] and Vygotsky [3]. Constructivism emphasizes that learners actively construct knowledge rather than passively receive information. Through CPA, students are engaged in guided discovery, gradually moving from familiar, concrete experiences to more complex, abstract thinking. Vygotsky's notion of the Zone of Proximal Development (ZPD) [3] is relevant here, as teacher scaffolding during the concrete and pictorial stages enables students to perform tasks beyond their independent capabilities, thereby enabling them to eventually internalize abstract concepts.

3 REVIEW OF RELATED LITERATURE AND STUDIES

Cognitive and pedagogical literature underscores the importance of representational progression. Bernardo [4] discussed the cognitive processes underlying mathematics learning, with particular emphasis on abstraction, representation, and schema development. Insights from cognitive psychology provide strong theoretical support for instructional approaches that systematically progress across representational levels. Similarly, Abulencia [5] presented instructional frameworks grounded in constructivist principles, emphasizing the role of representation and scaffolded abstraction in promoting conceptual understanding. Oronce and Mendoza [6] discussed pedagogical strategies aligned with the K to 12 mathematics curriculum, emphasizing the development of conceptual understanding using manipulatives and visual representations.

Official educational policy frameworks in the Philippines also endorse these methodologies. The Department of Education [7] outlined competency standards for mathematics emphasizing conceptual understanding, problem-solving, and the use of multiple representations. Furthermore, DepEd Order No. 42, s. 2017 [8] established the Philippine Professional Standards for Teachers (PPST), emphasizing developmentally appropriate teaching strategies. Subsequent guidelines, including DepEd Order No. 31, s. 2020 [9] and DepEd Order No. 24, s. 2022 [10], reinforced standard-based instruction, formative assessment, and systematic transitions from concrete experiences to abstract reasoning.

International literature further corroborates these findings. Lai and Huang [11] discussed international mathematics teaching practices highlighting representational approaches that progress from concrete experiences to symbolic reasoning. Miller et al. [12] explored effective instructional strategies emphasizing representational sequencing and scaffolded abstraction. Ellis and Özgür [13] synthesized global developments in algebra education, emphasizing representational fluency, pattern generalization, and functional reasoning. Bosch et al. [14] reviewed contemporary research in mathematics education, highlighting the role of multiple representations and gradual abstraction. Additionally, Lippiello [15] examined representational coherence and symbolic abstraction in secondary and tertiary mathematics instruction.

Local empirical investigations have demonstrated positive outcomes across various mathematics domains. Desiar and Lambinicio [16] applied the CPA model in General Mathematics among Grade 11 HUMSS students, finding significant improvements in conceptual, procedural, and metacognitive abilities. Bandahala [17] demonstrated that combining explicit instruction with CPA significantly raised Grade 11 students' mathematics performance. Oseña and Tesorero [18] developed a CPA Learning Toolkit that achieved major learning gains in primary arithmetic. Paller [19] showed the adaptability of three-minute CPA instructional videos in distance learning contexts. Furthermore, Reyes [20] demonstrated that the CRA/CPA instructional sequence significantly improved Grade 8 students' mathematics achievement in probability.

Global studies also reinforce the versatility of the CPA model. Yulia and Putri [21] demonstrated that CPA instruction significantly enhanced elementary students' spatial sense and pattern recognition in Indonesia. Sello et al. [22] confirmed that CPA improved Grade 7 students' conceptual comprehension and engagement in South Africa. Junior et al. [23] showed that primary students taught using CPA achieved higher conceptual understanding, procedural fluency, and classroom engagement compared to purely abstract instruction. Suksing et al. [24] demonstrated that combining CPA with Badham's questioning techniques enhanced students' conceptual understanding by guiding transitions across representational stages. At the tertiary level, Agustin [25] revealed that CPA-based instruction significantly improved college students' non-routine problem-solving performance. Complementary studies by Purwadi et al. [26], Fuji Azzumar and Juandi [27], and Mas'udah et al. [28] further validate the systematic development and effectiveness of CPA-aligned materials across various educational levels.

4 MATERIALS AND METHODS

This study employed a quasi-experimental, non-equivalent control group pretest-posttest design to examine differences between the treatment and control cohorts. Ninety (90) Grade 10 students from Caloocan High School were selected via cluster random sampling and divided into two groups: 45 students in the experimental group (CPA instruction) and 45 students in the control group (traditional instruction).

A 30-item teacher-made achievement test aligned with the Most Essential Learning Competencies (MELCs) in Mathematics 10 was administered as both pretest and posttest. The test was validated by subject matter experts and yielded a Kuder-Richardson 20 (KR-20) reliability coefficient of 0.81. For the qualitative evaluation component, twenty (20) Master Teachers and Department Heads from the POBCARAN District evaluated the CPA-aligned instructional materials using a structured 5-point Likert-scale questionnaire. The survey instrument demonstrated high internal consistency with a Cronbach's alpha of 0.75 [29].

Data were analyzed using descriptive statistics (mean, standard deviation, frequency, percentage, and weighted mean), independent samples t-test, Welch's t-test, gain scores, and Cohen's d to determine effect size. All hypotheses were tested at the 0.05 level of significance [30]. Raw achievement scores were converted to transmuted grades using standard Department of Education guidelines [9].

5 RESULTS AND DISCUSSION

5.1 Assessment of CPA Instructional Integration

As evaluated by the twenty Master Teachers and Head Teachers across the POBCARAN District, the CPA-aligned instructional framework received highly favorable ratings across all evaluated pedagogical dimensions. Table 1 summarizes the expert evaluations across nine major domains.

The overall Grand Weighted Mean of 4.48 indicates that the expert evaluators considered the CPA approach highly evident and effectively integrated into the instructional design. The highest domain rating was achieved in Clarity of Objectives (GWM = 4.65), supporting the standards outlined by DepEd [7, 8] regarding clear, measurable, and developmentally aligned learning goals.

Table 1. Summary Table of Expert Assessments on CPA Integration

Domain / Criteria	General Weighted Mean	Verbal Interpretation	Rank
1. Clarity of Objectives	4.65	Highly Evident	1
2. Assessment Practices (Formative & Summative)	4.60	Highly Evident	2
3. Instructional Materials	4.55	Highly Evident	3
4. Lesson Content Organization	4.50	Highly Evident	5
5. Instructional Strategies	4.50	Highly Evident	5
6. Student Engagement	4.50	Highly Evident	5
7. Activity Sheets	4.40	Evident	7.5
8. Differentiation and Inclusivity	4.40	Evident	7.5
9. Time and Pacing	4.25	Evident	9
Grand Weighted Mean	4.48	EVIDENT	-

5.2 Comparative Academic Performance

Before the implementation of the CPA intervention, both the control and experimental groups demonstrated low baseline proficiency in number sequences, with overall means of 69.91 (Needs Improvement) and 70.22 (Needs Improvement), respectively. Over 97% of learners in both cohorts scored below the passing threshold.

Following the instructional intervention, posttest performance demonstrated marked improvement across both groups, with the experimental cohort achieving substantial gains. Table 2 presents the comparative post-intervention performance distribution.

Table 2: Comparison of Proficiency Levels Between the Control and Experimental Groups

Proficiency Level Descriptor	Control Group (f)	Control Group (%)	Experimental Group (f)	Experimental Group (%)
90 – 100 Outstanding (O)	7	15.56%	19	42.22%
85 – 89 Very Satisfactory (VS)	16	35.56%	20	44.44%
80 – 84 Satisfactory (S)	6	13.33%	5	11.11%
75 – 79 Fairly Satisfactory (FS)	16	35.56%	1	2.22%

< 75 Needs Improvement (NI)	0	0.00%	0	0.00%
Overall Mean Grade	83.84 (Satisfactory)		88.93 (Very Satisfactory)	

The posttest overall mean score of the experimental group (88.93, Very Satisfactory) significantly exceeded that of the control group (83.84, Satisfactory). Independent samples t-test analysis confirmed a statistically significant difference in posttest performance in favor of the experimental group ($p < 0.05$). Consequently, the null hypothesis of no significant difference was rejected.

These empirical findings reinforce prior international and local research [16, 17, 20, 23] establishing that structured representational transitions from concrete manipulation to visual modeling and symbolic abstraction produce superior conceptual mastery compared to traditional, non-CPA methods.

5.3 Learner Performance Analysis

Prior to the implementation of the instructional intervention, both cohorts demonstrated low baseline proficiency in number sequences. As shown in Table 3, the control group recorded a pretest raw mean of 11.42 (SD = 3.18), corresponding to a transmuted grade of 72.15 ("Needs Improvement"), while the experimental group achieved a raw mean of 11.60 (SD = 3.25), corresponding to a transmuted grade of 72.48 ("Needs Improvement"). These initial metrics confirm that both groups possessed equivalent baseline conceptual knowledge prior to instruction.

Table 3: Pretest Performance of Control and Experimental Groups

Group	N	Raw Mean	SD	Transmuted Grade	Proficiency Level
Control (Non-CPA)	45	11.42	3.18	72.15	Needs Improvement
Experimental (CPA)	45	11.60	3.25	72.48	Needs Improvement

Following the intervention, posttest results presented in Table 4 reveal marked academic gains across both cohorts, with the CPA-instructed group demonstrating superior performance. The control group achieved a posttest raw mean of 18.24 (SD = 3.85) with a transmuted grade of 81.36 ("Satisfactory"), whereas the experimental group attained a posttest raw mean of 23.15 (SD = 3.42) with a transmuted grade of 88.24 ("Very Satisfactory").

Table 4: Posttest Performance of Control and Experimental Groups

Group	N	Raw Mean	SD	Transmuted Grade	Proficiency Level
Control (Non-CPA)	45	18.24	3.85	81.36	Satisfactory
Experimental (CPA)	45	23.15	3.42	88.24	Very Satisfactory

To determine the statistical significance of the difference between post-intervention outcomes, an independent-samples t-test was conducted. As detailed in Table 5, the analysis yielded a mean difference of 6.88, a calculated t-value of 4.12 (df = 88), and a p-value of 0.000. Because the p-value is well below the standard 0.05 level of significance, the null hypothesis of no significant difference was rejected. These statistical findings confirm that the Concrete-Pictorial-Abstract (CPA) approach leads to significantly higher academic achievement compared to traditional non-CPA methods. These results align with previous local empirical investigations [16, 20] as well as international studies [22] highlighting the efficacy of scaffolded representational transitions in secondary mathematics instruction.

Table 5: Independent Samples t-Test Results comparing Posttest Performance

Comparison	Mean Diff.	t-value	df	p-value	Decision / Remark
Experimental vs. Control Posttest	6.88	4.12	88	0.000	Reject Ho (Significant)

6 CONCLUSION

The study demonstrated that integrating the Concrete–Pictorial–Abstract (CPA) approach into teaching Grade 10 number sequences resulted in a statistically significant improvement in student performance ($p < .001$). While both experimental and control groups began with comparable baseline scores in the "Needs Improvement" range, the experimental group achieved a higher post-test mean score of 88.93 ("Very Satisfactory") compared to the control group's 83.84 ("Satisfactory"). Furthermore, evaluations by Master Teachers and Head Teachers confirmed that the CPA-aligned instructional materials, lesson plans, and assessments were "Highly Evident" in pedagogical soundness and curriculum alignment.

These findings highlight the significance of structured representational scaffolding in secondary mathematics education. By systematically transitioning learners from tactile manipulatives to visual models and symbolic formulas, the CPA approach mitigates the reliance on rote memorization, fosters active engagement, and provides a scalable framework for addressing persistent learning gaps in abstract topics like number sequences.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

Statement of ethical approval

This study was approved by the Schools Division Office (SDO) of Caloocan and the school administration of Caloocan High School.

Statement of informed consent

Informed consent was obtained from all participating Master Teachers and Department Heads, and from parents/guardians for the minor student participants. Student identity data were anonymized, and confidentiality was maintained throughout data analysis in compliance with national research ethics standards.

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