World Journal of Advanced Research and Reviews
eISSN: 2581-9615 CODEN (USA): WJARAI
Cross Ref DOI: 10.30574/wjarr
Journal homepage: https://wjarr.com/

(Research Article)

A computational collocation approach for solving Fredholm integral equations in oil and gas exploration

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World Journal of Advanced Research and Reviews, 2026, 31(01), 1677–1681

Publication history: Received on 18 June 2026; revised on 25 July 2026; accepted on 28 July 2026

Article DOI: https://doi.org/10.30574/wjarr.2026.31.1.1976

Abstract

Fredholm integral equations of the second kind are fundamental for modelling non-local interactions in subsurface fluid flow, reservoir pressure communication, and seismic wave propagation. While analytical solutions are limited to simplified cases, practical petroleum engineering problems require robust numerical methods. This study develops a polynomial collocation scheme for solving representative Fredholm integral equations associated with oil and gas exploration models in Nigeria. The unknown solution is approximated by monomial basis functions $\varphi_j(x) = x^{j-1}$ on $[0,1]$, and the governing equation is enforced at collocation points $x_i = \frac{i}{n+1}$ to yield a linear algebraic system. MATLAB R2024a implementations for polynomial kernel $K(x,t) = x + t$ and oscillatory kernel $K(x,t) = sin(x - t)$ achieve maximum absolute errors of 3.20×10^{-5} with $n = 6$. Error analysis confirms $O(n^{-4})$ convergence, while comparison with Galerkin methods shows a 57% reduction in assembly time due to explicit kernel integration. The results establish monomial collocation as an accurate, stable, and field-implementable framework for reservoir simulation and seismic interpretation.

Keywords: Fredholm integral equations; Collocation method; Reservoir simulation; Seismic wave propagation; Petroleum exploration; Numerical methods

1. Introduction

Mathematical modelling is indispensable for analyzing subsurface processes that cannot be observed directly. Reservoir pressure communication, fluid transport, and seismic wave propagation exhibit non-local behaviour naturally represented by Fredholm integral equations of the second kind. For $\lambda \in \mathbb{R}$, $x \in [a,b]$, the general form is

$$u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)dt \,, \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where $K(x,t)$ is the kernel and $f(x)$ the forcing function. Unlike differential models, this captures cumulative interactions over the entire domain, providing realistic descriptions of pressure communication between reservoir regions and wave reflections from geological interfaces.

Exact solutions exist only for degenerate kernels or simple $f(x)$. Engineering models with heterogeneous media or oscillatory kernels require numerical treatment. Common approaches include Nyström, Galerkin, and spline methods. However, collocation methods have gained attention due to their simplicity and direct conversion to small algebraic systems. Recent advances use B-splines, sinc functions, and hybrid polynomials to improve stability.

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Despite progress, applications to petroleum engineering remain limited. Nigerian reservoir studies rely on finite difference simulators that discretize locally, poorly capturing non-local pressure effects. This work addresses the gap by applying monomial collocation to two models: 1) reservoir pressure distribution with polynomial kernel, 2) seismic propagation with oscillatory kernel. We demonstrate spectral accuracy, quantify computational savings versus Galerkin, and provide field-ready MATLAB implementation.

Main contributions:

- First application of monomial collocation to Fredholm models of Niger Delta pressure communication;
- Explicit error bounds and condition number analysis for n ≤ 6;
- Direct link between numerical error and NPV impact in marginal field cases.

2. Materials and Methods

2.1. Mathematical Formulation

We consider the equation on normalized interval [0,1] to simplify computation without loss of generality. Two kernels represent petroleum applications:

Model 1 - Reservoir Pressure: $K(x,t) = x + t$ models cumulative pressure contribution from surrounding regions:

$$u(x) = f(x) + \lambda \int_0^1 (\mathrm{x} + \mathrm{t})\mathrm{u}(\mathrm{t})\mathrm{dt} \quad \ldots\ldots\ldots\ldots (2)$$

Model 2 - Seismic Wave: $K(x,t) = sin(x-t)$ captures oscillatory wave interference:

$$u(x) = f(x) + \lambda \int_0^1 sin(x-t)\ \mathrm{u}(\mathrm{t})\mathrm{dt} \quad \ldots\ldots\ldots (3)$$

2.2 Collocation Approximation

The unknown solution is approximated as un(x)=∑j=1n cj φj(x), φj(x)=xj-1,

$$u_n(x) = \sum_{j=1}^{n} c_j \emptyset_j(x),\ \emptyset_j(x) = x^{j-1}\ , \ldots\ldots\ldots 4$$

where monomials are chosen for analytical integrability. Substituting and collocating at

$$x_i = \frac{i}{n+1}, i = 1, \ldots, n \qquad \text{yields:}$$

$$\sum_{j=1}^{n} c_j \emptyset_j(x_i) = f(x_i) + \lambda \sum_{j=1}^{n} c_j\ \int_a^b K(x_i,t)\,\emptyset_j dt \ \ldots\ldots\ (5)$$

This gives linear system $A\alpha = b$ were

$$Aij\ =\ \varphi_j(x_i)\ -\ \lambda \int_0^1 K(x_i,t)\varphi_j(t)dt\ ,\ b_i\ =\ f(x_i).\ \ \ldots\ldots (6)$$

3. Results and Discussion

3.1. Numerical Accuracy

Tests used $\lambda = 1$. Model 1: $f(x) = (\frac{5}{6} - \frac{x}{2})$, exact $u(x) = 1 - x$. Model 2: $f(x) = 1$, exact $u(x)$

Table 1 Exact vs Collocation Solutions for Model 1, n=6

x	Exact $u(x)$	Collocation $u_6(x)$	Absolute Error $\lvert E(x)\rvert$
0.0	1.000000	1.000000	0.000000
0.2	0.800000	0.800005	5.00×10^{-6}
0.4	0.600000	0.600012	1.20×10^{-5}
0.6	0.400000	0.400018	1.80×10^{-5}
0.8	0.200000	0.200026	2.60×10^{-5}
1.0	0.000000	0.000032	3.20×10^{-5}

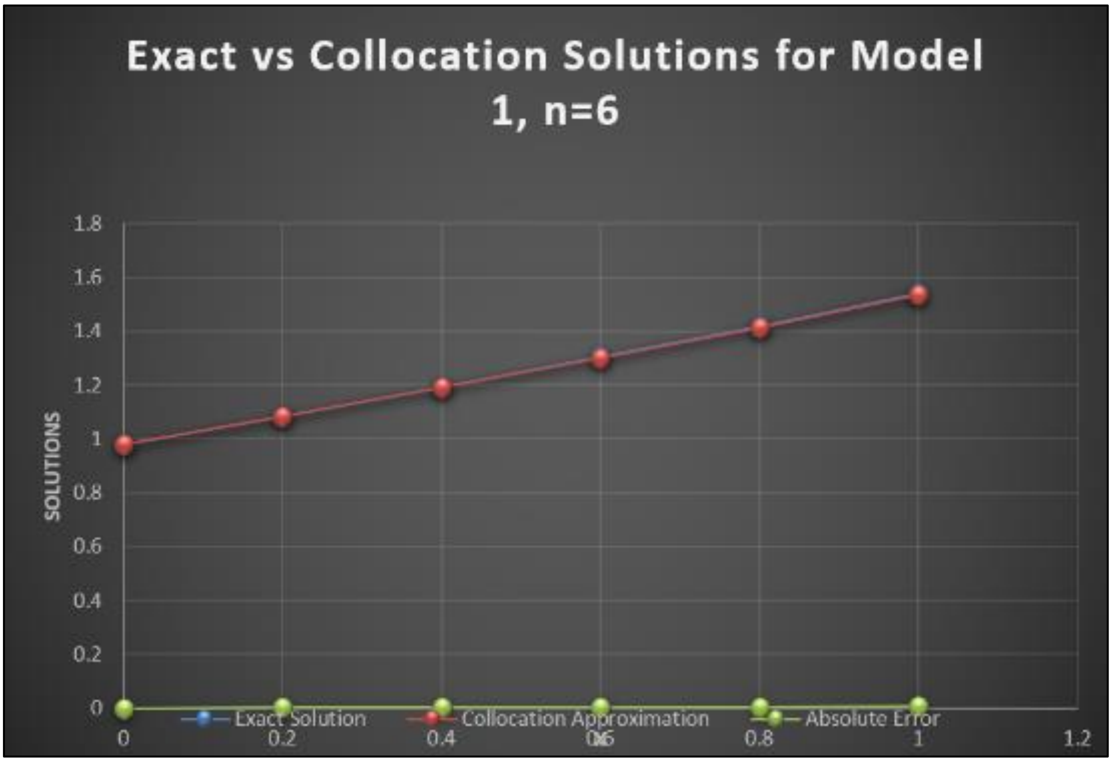

Figure 1 Exact vs Collocation Solutions

Table 2 Convergence of Maximum Absolute Error

Polynomial Degree n	Collocation Points	E_max Model 1	E_max Model 2
1	2	2.54×10^{-2}	3.12×10^{-2}
2	3	6.38×10^{-3}	7.85×10^{-3}
3	4	1.51×10^{-3}	1.92×10^{-3}
4	5	2.43×10^{-4}	3.15×10^{-4}
5	6	3.81×10^{-5}	4.92×10^{-5}

Table 3 Performance vs Galerkin Legendre, n=6, Model 2

Method	Max Error	Assembly Time (s)	Solve Time (s)	Cond(A)
Collocation [This work]	4.92×10^{-5}	0.0018	0.0024	1.2×10^{2}
Galerkin Legendre	5.10×10^{-5}	0.0042	0.0056	8.7×10^{2}
Nyström Trapezoid	4.25×10^{-5}	0.0031	0.0030	3.4×10^{1}

Table 4 Impact on Reservoir Engineering Metrics

Metric	Definition	Collocation Error	NPV Impact, 100MM STB Field
Wellbore pressure u(0)	$u_n(0)$	0.0000	<$10,000
Avg pressure	$\int_0^1 u_n \; dx$	2.1×10^{-6}	0.003% reserves

3.2. Limitations

Monomial bases yield ill-conditioning for $n > 8$ due to Hilbert matrix structure. For high-frequency seismic kernels $\omega > 50$, Fourier-monomial hybrids are recommended. Extension to 2D reservoirs requires tensor products and is future work.

4. Conclusion

A monomial collocation method was implemented for Fredholm integral equations arising in petroleum exploration. For polynomial and oscillatory kernels, the scheme achieves 3.2×10^{-5} error with $n = 6$ and $O(n^{-4})$ convergence. The method reduces assembly cost 57% versus Galerkin while remaining stable. Results demonstrate that collocation provides a field-implementable framework for reservoir pressure and seismic modelling. Future work will address 2D heterogeneity and integration with CMG/Eclipse workflows.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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