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Optimization of Cutting Forces in Hard Turning of AISI 420 Steel Using Grey Relational Analysis

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Abstract

This paper presents a multi-response optimization of cutting forces in the hard turning of AISI 420 stainless steel using Grey Relational Analysis (GRA). Machining experiments were performed on a CNC lathe by varying cutting speed and feed rate, while keeping the depth of cut constant. The three components of cutting forces—feed, tangential, and radial were measured. GRA approach was employed to convert the multi-response problem into a single optimization index. Experimental data were normalized and Grey Relational Coefficients (GRC) and Grades (GRG) were computed to evaluate overall performance. The optimal parameter setting was determined based on the maximum GRG. The results indicate that feed rate is the most influential parameter affecting cutting forces, followed by cutting speed. The optimal machining conditions significantly reduce cutting forces, improving machining performance. The proposed methodology proves to be an effective tool for multi-objective optimization in hard turning processes.

Keywords: Hard turning; GRA; Cutting Forces; AISI 420 stainless steel

1. Introduction

Hard turning has emerged as an effective alternative to conventional grinding processes for finishing hardened steels, offering advantages such as reduced machining time, lower production costs, and improved flexibility. Among various materials, AISI 420 stainless steel is widely used in industrial applications due to its high strength, corrosion resistance, and good wear properties [1]. However, machining of such hardened materials presents significant challenges, particularly in terms of high cutting forces, tool wear, and surface integrity. Cutting forces play a critical role in machining performance, as they directly influence tool life, dimensional accuracy, energy consumption, and surface quality [2]. Therefore, optimizing cutting forces is essential for improving the efficiency and sustainability of hard turning processes. The cutting forces are strongly affected by machining parameters such as cutting speed, feed rate, and depth of cut, making their proper selection a key factor in process optimization. In recent years, multi-response optimization techniques have gained increasing attention in machining research [3]. Traditional optimization methods often focus on a single performance characteristic, which may not adequately represent the overall machining performance. To overcome this limitation, Grey Relational Analysis (GRA), based on grey system theory, has been widely applied as an effective tool for solving multi-objective optimization problems. GRA enables the conversion of multiple performance characteristics into a single Grey Relational Grade (GRG), facilitating the identification of optimal machining conditions. Several studies have applied hybrid approaches combining the Taguchi method with Grey Relational Analysis to optimize machining parameters for different materials and responses [4] . These approaches have proven to be efficient in reducing experimental effort while providing reliable optimization result . However, limited studies have focused on the multi-response optimization of cutting forces in the hard turning of AISI 420 stainless steel. Therefore, the main objective of this study is to investigate and optimize cutting force components—feed force, tangential force, and radial force—during the hard turning of AISI 420 stainless steel using a hybrid Taguchi–GRA

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approach. The effects of cutting speed and feed rate are analyzed, and the optimal machining parameters are determined based on the Grey Relational Grade. The findings of this study aim to provide practical guidelines for improving machining performance and extending tool life in hard turning applications.

2. Experimental setup, procedure and equipment

Table 1 Process parameters and experimental conditions

	Level/ Factors	**High**	**Centre**	**Low**
	Coded no	**+1**	**0**	**-1**
[A]	Cutting speed (m/min)	170	135	100
[B]	Feed rate (mm/rev)	0.24	0.2	0.16
[C]	Depth of cut	0.2		

Table 2 Experimental plan and results

Standard	**Cutting speed (m/min)**	**Feed rate (mm/rev)**	**Feed force (N)**	**Tangential force (N)**	**Radial force (N)**
1	100	0.16	54	210	143
2	135	0.20	59.7	262	163
3	135	0.20	59.5	263	163
4	135	0.20	56.2	239	151
5	135	0.24	60	244	180
6	100	0.20	59	255	158
7	170	0.20	58	292	156
8	100	0.24	60	259	187
9	170	0.24	61.5	300	168
10	170	0.16	47	222	137
11	135	0.16	50	205	140

Figure 1 Alpha 1350 S CNC lathe and dynamometer

3. Analysis method

3.1. Grey rational analysis

Initiator of the Grey system theory (1982) widely used for measuring the degree of relationship between sequences by Grey relational grade [5]. In GRA, the experimental values of the measured quality characteristics are normalized in a range from zero to one [6]. This is known as grey relational generation. Then the grey relational coefficient (GRC) is calculated. The overall Grey relational grade is then computed by averaging the Grey relational coefficient corresponding to each performance characteristic. As a result, optimal combination of process parameters is evaluated considering the highest Grey relational grade by using the Taguchi method. The overall performance characteristic depends on the computation of the grey relational grade (GRG). Thus, a multiple response process optimization is transformed into a single objective problem [7]. The highest GRG will be evaluated as the optimal parametric combination. Typically the normalization process involves two concepts into the *Taguchi's* technique (nominal the smaller is the better and higher is the better) [8]. The "smaller is the better" concept is used for normalizing the forces and tool life by using Eq2.

$$x_i^*(k) = \frac{x_i(k) - min\, x_i(k)}{max\, x_i(k) - min\, x_i(k)} \ldots\ldots\ldots\ldots\ldots(1)$$

$$x_i^*(k) = \frac{max\, x_i(k) - x_i(k)}{max\, x_i(k) - min\, x_i(k)} \ldots\ldots\ldots\ldots(2)$$

The terms x_i (k) is the i^{th} series in the original value of *k*; order $x_i^*(k)$ is the i^{th} series and *k* order after normalization, $min x_i(k)$ is the minimum value in the i^{th} series, $max\, x_i(k)$ is the maximum value in the i^{th} series**.** Table 3 shows the normalized results of experimental results obtained for performances.

Table 3 Normalization (data pre-processing) of the experimental results

Exp no	Feed force	Tangential force	Radial force
1	0.517241	0.947368	0.88
2	0.124138	0.4	0.48
3	0.137931	0.389474	0.48
4	0.365517	0.642105	0.72
5	0.103448	0.589474	0.14
6	0.172414	0.473684	0.58
7	0.241379	0.084211	0.62
8	0.103448	0.431579	0
9	0	0	0.38
10	1	0.821053	1
11	0.793103	1	0.94

3.2. Grey rational coefficients (GRC)

Normalization creates a new matrix of difference vectors. From this matrix, a GRC is calculated, expressed as:

$$\vartheta_i(k) = \frac{(\Delta min + \xi \Delta max)}{(\Delta_{0i}(k) + \xi \Delta max)} \qquad (3)$$

The term $\vartheta i(k)$ denotes GRC for the *k* output parameter and $\Delta_{0i}(k) = |\, x_0^*(k) - xi^*(k)\,|$ is the deviation sequence. Lastly, $\Delta min = min|\, x_0^*(k) - x_i^*(k)|$ whereas $\Delta max = max\, |x_0^*(k) - x_i^*(k)|$ and **ζ**= weighting coefficient that is 0.5.

3.3. Grey Relational Grades (GRG)

Table 4 GRG for each experimental run

Expt. no	Grey Relational Coefficient			GRG (γ)	Rank
	Feed force	Tangential force	Radial force		
1	0.482759	0.052632	0.12	0.73999515	3
2	0.875862	0.6	0.52	0.43605002	9
3	0	0.610526	0.52	0.64681102	4
4	0.634483	0.357895	0.28	0.55485907	5
5	0.896552	0.410526	0.86	0.4249349	7
6	0.827586	0.526316	0.42	0.46909371	6
7	0.758621	0.915789	0.38	0.43953398	8
8	0.896552	0.568421	1	0.38644611	10
9	1	1	0.62	0.37103175	11
10	0	0.178947	0	0.9121447	1
11	0.206897	0	0.06	0.86672474	2

Finally, the GRG is obtained by averaging the GRC corresponding to each performance measures. Thus by applying (Eq 4), all GRGs can be computed

$$(\gamma)i = \frac{1}{n}\sum_{1}^{n}\vartheta_i(k) \quad (4)$$

The term γ denotes the Grey Relational Grade (GRG) while ,*n,* is the number of output parameters. Table 4 presents the grey rational coefficients and grades for each response.

4. Results and discussion

The experimental results were analyzed using Grey Relational Analysis (GRA) to evaluate the combined effect of machining parameters on cutting force components. The measured responses feed force, tangential force, and radial force were first normalized using the "smaller-the-better" criterion, as lower cutting forces are desirable in hard turning operations. The normalized data were used to compute the Grey Relational Coefficients (GRC), which quantify the relationship between the ideal and actual normalized responses. Subsequently, the Grey Relational Grade (GRG) was calculated for each experimental run by averaging the corresponding GRC values. The GRG represents the overall performance index for multi-response optimization. From Table 4, it is observed that experiment number 10 achieved the highest GRG value (0.912), followed by experiment number 11 (0.867), indicating that these parameter combinations provide the best overall performance in minimizing cutting forces. In contrast, experiment number 9 exhibited the lowest GRG value, reflecting poor machining performance under those conditions. The response table for GRG (Table 5) shows the effect of machining parameters at different levels. It is evident that the feed rate has the most significant influence on the cutting forces, as indicated by the highest delta value (Max–Min), followed by cutting speed. This suggests that reducing feed rate leads to a considerable decrease in cutting force components.. The optimal combination of machining parameters was identified based on the highest GRG values. The results indicate that low feed rate and high cutting speed produce minimum cutting forces. This can be attributed to reduced material load per revolution and improved cutting conditions at higher speeds. Overall, the application of Grey Relational Analysis successfully transformed the multi-response optimization problem into a single performance index, enabling efficient identification of optimal machining parameters. The findings confirm that GRA is a reliable and effective method for optimizing machining performance in hard turning of AISI 420 stainless steel.

Table 5 Response table for GRG

Machining parameters	Grey Relational Grade			Main Effect Max-Min	Rank
	Level 1	Level 2	Level 3		
Cutting speed	0.531844988	0.585875948	0.5742368	0.054031	2
Feed rate	0.83962153	0.509269559	0.3941376	0.4454839	1

Total mean value of GRG = 0.568; * Levels for optimum GRG

5. Conclusion

In conclusion, the present study demonstrated the effectiveness of a hybrid Taguchi–Grey Relational Analysis approach for multi-response optimization of cutting forces in the hard turning of AISI 420 stainless steel. The methodology successfully integrated multiple performance characteristics into a single optimization framework, enabling precise evaluation of machining performance. The results confirmed that feed rate is the dominant factor influencing cutting forces, while cutting speed plays a secondary but significant role. The optimal parameter setting, characterized by low feed rate and high cutting speed, achieved a substantial reduction in cutting force components.

The proposed approach offers a robust and practical tool for machining optimization and can be extended to other materials and performance measures. Future work may focus on incorporating additional responses such as surface roughness and tool wear to further enhance the comprehensiveness of the optimization process.

Compliance with ethical standards

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Disclosure of conflict of interest

The author declared that there was no conflict of interest during the cause of this study and producing and submitting this manuscript for publications.

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