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# A mathematical model of insecurity dynamics in Nigeria driven by weak governance and institutional failure

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## Abstract

This study develops a deterministic mathematical model to examine insecurity in Nigeria as a structurally reinforced outcome of weak governance and socio-economic vulnerability. The model incorporates a governance feedback mechanism linking institutional breakdown to increased recruitment into insecurity, thereby capturing the cyclical interaction between vulnerability and instability. Analytical results establish a basic reproduction number $R_0$ that determines the persistence of insecurity. The insecurity-free equilibrium is stable when $R_0 < 1$, whereas a unique endemic equilibrium exists and is globally stable when $R_0 > 1$. Sensitivity analysis identifies governance failure and recruitment intensity as dominant drivers of persistent insecurity, while recovery and reintegration capacity significantly reduce long-term instability. Numerical simulations further demonstrate that modest improvements in institutional effectiveness can substantially lower endemic insecurity levels. The findings provide a quantitative framework for understanding insecurity as a governance-dependent transmission process and highlight structural reform as essential for sustainable security outcomes and to the best of the authors' knowledge, this is the first model to incorporate governance feedback in insecurity dynamics in Nigeria.

**Keywords:** Governance failure; Insecurity dynamics; Mathematical modeling; Reproduction number; Sensitivity analysis

## 1 Introduction

Insecurity in Nigeria has evolved from isolated disturbances into a persistent structural challenge affecting governance effectiveness, economic productivity, and social cohesion [12, 20]. The recurrent nature of insurgency, banditry, and organized violence suggests that insecurity is not merely episodic but systemically reinforced. Weak institutional capacity, ineffective public service delivery, and governance breakdown have frequently been associated with the persistence of instability [2, 10]. At the same time, socio-economic vulnerability, particularly among unemployed youth, creates a recruitment pool that sustains insecurity dynamics [11].

Mathematical modelling provides a structured way to represent complex social processes through interacting compartments and transition mechanisms [5]. Compartmental frameworks have been successfully applied to epidemiology, crime diffusion, and radicalization processes, where spread occurs through exposure and recruitment pathways. However, most existing models treat governance quality as static or external, rather than as an endogenous factor that both influences and is influenced by insecurity. This creates a conceptual gap in understanding how institutional failure can amplify instability in a feedback-driven manner [1, 9].

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The motivation for this study is therefore to conceptualize insecurity as a systemic transmission process in which governance breakdown directly reinforces vulnerability and recruitment. By modifying the classical compartmental structure to include a governance feedback parameter, the model captures the cyclical relationship between institutional weakness and insecurity persistence. This approach allows insecurity to be analyzed through a threshold framework governed by a basic reproduction number, thereby identifying conditions under which instability either fades or becomes entrenched.

This study conceptualizes insecurity as a governance-dependent transmission process in which institutional failure reinforces vulnerability and recruitment. By extending classical compartmental modeling frameworks to include a governance feedback mechanism, the model captures the dynamic interplay between institutional weakness and insecurity persistence. This approach enables the identification of threshold conditions governing stability and provides quantitative insight into how governance reforms can alter long-term security outcomes.

## 2 Model Formulation and Assumptions

### 2.1 Model Formulation Description

The insecurity model partitions the total population N(t) into four interacting compartments namely; S(t), which represents susceptible individuals who are not yet vulnerable to insecurity, V(t), which represents vulnerable individuals at risk of recruitment, I(t), which represents individuals actively engaged in insecurity, and R(t), which represents recovered or reintegrated individuals.

Individuals enter the susceptible class at a constant rate $\Lambda$. Susceptible individuals become vulnerable at rate $\alpha$, reflecting socio-economic exposure and institutional weakness. They exit naturally at rate $\mu$. The model incorporates a governance breakdown parameter $\kappa$, which captures the reinforcing effect of insecurity on vulnerability. Active insecurity amplifies vulnerability through institutional erosion at rate $\kappa I$, representing institutional erosion and recruitment amplification. Vulnerable individuals transition into the insecure class at rate $\beta$. This transition reflects recruitment, radicalization, or criminal engagement [3]. Insecure individuals recover at rate $\gamma$ through rehabilitation, reintegration, or security intervention mechanisms. Recovered individuals may exit at rate $\delta$ or experience natural turnover at rate $\mu$. All compartments experience natural removal at rate $\mu$.

### 2.2 Model Assumptions

The modified insecurity model is developed under the following assumptions:

- The total population is partitioned into four mutually exclusive compartments: susceptible S(t), vulnerable V(t), insecure I(t), and recovered R(t).
- Individuals enter the susceptible class at a constant rate $\Lambda$, and all compartments experience natural exit at rate $\mu$.
- Susceptible individuals become vulnerable at rate $\alpha$ due to socio-economic exposure and institutional weaknesses.
- Vulnerable individuals are recruited into insecurity at rate $\beta$.
- Active insecurity reinforces vulnerability through a governance feedback mechanism at rate $\kappa$.
- Insecure individuals recover or are reintegrated at rate $\gamma$.
- Recovered individuals may exit the system at rate $\delta$.
- All parameters are assumed positive constants, and the system is formulated as a closed deterministic model without stochastic perturbations.

These assumptions enable insecurity to be modeled as a structured transmission process influenced by governance dynamics.

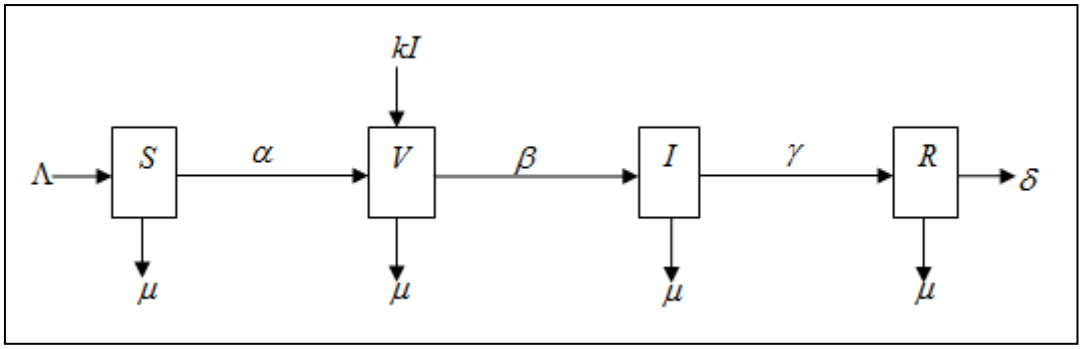


**Figure 1** Flow Diagram of the Insecurity–Governance Transmission Model.

## 3 Mathematical Analysis of the Insecurity–Governance Model

### 3.1 Insecurity–Governance Model Equations

$$\frac{dS}{dt} = \Lambda - \alpha S - \mu S, \tag{1}$$

$$\frac{dV}{dt} = \alpha S + \kappa I - (\beta + \mu)V, \tag{2}$$

$$\frac{dI}{dt} = \beta V - (\gamma + \mu)I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I - (\delta + \mu)R. \tag{4}$$

**Table 1** Insecurity Model Variables and Descriptions

| Variable | Description |
|---|---|
| $S(t)$ | Secure population. |
| $V(t)$ | Vulnerable population (exposed due to broken systems, poverty, weak governance). |
| $I(t)$ | Insecure population (criminality, insurgency, banditry). |
| $R(t)$ | Recovered/Reintegrated population. |

**Table 2** Insecurity Model Parameters and Descriptions

| Parameter | Description |
|---|---|
| $\Lambda$ | Recruitment rate into secure population. |
| $\alpha$ | Rate of transition from secure to vulnerable due to weak governance. |
| $\beta$ | Rate at which vulnerability leads to insecurity. |
| $\gamma$ | Governance response and rehabilitation rate. |
| $\delta$ | Loss of recovery due to governance relapse. |

| | |
|---|---|
| $\mu$ | Natural death/migration rate. |
| $\kappa$ | Rate at which insecurity worsens governance and creates new vulnerable individuals. |

### 3.2 Positivity of the Insecurity–Governance Model

**Theorem 1.**

Solutions of the insecurity model equations (1)-(4) remain non-negative for all t ≥ 0, provided the initial conditions are non-negative.

**Proof:**

We will establish the non-negativity of solutions by analyzing the behavior of the governing vector field along the boundary of the non-negative state space.

From equation (1): $\frac{dS}{dt} = \Lambda - (\alpha + \mu)S$ which has an explicit solution

$$S(t) = S(0)e^{-(\alpha+\mu)t} + \frac{\Lambda}{\alpha+\mu}\left(1 - e^{-(\alpha+\mu)t}\right).$$

Since $S(0) \geq 0,\ \Lambda > 0,$ and $e^{-(\alpha+\mu)t} > 0.$

Therefore, $S(t) \geq 0,$ for all $t \geq 0.$

Also from equation (2): $\frac{dV}{dt} = \alpha S + \kappa I - (\beta + \mu)V,$

Considering the boundary V = 0: $\frac{dV}{dt} = \alpha S + kI \geq 0,$ because $S(t) \geq 0,\ I(t) \geq 0.$

Thus, the flow points into the positive region, and V(t) cannot become negative. Hence, $V(t) \geq 0$

for all $t \geq 0.$

From equation (3): $\frac{dI}{dt} = \beta V - (\gamma + \mu)I,$

At the boundary I = 0: $\frac{dI}{dt} = \beta V \geq 0,$ since $V(t) \geq 0$. Therefore, trajectories cannot cross into the negative I-region. $I(t) \geq 0$ for all $t \geq 0.$

Finally from equation (4): $\frac{dR}{dt} = \gamma I - (\delta + \mu)R,$

At R = 0: $\frac{dR}{dt} = \gamma I \geq 0,$ because $I(t) \geq 0$. Thus, $R(t) \geq 0$ for all $t \geq 0.$

The positive invariance of the state Space will be $\Re_+^4 = \left\{(S,V,I,R) \in \Re^4 : S,V,I,R \geq 0\right\}$ for all $t \geq 0.$

Since all inflow terms are non-negative on the boundary, no variable has a negative source term independent of itself and the region $\Re_+^4$ is positively invariant. Then, the positivity result guarantees that the insecurity model with governance feedback is mathematically well-posed and socially meaningful, ensuring that all population compartments remain non-negative for all time.

### 3.3 Invariant Region of the Insecurity–Governance Model

**Theorem 2.**

If μ > κ, then the region Ω is positively invariant for the insecurity model.

**Proof:**

Let the total population be given by $N(t) = S(t) + V(t) + I(t) + R(t).$ (5)

Differentiating equation (5) with respect to time gives;

$$\frac{N(t)}{dt} = \frac{S(t)}{dt} + \frac{V(t)}{dt} + \frac{I(t)}{dt} + \frac{R(t)}{dt}. \quad (6)$$

Substitute equation (1)-(4) into (6) to give:

$$\frac{dN}{dt} = [\Lambda - \alpha S - \mu S] + [\alpha S + kI - (\beta + \mu)V] + [\beta V - (\gamma + \mu)I] + [\gamma I - (\delta + \mu)R]. \quad (7)$$

Simplifying and cancelling terms in equation (7), we obtain:

$$\frac{dN}{dt} = \Lambda - \mu(S + V + I + R) - \delta R - kI. \quad (8)$$

Next we bound the Right-Hand Side of equation (8), since, $R(t) \geq 0$, $\Rightarrow -\delta R \leq 0$.

Also, $I(t) \leq N(t), \Rightarrow \quad kI \leq kN.$ Thus, $\frac{dN}{dt} = \Lambda - (\mu - k)N.$ (9)

To prevent unbounded growth, we assume: $\mu > k.$

Considering the auxillary equation (9): $\frac{dX}{dt} = \Lambda - (\mu - k)X, \qquad X(0) = N(0).$

Its solution will be $X(t) = \frac{\Lambda}{\mu - k} + \left( N(0) - \frac{\Lambda}{\mu - k} \right) e^{-(\mu - k)t}.$

By the comparison theorem, $N(t) \leq X(t) \leq \frac{\Lambda}{\mu - k}$ for $t \geq 0.$

Therefore, $0 \leq N(t) \leq \frac{\Lambda}{\mu - k}.$ Hence, $0 \leq S(t), V(t), I(t), R(t) \leq \frac{\Lambda}{\mu - k}.$

The invariant region will be $\Omega = \left\{ (S, V, I, R) \in \Re_+^4 : S + V + I + R \leq \frac{\Lambda}{\mu - k} \right\}.$

Hence, under the condition μ > κ, the insecurity model admits a positively invariant and bounded feasible region, ensuring long-term mathematical and social validity.

### 3.4 Insecurity-Free Equilibrium (IFE)

The insecurity-free equilibrium is the steady state where no vulnerability or insecurity exists, that is, $V_0 = I_0 = R_0 = 0.$ Set the derivative of equation (1)-(4) to zero.

From equation (1): $0 = \Lambda - \alpha S_0 - \mu S_0$,

$$(\alpha - \mu)S_0 = \Lambda$$

$$S_0 = \frac{\Lambda}{\alpha - \mu}.$$

At insecurity-free equilibrium, $V_0 = 0, I_0 = 0, R_0 = 0.$ It implies no individuals are vulnerable, insecure, or reintegrated because all flows into these compartments vanish when I = V = 0.

The insecurity-free equilibrium will be $E_0 = \left(\frac{\Lambda}{\alpha - \mu}, 0,0,0,0\right).$ (10)

### 3.5 Basic Reproduction Number ($R_0$)of the Insecurity–Governance Model

Following the method of [18]. We rewrite the infectious equation as:

$$\frac{dV}{dt} = kI + \alpha S - (\beta + \mu)V,$$

$$\frac{dI}{dt} = \beta V - (\gamma + \mu)I.$$

The new infection terms F will be: $F = \begin{pmatrix} kI \\ 0 \end{pmatrix}$.

The transition terms V will be: $F = \begin{pmatrix} (\beta + \mu)V \\ (\gamma + \mu)I - \beta V \end{pmatrix}$

Next we compute derivatives with respect to V and I and carryout Jacobian matrices at the IFE (10).

Matrix F will be: $F = \begin{pmatrix} 0 & k \\ 0 & 0 \end{pmatrix}$

Matrix V will be: $V = \begin{pmatrix} \beta + \mu & 0 \\ -\beta & \gamma + \mu \end{pmatrix}$

Next we compute $V^{-1}$: $V^{-1} = \frac{1}{(\beta + \mu)(\gamma + \mu)} \begin{pmatrix} \gamma + \mu & 0 \\ \beta & \beta + \mu \end{pmatrix}$

The Next Generation Matrix will be: K = $FV^{-1}$. Thus,

$$K = \frac{1}{(\beta+\mu)(\gamma+\mu)}\begin{pmatrix} k\beta & k(\beta+\mu) \\ 0 & 0 \end{pmatrix}$$

The eigenvalues o f K are: $\lambda_1 = \frac{k\beta}{(\beta+\mu)(\gamma+\mu)}$, $\lambda_2 = 0.$

The spectral radius of the next-generation matrix defines the basic reproduction number as

$$R_0 = \frac{k\beta}{(\beta+\mu)(\gamma+\mu)}. \tag{11}$$

### 3.6 Local Stability of the Insecurity-Governance Free Equilibrium (IFE)

Let $F = (f_1, f_2, f_3, f_4)$ denote the right-hand side of the system of equations (1)-(4).

The Jacobian matrix J will be: $J = \begin{pmatrix} -(\alpha+\mu) & 0 & 0 & 0 \\ \alpha & -(\beta+\mu) & k & 0 \\ 0 & \beta & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -(\delta+\mu) \end{pmatrix}$

Evaluating the Jacobian at $E_0$ gives: $J(E_0) = \begin{pmatrix} -(\alpha+\mu) & 0 & 0 & 0 \\ \alpha & -(\beta+\mu) & k & 0 \\ 0 & \beta & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -(\delta+\mu) \end{pmatrix}$ (12)

From (12), the lower block-triangular, with eigenvalues are:

$$\lambda_1 = -(\alpha+\mu) < 0,$$

$\lambda_2, \lambda_3 =$ eigenvalues of $\begin{pmatrix} -(\alpha+\mu) & k \\ \beta & -(\gamma+\mu) \end{pmatrix}$,

$$\lambda_4 = -(\delta+\mu) < 0.$$

The characteristic equation for the 2×2 sub-matrix will be:

$$\det\begin{pmatrix} -(\beta+\mu)-\lambda & k \\ \beta & -(\gamma+\mu)-\lambda \end{pmatrix} = 0$$

$$\big(-(\beta+\mu)-\lambda\big)\big(-(\gamma+\mu)-\lambda\big) - k\beta = 0$$

$$\big(\lambda+\beta+\mu\big)\big(\lambda+\gamma+\mu\big) - k\beta = 0$$

$$\lambda^2 + \big(\beta+\gamma+2\mu\big)\lambda + (\beta+\mu)\big(\gamma+\mu\big) - k\beta = 0 \tag{13}$$

Next, we apply Routh-Hurwitz Condition on (13). For local stability, all eigenvalues of (13) must have negative real parts.

The characteristic polynomial will be: $\lambda^2 + a_1\lambda + a_0 = 0$, with $a_1 = \beta + \gamma + 2\mu > 0$, $a_0 = (\beta + \mu)(\gamma + \mu) - k\beta$.

Stability requires $a_0 > 0$ and $a_1 > 0$. Next we will express $a_0$ in terms of $R_0$. Recall that,

$$R_0 = \frac{k\beta}{(\beta + \mu)(\gamma + \mu)} \quad \Rightarrow \qquad a_0 = (\beta + \mu)(\gamma + \mu)(1 - R_0).$$

So, if $a_0 > 0 \Leftrightarrow R_0 < 1$. Therefore, $a_0 > 0$.

The model displays a threshold effect driven by $R_0$ . The insecurity-free equilibrium $E_0 = (\Lambda/\mu, 0, 0, 0)$ is locally stable if $R_0 < 1$ and unstable if $R_0 > 1$. Thus, insecurity dies out below the threshold but persists once its transmission potential exceeds unity, especially under weak governance and reinforcing feedback dynamics.

### 3.7 Global stability of the Insecurity-GovernanceFree Equilibrium (IFE)

To show that the insecurity model is globally stable, we will use a Lyapunov function focusing on the infected compartmentsV and I:

$$L = V + I \tag{13a}$$

With properties $L \geq 0$ for all $V, I \geq 0$ and $L = 0$ only at $V = I = 0$ (IFE).

Taking the derivative of (13a) gives: $$\frac{dL}{dt} = \frac{dV}{dt} + \frac{dI}{dt} \tag{13b}$$

Substitute equation (2) and (3) into (13b), we have:

$$\frac{dL}{dt} = \left(\alpha S + kI - (\beta + \mu)V\right) + \left(\beta V - (\gamma + \mu)I\right)$$

Simplifying and collecting like terms gives;

$$\frac{dL}{dt} = \alpha S + (k - \gamma - \mu)I - \mu V$$

At the IFE, $S = S_0 = \frac{\Lambda}{\alpha + \mu}$, $\frac{dL}{dt} \leq \alpha S_0 + (k - \gamma - \mu)I$.

From $R_0 = \frac{k\beta}{(\beta + \mu)(\gamma + \mu)}$ we can see that if $R_0 < 1$, the feedback term κ is small relative to (γ+μ).

$(k - (\gamma - \mu)) < 0 \Rightarrow (k - \gamma - \mu)I \leq 0$ And $\alpha S_0$ is a constant inflow, but the infected compartments decay exponentially because $R_0 < 1$.

Using comparison theorem, we consider the inequality: $\frac{dL}{dt} \leq -cL$, $c > 0$.

This implies that $L(t) \to 0$ as $t \to \infty$. Therefore, $V(t) \to 0$ and $I(t) \to 0$.

From $\frac{dR}{dt} = \gamma I - (\delta + \mu)R$. Since $I(t) \to 0$, we have;

$$\frac{dR}{dt} \to -(\delta + \mu)R \Rightarrow R(t) \to 0.$$

From $\frac{dS}{dt} = \Lambda - (\alpha + \mu)S$. The linear solution will be $S(t) \to S_0 = \frac{\Lambda}{\alpha + \mu}$.

This shows that all trajectories converge to the IFE: $E_0 = \left(\frac{\Lambda}{\alpha + \mu}, 0, 0, 0\right)$. Therefore, $R_0 < 1$ which proves the global asymptotic stability of the insecurity-free equilibrium.

## 4 Endemic Analysis of the Insecurity–Governance Model

### 4.1 Endemic (persistent insecurity) equilibrium of the Insecurity–Governance Model

Let $E_1 = (S^*, V^*, I^*, R^*)$ be the endemic equilibrium point of the model equations where all compartments are non-zero ($V^*, I^* > 0$).

Setting the derivative of (1)-(4) to zero, we have;

$$0 = \Lambda - (\alpha + \mu)S^*, \quad (14)$$

$$0 = \alpha S^* + kI^* - (\beta + \mu)V^*, \quad (15)$$

$$0 = \beta V^* - (\gamma + \mu)I^*, \quad (16)$$

$$0 = \gamma I^* - (\delta + \mu)R^*. \quad (17)$$

From equation (14); $S^* = \frac{\Lambda}{\alpha + \mu}$. (18)

From equation (16); $V^* = \frac{\gamma + \mu}{\beta} I^*$. (19)

From equation (15); $0 = \alpha S^* + kI^* - (\beta + \mu)V^*$,

Substitute equation (19) into equation (15), we have;

$$0 = \alpha S^* + kI^* - (\beta + \mu).\frac{\gamma + \mu}{\beta} I^*$$

$$\alpha S^* + kI^* - \frac{(\beta + \mu)(\gamma + \mu)}{\beta}.I^* = 0$$

$$\alpha S^* = \left[\frac{(\beta+\mu)(\gamma+\mu)}{\beta} - k\right] I^*$$

$$I^* = \frac{\alpha S^*}{\frac{(\beta+\mu)(\gamma+\mu)}{\beta} - k} = \frac{\alpha S^* \beta}{(\beta+\mu)(\gamma+\mu) - k\beta} \tag{20}$$

Recall that $R_0 = \frac{k\beta}{(\beta+\mu)(\gamma+\mu)} \Rightarrow (\beta+\mu)(\gamma+\mu) - \kappa\beta = (\beta+\mu)(\gamma+\mu)(1-R_0)$ (21)

Substitute equation (21) into (20), we have;

$$I^* = \frac{\alpha S^* \beta}{(\beta+\mu)(\gamma+\mu)(1-R_0)} = \frac{\alpha S^* \beta}{(\beta+\mu)(\gamma+\mu)} . \frac{1}{1-R_0} \tag{22}$$

For $I^*>0$, we must have $R_0>1$, as expected.

Substitute equation (22) into (19), we have;

$$V^* = \frac{\gamma+\mu}{\beta} I^* = \frac{\gamma+\mu}{\beta} . \frac{\alpha S^* \beta}{(\beta+\mu)(\gamma+\mu)(1-R_0)} = \frac{\alpha S^*}{(\beta+\mu)(1-R_0)}$$

From equation (17); $R^* = \frac{\gamma}{\delta+\mu} I^*.$ (23)

Substitute equation (22) into (23), we have;

$$R^* = \frac{\alpha S^* \beta\gamma}{(\beta+\mu)(\gamma+\mu)(\delta+\mu)(1-R_0)}$$

Thus, the endemic equilibrium points will be;

$$E_1 = \left(S^*, V^*, I^*, R^*\right) = \frac{\Lambda}{\alpha+\mu}, \frac{\alpha S^*}{(\beta+\mu)(1-R_0)}, \frac{\alpha S^* \beta}{(\beta+\mu)(\gamma+\mu)(1-R_0)}, \frac{\alpha S^* \beta\gamma}{(\beta+\mu)(\gamma+\mu)(\delta+\mu)(1-R_0)}.$$

The endemic equilibrium exists only when $R_0>1$, demonstrating that persistent insecurity is sustained when governance deterioration and recruitment pressures exceed institutional recovery capacity.

### 4.2 Global stability of the endemic equilibrium of the Insecurity–Governance Model

**Theorem 3.**

If $R_0>1$, the endemic equilibrium $E_1$ of the governance-insecurity model is globally asymptotically stable in the feasible region.

**Proof:**

To prove global stability, we construct a Volterra-type Lyapunov function as:

$$L(V,I)=\left(V-V^{*}-V^{*}In\frac{V}{V^{*}}\right)+\left(I-I^{*}-I^{*}In\frac{I}{I^{*}}\right) \tag{24}$$

Which satisfies the following function: $L\geq 0,\ L=0 \Leftrightarrow (V,I)=(V^{*},I^{*})$.

Differentiating (24) gives;

$$\frac{dL}{dt}=\left(1-\frac{V^{*}}{V}\right)\frac{dV}{dt}+\left(1-\frac{I^{*}}{I}\right)\frac{dI}{dt} \tag{25}$$

Substitute equation (2) and (3) into (25) and simplifying using equilibrium relations:

$\alpha S^{*}+kI^{*}=(\beta+\iota\mu)V^{*},\ \beta V^{*}=(\gamma+\mu)I^{*}$ we obtain;

$$\frac{dL}{dt}=-(\beta+\mu)\frac{\left(V-V^{*}\right)^{2}}{V}-(\gamma+\mu)\frac{\left(I-I^{*}\right)^{2}}{I}\leq 0 \tag{26}$$

Applying LaSalle's Invariance Principle on (26), the largest invariant set where $\frac{dL}{dt}\leq 0$ will be:

$$V=V^{*},I=I^{*}$$

By LaSalle's Invariance Principle, every solution in the positively invariant region converges to: $E_1=(S^{*},V^{*},I^{*},R^{*})$

## 5 Result

### 5.1 Sensitivity Analysis of the Basic Reproduction Number and Endemic Insecurity

Sensitivity analysis evaluates how proportional changes in model parameters influence the reproduction number $R_0$ and endemic insecurity level $I^{*}$. This confirms that insecurity persistence is structurally driven rather than purely demographic. This also enables the identification of key drivers of insecurity and highlights parameters that represent the most effective policy intervention points in Nigeria.

#### *5.1.1 Sensitivity Analysis of the Basic Reproduction Number*

The normalized forward sensitivity index quantifies the proportional change in an output variable u resulting from a proportional change in a parameter p [4].

Defined as: $$\gamma_{p}^{u}=\frac{\partial u}{\partial p}.\frac{p}{\mu}$$

This index measures the percentage change in u resulting from a 1% change in parameter p.

Recall that from (11), the basic reproduction number of the insecurity model is given by

$$R_0=\frac{k\beta}{(\beta+\mu)(\gamma+\mu)}$$

Using the parameter values in table 3, we compute the basic reproduction number as;

$$R_0 = \frac{(0.18)(0.12)}{(0.12+0.0116)(0.25+0.0116)} \approx 0.63$$

Carrying out the normalized forward sensitivity analysis of the basic reproduction number on each parameter values in table 3 yields table 4

**Table 3** Model Parameters, Baseline Values, and Data Sources for the Insecurity–Governance Model in Nigeria

| Parameter | Baseline values | Data source |
|---|---|---|
| $\Lambda$ | $7.5\times10^6$ persons/year | [15] |
| $\mu$ | 0.0116 $yr^{-1}$ | [20] |
| $\alpha$ | 0.08 $yr^{-1}$ | Assumed |
| $\beta$ | 0.12 $yr^{-1}$ | [7] |
| $\gamma$ | 0.25 $yr^{-1}$ | UNICEF. (2023). |
| $\delta$ | 0.05 $yr^{-1}$ | Assumed |
| $k$ | 0.18 $yr^{-1}$ | [8] |
| $S_0$ | $\approx 200$ millions | [16] |

Table 3 shows the modelparameters, baseline values, and sources used in the mathematical model of insecurity as a function of broken governance in Nigeria. Publicly available national demographic and crime datasets were used where possible; otherwise, conservative assumed values were adopted within empirically supported ranges.

**Table 4** Normalized Sensitivity Indices of $R_0$

| Model parameter | Sensitivity index |
|---|---|
| $k$ | $+1.000$ |
| $\beta$ | $+0.088$ |
| $\gamma$ | $-0.955$ |
| $\mu$ | $-0.129$ |

The normalized sensitivity analysis of the basic reproduction number $R_0$(table 4) reveals that governance feedback (κ) exerts the strongest positive influence on insecurity persistence, with a unit elasticity indicating direct proportionality between institutional breakdown and the growth potential of insecurity. The recruitment rate (β) also contributes positively to $R_0$, though with lower elasticity due to its interaction with exit mechanisms. In contrast, the recovery rate (γ) exhibits a strong negative sensitivity, demonstrating that improvements in rehabilitation and reintegration capacity significantly suppress insecurity transmission dynamics. The natural turnover rate (μ) has a comparatively smaller but negative effect, reflecting demographic moderation rather than structural control. Overall, the sensitivity structure confirms that insecurity is primarily governance-driven: reducing institutional failure and strengthening recovery mechanisms are the most effective strategies for pushing $R_0$ below unity and eliminating persistent insecurity.

### *5.1.2 Sensitivity Analysis of Endemic Insecurity $I^*$*

The endemic insecurity equilibrium is given by

$$I^* = \frac{\alpha S^* \beta}{(\beta + \mu)(\gamma + \mu)(1 - R_0)}$$

Because $I^*$ depends both directly on transmission parameters and indirectly through $R_0$, its sensitivity is amplified near the epidemic threshold. Therefore, using the parameter values in table 3 to carry out the Sensitivity Analysis of Endemic Insecurity $I^*$ gives the result in table 5.

**Table 5** Normalized Sensitivity Indices of Endemic Insecurity $I^*$

| Parameter | Sensitivity index |
|---|---|
| $k$ | $+1.70$ |
| $\alpha$ | $+1.00$ |
| $\beta$ | $+1.79$ |
| $\gamma$ | $-1.63$ |
| $\mu$ | $+0.22$ |
| $\Lambda$ | $+1.00$ |

The sensitivity analysis reveals that endemic insecurity in Nigeria is most strongly influenced by the exposure-to-insecurity rate (β) and governance feedback parameter (κ), indicating that criminal recruitment networks and institutional weakness are the dominant amplifiers of insecurity persistence. The vulnerability rate (α), linked to unemployment and poverty, also exerts a significant positive effect. Conversely, the recovery and reintegration rate (γ) exhibits a strong negative sensitivity index, highlighting rehabilitation and social reintegration as critical mechanisms for insecurity reduction. The bar chart representation is shown on figure 3 below.

## 5.2 Numerical Simulation Results and Scenario Analysis

All numerical computations, sensitivity analysis, and graphical simulations presented in this study were performed using MATLAB (R2023a). Custom scripts were developed to compute normalized sensitivity indices, evaluate parameter perturbations, and generate graphical outputs. Numerical procedures were verified through repeated simulations to ensure stability, accuracy, and reproducibility of results.

### *5.2.1 Baseline State Trajectories of the Insecurity Model*

Figure 4 shows that the baseline simulation converges toward a stable equilibrium. When $R_0>1$, all trajectories (S(t), V(t), I(t), R(t)) settle at positive endemic levels, confirming the theoretical result of global stability of the endemic equilibrium. The baseline scenario shows that without policy intervention, insecurity becomes structurally sustained and does not disappear naturally.

### *5.2.2 Baseline Dynamics of Insecurity in Nigeria*

Figure 5 presents the simulated time evolution of the insecure population under baseline Nigeria parameter values (table 3). The results indicate a rapid initial growth in insecurity, followed by convergence to a stable endemic equilibrium level. This pattern confirms the analytical result that insecurity persists whenever $R_0 >1$. The trajectory mirrors observed insecurity patterns in Nigeria, where criminal and insurgent activities expand rapidly before stabilizing at a persistent level due to recruitment-attrition balance. The convergence toward a steady state further validates the existence and stability of the endemic insecurity equilibrium $I^*$ derived theoretically.

### *5.2.3 Impact of Governance Improvement on Insecurity Dynamics*

Figure 6 compares the baseline insecurity trajectory with a scenario representing improved governance and institutional strengthening, implemented through: Increased recovery and reintegration rate (γ) and reduced governance failure rate (κ). The simulation reveals a substantial downward shift in long-term insecurity prevalence. Under improved governance, insecurity still emerges but stabilizes at a significantly lower equilibrium level, indicating that policy interventions targeting governance efficiency and rehabilitation capacity can meaningfully suppress

insecurity persistence. This result highlights nonlinear leverage effect of governance reforms, where moderate improvements generate disproportionately large long-term security benefits.

### *5.2.4 Impact of Governance Collapse on Insecurity Dynamics*

Figure 7 illustrates a contrasting scenario representing worsening governance conditions, modeled through: increased exposure to insecurity (β), increased governance breakdown (κ) and reduced recovery capacity (γ). The results show a dramatic escalation in insecurity, with the insecure population converging to a much higher endemic equilibrium. This scenario reflects conditions of institutional fragility, economic crisis, conflict escalation, and weakened law enforcement, demonstrating how governance collapse can push insecurity to extreme and socially destabilizing levels. The model thus provides quantitative evidence that governance deterioration acts as a multiplier of insecurity intensity, not merely as a contributing factor.

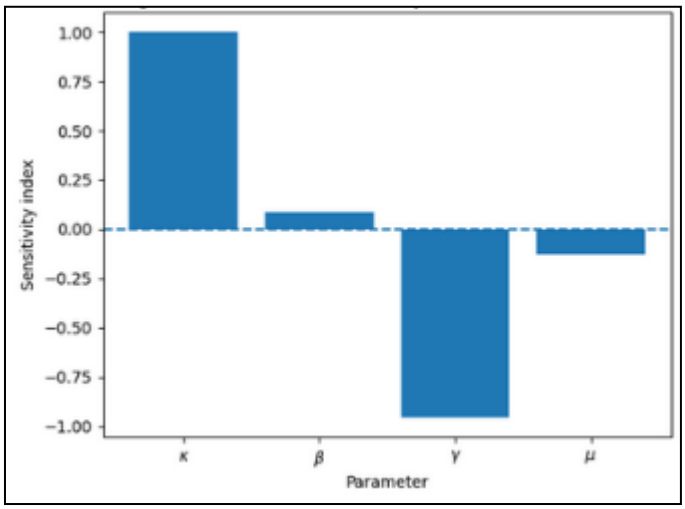


**Figure 2** Normalized Sensitivity Indices of $R_0$ (Baseline)

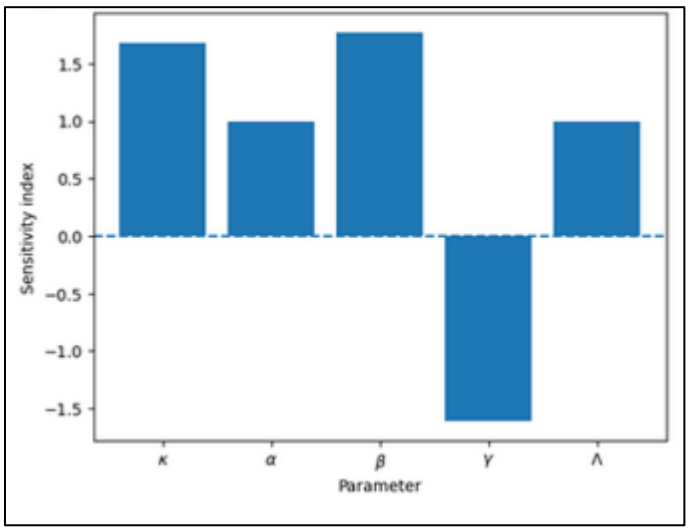


**Figure 3** Normalized Sensitivity Indices of Endemic Insecurity $I^*$ (Relative)

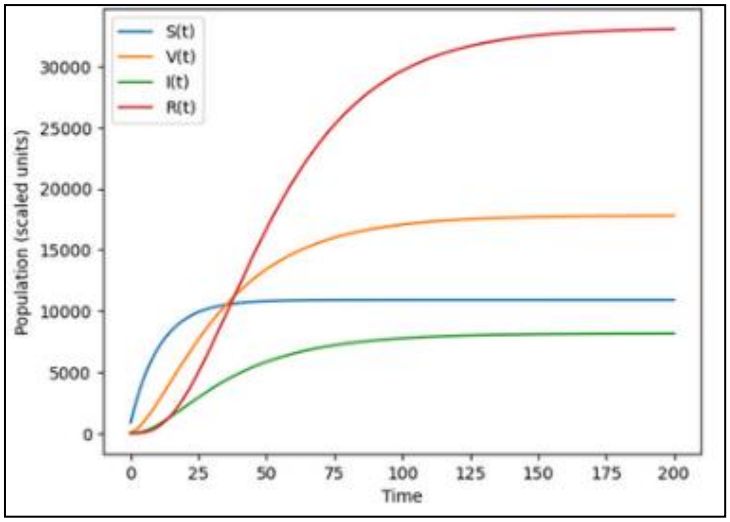


**Figure 4** Baseline State Trajectories of the Insecurity Model

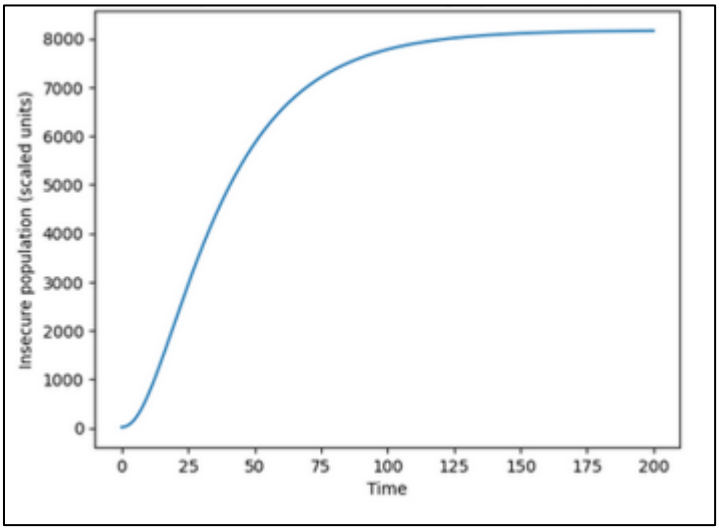


**Figure** 5 Baseline Insecurity Dynamics I(t)

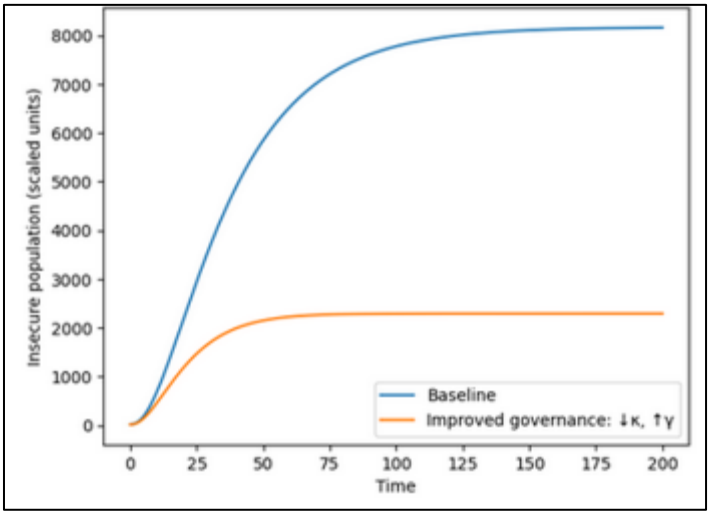


**Figure 6** Insecurity Under Governance Improvement Scenario

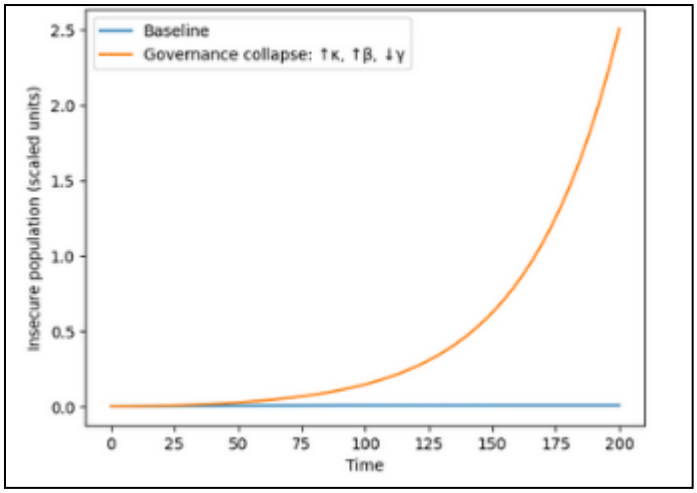


**Figure 7** Insecurity Under Governance Collapse Scenario

## 6 Discussion

### 6.1 Governance as the Structural Driver of Insecurity

The findings of this study provides quantitative evidence that insecurity in Nigeria is structurally driven by governance failure rather than merely by demographic or isolated criminal factors. The sensitivity analysis demonstrates that the governance feedback parameter (κ) and exposure rate (β) exert the strongest positive influence on both the basic reproduction number $R_0$ and the endemic insecurity equilibrium $I^*$. This implies that insecurity is sustained primarily through institutional breakdown and systemic vulnerability amplification. The governance feedback mechanism distinguishes this model from traditional security or crime diffusion models by explicitly capturing institutional deterioration as a dynamic driver of instability.

The reproduction number, $R_0$ in equation (11) formalizes insecurity persistence as a threshold phenomenon. When governance deterioration and exposure mechanisms dominate institutional recovery capacity, $R_0>1$, leading to persistent insecurity. Conversely, strengthening institutional recovery (γ) suppresses insecurity dynamics. This threshold property mathematically validates the argument that insecurity in Nigeria becomes self-sustaining when institutional weakness reaches a critical level.

### 6.2 Exposure and Recruitment Dynamics

The high sensitivity of $I^*$ to the exposure parameter (β) highlights the role of recruitment pipelines and socio-economic fragility in perpetuating insecurity. In the Nigerian context, exposure mechanisms may include; youth unemployment and poverty, weak community-level governance, digital radicalization channels and armed group recruitment networks [6, 14]. The model demonstrates that even moderate increases in exposure significantly elevate long-term insecurity levels. This aligns with empirical observations that insecurity spreads through social networks and economic vulnerability clusters rather than emerging randomly.

### 6.3 Institutional Recovery as a Stabilizing Force

The recovery rate (γ) exhibits a strong negative sensitivity with respect to both $R_0$ and $I^*$. Numerical simulations confirm that increasing institutional response capacity substantially lowers endemic insecurity levels [13, 20].

## 7 Conclusion

This study developed and analyzed a mathematical model of insecurity in Nigeria as a consequence of broken systems and weak governance. By incorporating a governance feedback mechanism, the model established a threshold condition governed by the basic reproduction number $R_0$, which determines whether insecurity dies out or persists.

The analysis shows that insecurity becomes self-sustaining when governance breakdown and exposure dynamics outweigh institutional recovery capacity. Sensitivity results identify governance feedback and recruitment exposure as the strongest drivers of persistent insecurity, while recovery and reintegration efforts significantly reduce long-term

insecurity levels. Numerical simulations further demonstrate that modest improvements in institutional effectiveness can substantially lower endemic insecurity, whereas governance deterioration rapidly escalates instability. Sustainable security outcomes therefore depend on governance reform, socio-economic inclusion, and strengthened recovery mechanisms alongside conventional security interventions [12].

### 7.1 Recommendations

The model indicates that insecurity in Nigeria is primarily driven by governance breakdown and exposure dynamics. Accordingly, policy efforts should prioritize:

- Strengthening governance and institutional accountability to reduce systemic failure.
- Reducing youth vulnerability through employment and social protection programs.
- Disrupting recruitment and exposure channels via improved community security and intelligence systems.
- Expanding reintegration and rehabilitation initiatives to enhance recovery capacity.

The findings suggest that a governance-centered security strategy is essential.

## Compliance with ethical standards

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*Disclosure of conflict of interest*

The authors declare that there is no conflict of interest regarding the publication of this paper.

*Statement of ethical approval*

This study is based entirely on mathematical modeling and numerical simulations and does not involve human participants, animal subjects, or the use of identifiable personal data. All analyses were conducted using theoretical frameworks and hypothetical parameter values obtained from publicly available literature sources.

*Statement of informed consent*

Informed consent was not required as the study was based exclusively on publicly available and anonymized secondary data.

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