World Journal of Advanced Research and Reviews
eISSN: 2581-9615 CODEN (USA): WJARAI
Cross Ref DOI: 10.30574/wjarr
Journal homepage: https://wjarr.com/

(Research Article)

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Estimation of tuber weight using ratio estimator based on ranked set sampling

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World Journal of Advanced Research and Reviews, 2026, 30(03), 2228–2238

Publication history: Received on 22 May 2026; revised on 26 June 2026; accepted on 29 June 2026

Article DOI: https://doi.org/10.30574/wjarr.2026.30.3.1795

Abstract

Ranked Set Sampling (RSS) was first proposed by McIntyre (1952) [1] to enhance the efficiency of the population mean. In statistical inferences, the estimation of population parameters using information obtained from a sample is an important method. This involves choosing an appropriate sampling method to collect the data. RSS is an efficient sampling method used for data collection [2]. It is a cost-effective sampling technique when the variable of interest is expensive or difficult to measure, but it could be ranked easily at a negligible cost. Under equal allocation, RSS performs better than simple random sampling (SRS). The performance of RSS further improves when appropriate unequal allocation is used instead of equal allocation. RSS presumes that the sampling units are correctly ranked with respect to variable of interest either before or after its quantification. This is called perfect ranking scenario, but this may not be possible always while dealing with real life situations. In these situations, one could take help of some other characteristics for ranking, which is supposed to be inexpensive, easily available, and highly correlated with the main characteristic of interest. Unlike perfect ranking scenario, the ranking so obtained may be referred to as a concomitant ranking because of its dependence on a concomitant variable. In the terminology of classical sampling, this characteristic is known as an auxiliary variable [3].

Here, on the availability of concomitant variables, ratio estimation based on ranked set sampling is proposed. In the present paper, ratio estimator of the population mean based on RSS is suggested, and it is shown that the ratio estimator based on RSS is more efficient than the estimators based on SRS. These procedures are illustrated using a real data set regarding the yield of tuber (potato). The technique is more useful to those who look for cost-effective sampling technique for estimating agricultural products that are grown underground, such as potatoes, ginger, turmeric, garlic, onions, beetroot, peanuts, etc.

Keywords: Perfect ranking; Concomitant ranking; Bias; Efficiency; Relative savings

1. Introduction

Ranked Set Sampling (RSS) was introduced by McIntyre (1952)[1] to increase an efficiency of the population mean. The method is applicable if the actual measurement of the characteristics of interest is expensive or difficult to measure but ranking of the sampled units can be done easily without having their actual measurements. It is a cost- effective sampling procedure and more efficient than the corresponding estimator based on Simple Random Sampling (SRS). RSS performs better than simple random sampling (SRS) under equal allocation, but the performance of RSS again improves if appropriate unequal allocation is used in place of equal allocation. The first theoretical results about this method were given by Takahasi and Wakimoto (1968) [4]. He suggested that the sample mean based on RSS is an unbiased estimator of the population mean and shown that the estimator based on RSS is more efficient than SRS. There have been many modifications and improvements made on RSS method originated by McIntyre (1952) [1], which is applicable in a wider range of fields than originally intended [Chen *et al.*, 2004] [5]. In real life situations, there are some cases where the characteristics of interest are difficult to measure or rank them but a concomitant variable, which is highly correlated

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with the main characteristic of interest, can be easily ranked and used for the ranking of the sampling units. For example, while ranking households with respect to their income, one may take into account the type, quality, and appearance of the houses in which they live in or the number of adult members in the households, etc., when perfect ranking is not possible directly. Unlike a perfect ranking scenario, the ranking so obtained may be referred to as a concomitant ranking because of its dependence on a concomitant variable [3]. Here, on the availability of concomitant variables, ratio estimation based on ranked set sampling is proposed. In the present paper, ratio estimator of the population mean based on RSS is suggested and it is shown that the ratio estimator based on RSS is more efficient than the estimators based on SRS. These procedures are illustrated using a real data set regarding the yield of tuber (potato). The technique is more useful to those who look for cost-effective sampling technique for estimating agricultural products that are grown underground, such as potatoes, ginger, turmeric, garlic, onions, beetroot, peanuts, etc.

Here, section 2 explores the RSS with equal allocation, and section 3 outlines the RSS with an unequal allocation for asymmetrical population. Section 4 suggests the application of ratio estimator based on SRS method. In section 5, we proposed the ratio estimator based on RSS method and in section 6 we illustrate the efficiency of ratio estimator based on RSS method for estimating the tuber weight based on real data set taken from The Central Potato Research Station (Indian Council of Agricultural Research), Sahaynagar, Patna, Bihar, India. In section 7, the conclusions based on results obtained in section 6 of the present work are included.

2. RSS with Equal Allocation

In RSS procedure with equal allocation, m^2 units are randomly selected from an infinite population, and then the selected units are arranged in *m* sets, each consisting of *m* units. After ranking the units of each set separately with respect to the variable of interest, the unit with the lowest rank is quantified from the first set, the unit with the second lowest rank is quantified from the second set and the process is continued until the unit possessing rank *m* is quantified from the m^{th} set. Thus, in this way, *m* units are quantified out of the initially m^2 units selected from an infinite population. The whole process is repeated *r* times in order to get a large sample size *n*. This provides a ranked set sample of size *n* = *mr*. We call *m*, as the set size and *r*, the number of cycles. As one gets the same number of quantifications *r* for each rank order, this procedure is called RSS with equal allocation (RSSWEA) [Norris, Patil and Sinha, 1995] [6].

In general, suppose that $X_{(i:m)j}$ denotes the i^{th} order statistic based on perfect ranking in the j^{th} cycle, for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,r$. It is to be noted that these are not *iid* in general, but for a given value of *i* these are so with $E(X_{(i:m)j}) = \mu_{(i:m)}$ and $Var(X_{(i:m)j}) = \sigma^2_{(i:m)}$. The McIntyre's estimator $\hat{\mu}_{MRSS}$ of the population mean μ is defined as follows:

$$\hat{\mu}_{MRSS} = \frac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)} \qquad \text{where,} \quad \hat{\mu}_{(i:m)} = \frac{1}{r}\sum_{j=1}^{r}X_{(i:m)j}$$

Here, $E(\hat{\mu}_{(i:m)}) = \mu_{(i:m)}$; $E(\hat{\mu}_{MRSS}) = \mu$ and $Var(\hat{\mu}_{(i:m)}) = \frac{\sigma^2_{(i:m)}}{r}$.

Thus, we get, $$Var(\hat{\mu}_{MRSS}) = Var\left[\frac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)}\right] = \frac{1}{m^2}\sum_{i=1}^{m}Var(\hat{\mu}_{(i:m)}) = \frac{1}{m^2}\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r}$$

Therefore, $$Var(\hat{\mu}_{MRSS}) = \frac{1}{m^2 r}\sum_{i=1}^{m}\sigma^2_{(i:m)} .$$

The estimate of σ^2 for simple random sampling in case of equal allocation is given by,

$$\hat{\sigma}^2_{MRSS} = \left[\frac{mr-m+1}{m^2 r(r-1)}\right]\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i:m)j} - \hat{\mu}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\hat{\mu}_{(i:m)} - \mu_{MRSS})^2$$

2.1. Relative Precision (RP) for an Equal Allocation

The relative precision (RP) of MRSS estimator $\hat{\mu}_{MRSS}$ as compared with SRS estimator $\hat{\mu}_{SRS}$ with the same sample size n is obtained as:

$$RP = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSS})}$$

As $Var(\hat{\mu}_{SRS}) = \frac{\sigma^2}{mr}$, this leads to

$$RP = \frac{\sigma^2}{\overline{\sigma^2}} \qquad \text{where } \overline{\sigma^2} = \frac{\sum_{i=1}^{m} \sigma^2_{(i:m)}}{m}$$

An equivalent and useful measure could be relative cost (RC) and relative savings (RS). These are defined as $RC = \frac{1}{RP}$ and $RS = 1- RC$

The range of RP is $1 \le RP \le \frac{m+1}{2}$ and that of RS is $0 \le RS \le \frac{m-1}{m+1}$.

3. RSS with Unequal Allocation

For unequal allocation McIntyre (1952) [1] suggested that the sample size of each rank order be proportional to its standard deviation while sampling with asymmetrical populations. It implies that if r_i denotes the number of the sets having quantified units with rank i, then $r_i \alpha \sigma_{(i:m)}$ for $i = 1,2,\ldots,m$. Thus, the expression for r_i is given as

$$r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m} \sigma_{(i:m)}}; \quad i = 1,2,\ldots,m.$$

The RSS estimator $\hat{\mu}_{MRSSUA}$ of the population mean μ based on an unequal allocation of samples is given by

$$\hat{\mu}_{MRSSUA} = \frac{1}{m} \sum_{i=1}^{m} \frac{T_i}{r_i} \qquad \text{and} \qquad Var(\hat{\mu}_{MRSSUA}) = \frac{1}{m^2} \sum_{i=1}^{m} \frac{\sigma^2_{(i:m)}}{r_i}$$

where T_i represents the sum of the quantifications of the r_i units having i^{th} rank order. On putting the value of r_i into the expression for $Var(\hat{\mu}_{MRSSUA})$ we have

$$Var(\hat{\mu}_{MRSSUA}) = \frac{(\overline{\sigma})^2}{n}; \qquad \text{where, } \overline{\sigma} = \frac{\sum_{i=1}^{m} \sigma_{(i:m)}}{m}$$

The estimate of σ^2 for simple random sampling in case of unequal allocation is given by,

$$\hat{\sigma}^2_{MRSSUA} = \sum_{i=1}^{m}\left[\frac{m(r_i-1)+1}{m^2 r_i (r_i-1)}\right]\sum_{j=1}^{r_i}(X_{(i:m)j} - \overline{X}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\overline{X}_{(i:m)} - \overline{X}_{(m)r})^2$$

3.1. Relative Precision (RP) for Unequal Allocation

The relative precision (RP_{ua}) of $\hat{\mu}_{MRSSUA}$ relative to $\hat{\mu}_{SRS}$ with the same number of quantifications is given by

$$RP_{ua} = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSSUA})}$$

This yields that $$RP_{ua} = \frac{\sigma^2/n}{\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r_i}/m}$$

This could also be written as $$RP_{ua} = \left(\frac{\sigma}{\overline{\overline{\sigma}}}\right)^2.$$

This proves that $RP_{ua} \geq RP$. Takahasi and Wakimoto (1968) [4] show that $0 \leq RP_{ua} \leq m$. For the above results we can see Sinha (2005) [3].

4. Ratio Estimator based on SRS Method

There are some practical situations where the variable of interest *Y* is difficult to measure and to rank them but the concomitant variable *X*, which is highly correlated with *Y*, can be easily ranked and be used for ranking of the sampling units. This idea was first considered by Stokes (1977) [7]. Let the variable of interest *Y* and the concomitant variable *X* is correlated with the coefficient of correlation ρ. The population ratio of these two variable is $R = \frac{\mu_x}{\mu_y}$ and its estimator is $\hat{R} = \frac{\bar{y}}{\bar{x}}$. Where μ_y and μ_x are the population means of the variables *Y* and *X* respectively, $\bar{x}$ and $\bar{y}$ are the sample means for μ_x and μ_y respectively. The ratio estimator is biased, and bias is negligible when the estimator is approximated using Taylor series expansion to the first degree. The approximated variance of $\hat{R}$ is

$$Var(\hat{R}) \cong (R^2/n)\left(V_x^2 + V_y^2 - 2\rho_{xy}V_xV_y\right)$$

where $V_x = \sigma_x/\mu_x$, $V_y = \sigma_y/\mu_y$ and

$$\rho_{xy} = \sum_{i=1}^{N}(X_i - \mu_x)(Y_i - \mu_y)\Big/N\sigma_x\sigma_y$$

σ_x and σ_y are the standard deviations of the variables *X* and *Y* respectively.

There are many ratio-type estimators based on SRS that has been proposed. Some of these estimators are proposed by Kadilar and Cingi (2004) [8]. These estimators are in the form

$$\hat{\mu}_{SRS} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(\alpha\bar{x} + \gamma)}(\alpha\mu_x + \gamma)$$

where $\beta = \sigma_{xy} / \sigma_x^2$.

They suggested utilizing some known parameters of the concomitant variable *X*.

If $\alpha = 1$ and $\gamma = 0$, the estimator will be

$$\hat{\mu}_{SRS1} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{\bar{x}}\mu_x$$

If $\alpha = 1$ and $\gamma = V_x$, then the estimator will be

$$\hat{\mu}_{SRS2} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(\bar{x} + V_x)}(\mu_x + V_x)$$

where V_x is the coefficient of variation defined as $V_x = \sigma_x / \mu_x$

If $\alpha = 1$ and $\gamma = K_x$, then the estimator will be

$$\hat{\mu}_{SRS3} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(\bar{x} + K_x)}(\mu_x + K_x)$$

where K_x is the coefficient of Kurtosis defined as $K_x = \mu_{x4} / \mu_{x2}^2$,

where $\mu_{xr} = E(X - \mu_x)^r$.

If $\alpha = K_x$ and $\gamma = V_x$, then the estimator will be

$$\hat{\mu}_{SRS4} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(K_x\bar{x} + V_x)}(K_x\mu_x + V_x)$$

If $\alpha = V_x$ and $\gamma = K_x$, then the estimator will be

$$\hat{\mu}_{SRS5} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(V_x\bar{x} + K_x)}(V_x\mu_x + K_x)$$

The mean square errors (MSE) of the above estimators are approximately

$$MSE(\hat{\mu}_{SRSi}) \cong \frac{1-f}{n}\left[R_i^2\sigma_x^2 + \sigma_y^2(1-\rho^2)\right] \text{ for } i = 1,2,3,4,5.$$

where $R_1 = \frac{\mu_y}{\mu_x}$, $R_2 = \frac{\mu_y}{\mu_x + V_x}$, $R_3 = \frac{\mu_y}{\mu_x + K_x}$,

$$R_4 = \frac{\mu_y K_x}{\mu_x K_x + V_x}, \qquad \text{and} \qquad R_5 = \frac{\mu_y}{V_x \mu_x + K_x}.$$

In the present paper it is suggested to use a similar form of estimators as above based on RSS. Here, it is assumed that the population mean of the auxiliary variable is known priory. For some estimators, we need to know other parameters such as coefficient of variation and coefficient of kurtosis. We also assume that the relation between *X* and *Y* is positive and approximately linear.

4.1. Sampling Method

Let *Y* be the variable of interest and *X* be a suitable concomitant variable that is correlated with *Y* and easy to rank. The summary of the RSS procedure is as follows:

- In RSS procedure m^2 bivariate units (*X,Y*) are randomly selected from an infinite population.
- Then the selected units are arranged into *m* sets, each consisting of size *m* units.
- From the first set, the smallest *X* and the associated *Y* are measured. From the second set, the second smallest *X* and the associated *Y* are measured. The process is continued until the m^{th} set, where largest *X* and associated *Y* are measured.
- Thus, in this way, *m* units of *X* and their associated *Y* values are quantified from the initially m^2 units drawn from an infinite population.
- The whole process is repeated *r* times in order to get a large sample size *n*.
- This provides a required ranked set sample of size *n* = *mr*. We refer to *m* as the set size and *r* as the number of cycles.

It is assumed that ranking on the auxiliary variable *X* is perfect. The associated variable *Y* is then with error unless the relation between *X* and *Y* is perfect. Let us denote $\left(X_{j(i)}, Y_{j[i]}\right)$ as the pair of i^{th} order statistics of X and associated unit *Y* in the j^{th} cycle. Then the ranked set sample is

$$\left(X_{1(1)}, Y_{1[1]}\right), \ldots, \left(X_{1(m)}, Y_{1[m]}\right),$$

$$\left(X_{2(1)}, Y_{2[1]}\right), \ldots, \left(X_{2(m)}, Y_{2[m]}\right),$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\left(X_{r(1)}, Y_{r[1]}\right), \ldots, \left(X_{r(m)}, Y_{r[m]}\right)$$

Then we define the sample mean based on RSS by

$$\bar{x}^* = \frac{1}{n}\sum_{i=1}^{n} x_{(i)} \text{ and } \bar{y}^* = \frac{1}{n}\sum_{i=1}^{n} y_{[i]} \text{ with variances are}$$

$$Var\left(\bar{X}^*\right) = \frac{\sigma_x^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{x(i)} - \mu_x\right)^2,$$

$$Var\left(\bar{Y}^*\right) = \frac{\sigma_y^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{y[i]} - \mu_y\right)^2 \text{ and}$$

$$Cov\left(\bar{X}^*, \bar{Y}^*\right) = \frac{1}{m}\sigma_{xy} - \frac{1}{m^2}\sum_{i=1}^{m} T_{xy[i]}$$

with $T_{xy[i]} = \left(\mu_{x(i)} - \mu_x\right)\left(\mu_{y[i]} - \mu_y\right).$

5. Ratio Estimator based on RSS Method

Samawi & Muttlak (1996) [9] proposed a ratio estimator based on RSS as $\hat{R} = \frac{\bar{y}^*}{\bar{x}^*}$ where $\bar{x}^* = \frac{1}{mr}\sum_{k=1}^{r}\sum_{i=1}^{m} X_{k(i)}$ and $\bar{y}^* = \frac{1}{mr}\sum_{k=1}^{r}\sum_{i=1}^{m} Y_{k(i)}$ and the ratio estimator of the population mean of *Y* is $\hat{\mu}_1 = \hat{R}\,\mu_x$.

Using one degree of Taylor series expansion, they showed that this estimator is more efficient than that from SRS with similar form.

Based on RSS, ratio-type estimator is suggested for the mean in the form

$$\hat{\mu}_{RSS} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(\alpha\bar{x}^* + \gamma)}(\alpha\mu_x + \gamma)$$

where α and γ are positive constants, and $\beta = \frac{\sigma_{xy}}{\sigma_x^2}$.

Let us take some special cases of this kind of ratio estimator. If the coefficient of variation and kurtosis of the concomitant variable are available, we may choose these parameters to be values for α and λ in the estimator above. For examples:

If $\alpha = 1$ and $\gamma = 0$, then the estimator will be

$$\hat{\mu}_{RSS1} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{\bar{x}^*}\mu_x$$

If $\alpha = 1$ and $\gamma = V_x$, then the estimator will be

$$\hat{\mu}_{RSS2} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(\bar{x}^* + V_x)}(\mu_x + V_x)$$

If $\alpha = 1$ and $\gamma = K_x$, then the estimator will be

$$\hat{\mu}_{RSS3} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(\bar{x}^* + K_x)}(\mu_x + K_x)$$

If $\alpha = K_x$ and $\gamma = V_x$, then the estimator will be

$$\hat{\mu}_{RSS4} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(K_x\bar{x}^* + V_x)}(K_x\mu_x + V_x)$$

If $\alpha = V_x$ and $\gamma = K_x$, then the estimator will be

$$\hat{\mu}_{RSS5} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(V_x\bar{x}^* + K_x)}(V_x\mu_x + K_x)$$

Here, $\hat{\mu}_{SRS} = \frac{\bar{y} + \beta(\mu_x - \bar{x})}{(\alpha\bar{x} + \gamma)}(\alpha\mu_x + \gamma)$ and

$$\hat{\mu}_{RSS} = \frac{\bar{y}^* + \beta(\mu_x - \bar{x}^*)}{(\alpha\bar{x}^* + \gamma)}(\alpha\mu_x + \gamma)$$

Using the first order of Taylor series expansion of $\hat{\mu}_{RSS}$ about μ_x, μ_y, it has been proved that

$\text{MSE}(\hat{\mu}_{RSS}) \cong Var(\hat{\mu}_{RSS})$ as the bias in the expansion is zero and

$$MSE(\hat{\mu}_{RSS}) \le MSE(\hat{\mu}_{SRS})$$

Using the second order bivariate Taylor series expansion of $h(X,Y)$ about μ_x, μ_y, it has been proved that

$Bias(\hat{\mu}_{RSS}) < Bias(\hat{\mu}_{SRS})$. For the above results we can see Al-Hadhrami (2009) [10].

6. Illustration

For an illustration, a real data set of four characteristics of potato plant fresh haulm weight (in kg), fresh root weight (in kg), tuber (potato) weight (in kg) and chlorophyll measurements of the plant each having 96 observations are taken from Central Potato Research Station, Sahaynagar, Patna - 801506. For estimating tuber weight, we have used concomitant ranking scenario. For selecting concomitant variables for ranking, we need to choose those variables that are highly correlated with the tuber weight. For this we performed the correlation analysis using Statistical Software, Minitab 16. The output of Correlation Analysis is given below.

6.1. Correlation Matrix

Correlations: Fresh Haulm Weight (Kg), Fresh Root Weight (gm), Tubers Weight (Kg), Chlorophyll Measure

	Fresh Haulm Weight(kg)	Fresh Root Weight(gm)	Tubers Weight(kg)
Fresh Root Weight(gm)	0.085 0.410		
Tubers Weight(kg)	0.219 0.032	0.016 0.875	
Chlorophyll Measure	0.049 0.637	0.007 0.945	0.349 0.000

Cell Contents: Pearson correlation
P-Value

After examining the Correlation Matrix, it was found that the chlorophyll level is highly correlated with the tuber weight. So, the chlorophyll level is taken as concomitant variable. The main characteristic of interest is tuber weight, while other characteristic, chlorophyll level, is used as a concomitant variable for ranking the plots with respect to the main characteristic of interest.

Let us assume the main characteristic of interest is tuber weight as Y and concomitant variable chlorophyll level as X which is correlated with a correlation coefficient ρ. It is also assume that X and Y follows bivariate normal distribution with parameters $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and ρ.

Let us take samples using SRS and RSS from a bivariate normal distribution with parameters $\mu_x = 48, \mu_y = 24.36, \sigma_x^2 = 2.18, \sigma_y^2 = 2.78$ and the coefficient of kurtosis of the variable *X* is taken as $K_x = 3$. From this distribution we have generated 1000 samples based on SRS and another 1000 samples using RSS. After that the estimate of μ_y is calculated for each sample, and then the average of $\hat{\mu}_y$'s and mean square errors are computed respectively using $\hat{E}(\hat{\mu}_y) = \frac{1}{1000}\sum_{i=1}^{1000}\hat{\mu}_{yi}$ and $M\hat{S}E(\hat{\mu}_y) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\mu}_{yi} - \mu_y)^2$ using simulation method. For different values of ρ and *m*, estimate of μ_y, the efficiency of RSS estimator and relative savings (RS) are computed as shown in the Table 1, Table 2 and Table 3.

Table 1 Expectation of RSS Estimator of μ_y

ρ	m	$\hat{E}(\hat{\mu}_{RSS1})$	$\hat{E}(\hat{\mu}_{RSS2})$	$\hat{E}(\hat{\mu}_{RSS3})$	$\hat{E}(\hat{\mu}_{RSS4})$	$\hat{E}(\hat{\mu}_{RSS5})$
0.349	3	24.36	24.27	24.39	23.87	24.34
	4	23.78	23.84	24.52	23.42	23.97
	5	23.92	24.23	23.89	23.83	23.94
0.320	3	24.35	23.86	24.27	24.38	24.41
	4	23.94	23.92	23.98	24.27	23.87
	5	24.23	23.93	23.97	23.96	23.84
0.291	3	24.34	23.95	24.42	24.37	23.89
	4	23.77	23.92	23.88	23.79	24.38
	5	23.82	23.89	23.97	23.94	24.36

Table 2 Efficiency of RSS Estimator with respect to SRS Estimator, $Eff_i = MSE(\hat{\mu}_{SRSi}) \big/ MSE(\hat{\mu}_{RSSi})$

ρ	m	Eff_1	Eff_2	Eff_3	Eff_4	Eff_5
0.349	3	1.70	1.72	1.74	1.68	1.79
	4	1.98	1.87	1.93	1.94	1.89
	5	2.21	2.19	2.27	2.24	2.26
0.320	3	1.52	1.57	1.54	1.49	1.47
	4	1.68	1.64	1.67	1.59	1.57
	5	1.87	1.84	1.78	1.86	1.72
0.291	3	1.45	1.38	1.34	1.43	1.49
	4	1.58	1.54	1.49	1.53	1.57
	5	1.67	1.69	1.63	1.59	1.64

Table 3 Relative Savings (RS) when RSS Estimator is used in comparison to SRS Estimator

ρ	m	$RS_1(\%)$	$RS_2(\%)$	$RS_3(\%)$	$RS_4(\%)$	$RS_5(\%)$
0.349	3	41	42	43	40	44
	4	49	47	48	48	47
	5	55	54	56	55	56
0.320	3	34	36	35	33	32

	4	40	39	40	37	36
	5	47	46	44	46	42
0.291	3	31	28	25	30	33
	4	37	35	33	35	36
	5	40	41	39	37	39

7. Conclusion

The results obtained in Table 1, Table 2 and Table 3 show that the bias of the estimators is small and is independent of correlation coefficient ρ , the fluctuations are only due to simulation. RSS estimators are considered to be more efficient than the estimators based on SRS of the similar form. The efficiency of RSS estimator decreases as the correlation coefficient ρ decreases and in this way, the relative savings (RS) in percentage also decreases. But the efficiency of the proposed estimator increases as the set size m increases, and also the percentage relative savings (RS) increases as the set size m increases. It appears that the proposed estimator is more efficient to those who look for cost-effective sampling procedure for estimating tuber weight that are grown under the land surface like ginger, turmeric, garlic, onions, beetroot, peanuts, etc.

Compliance with ethical standard

Acknowledgments

The author sincerely thanks the administration of Marwari College, Bhagalpur, Tilka Manjhi Bhagalpur University, Bhagalpur – 812007 (INDIA), for providing me the conducive environment during the data analysis and writing of this paper.

Disclosure of Conflict of Interest

The author declares no competing interests.

Statement of Ethical Approval

The proposed study is exempt from formal institutional ethics committee approval, as it does not involve human participants or live animals.

Statement of Informed Consent

There is no need for a statement of informed consent, as the objective of the research is agricultural products that are grown underground.

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