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# Approximate Analytical Solution of an Extended COVID-19 Model Incorporating Two-Dose Vaccination and Physical Control Measures

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## Abstract

The COVID-19 pandemic, which severely disrupted the global economy, remains a vital area of study for effective preparedness against future epidemics. The emergence of different variants has led to successive waves of the disease, prompting the development of numerous mathematical models. This study investigates an extended COVID-19 model that incorporates both first and second doses of vaccination as control strategies, alongside previously established physical preventive measures. The model is demonstrated to be mathematically and epidemiologically well-posed, with the existence and uniqueness of solutions to the state system established prior to analysis. Using the Next-Generation Matrix method, the control reproduction number was derived. Analytical results indicate that the disease-free equilibrium is locally and globally asymptotically stable when the control reproduction number, RcR_cRc, is less than one, and unstable when it exceeds one. Sensitivity analysis was conducted to determine the influence of key parameters on Rc. Findings highlight that improving compliance with hand sanitizing, social distancing, mask usage, testing, isolation, and vaccination significantly aids disease control. Conversely, reducing the rate of contact with exposed individuals, infectiousness development, and transmission probabilities also contributes to containment. Numerical simulations further illustrate the impact of these control measures, emphasizing the effectiveness of vaccination and adherence to physical protocols. The study recommends promoting vaccination and reinforcing compliance with physical preventive measures to mitigate the spread of COVID-19.

**Keywords:** Dual Dose Vaccination; Mathematical modeling; Physical Control Measures; Disease Dynamics; Covid-19; Approximate Analytical Solution

## 1. Introduction

The 2019–20 coronavirus pandemic was caused by the infectious disease corona virus (COVID-19), which was initially discovered in Wuhan, the Chinese capital, in 2019 [48, 38]. On December 31, 2019, it was first reported to the World Health Organization [48].Fever, a dry cough, and breathing problems are the most typical symptoms of Covid-19, while muscle pain, sputum production, diarrhea, and sore throat are less typical [38].Since the discovery of this virus, numerous investigations and studies, including mathematical models to comprehend the origin and transmission of the virus, have been conducted.

Nigeria is one of the 210 countries affected globally. The first case was confirmed in Lagos State on 27 February 2020. This index case was a 44-year old man, an Italian citizen who returned from Milan, Italy, on 24 February and presented at a health facility on 26 February 2020 [38]. As of 3 May 2020, 2,558 cases have been reported in the country across 35 states and the Federal Capital Territory (FCT). Of these numbers, 1,767 (69 %) are male, the age-group 21 – 30 years

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were the most affected (23%), 210 (8%) had international travel history; 400 (15.6%) cases have been discharged, and 87 deaths were recorded, bringing the case fatality rate (CFR) of confirmed cases to 3.4%, with a range from 0- 15.2% by region. Prior to report of the COVID-19 outbreak in Africa, the WHO identified a strong link between the continent and China and has sent out guidelines on preparedness for the outbreak. Nigeria is one of the thirteen top countries identified as high risk for COVID-19 importation based on either direct link or high travel volume to and from China. The WHO also advised that countries develop capacity to promptly detect cases that will enable them to contain the outbreak early so that the health system is not overwhelmed [38, 48].It is worth noting that as of April 13, 2024, the corona virus tracker is no longer being updated due to the unfeasibility of providing statistically valid global totals, as some countries have now stopped reporting. However, historical data remain accessible. As at last updated in April 2024 Nigeria has the following records: Reported cases: 267,188. Deaths: 3155. Recovered: 259,953[35. 38,48].

Covid-19 is still present in Nigeria. As of May 29, 2025, Nigerian recorded 1,565 new covid-19 cases, bringing the total to 95,934confirmed cases. Lagos State has been significantly affected, with 807 new cases reported on the same date [35. 38,48]. The [38] continues to monitor the situation, reporting six Covid-19 deaths in the last 24 hours.

## 2. Material and methods

In this work, we present a deterministic mathematical model with ten (10) human compartments made up of Vaccination $V_1$, Vaccination $V_2$, Susceptible humans, $S$, Exposed humans, $E$, Quarantined Humans, $Q$, Undetected Asymptomatic Infectious Humans, Undetected symptomatic infectious humans, $I$, Undetected symptomatic infectious humans under self-medication, $M$, Detected and hospitalized infectious humans (via testing), $I_H$ and Recovered humans, $R$. The model is set up to show the effects of two doses of vaccination on the health burden of the disease.

The following assumptions are made:

- Demographic features such as natural birth and death rates are incorporated into the model as can be seen in most recent works such as [34, 36, 40, 41,45].
- A proportion of the susceptible humans recruited into the system is taken to be vaccinated and denoted by $a$.
- The susceptible humans take the first dose of covid-19 at the rate $\kappa_1$ while those with the initial vaccination receives the second dose at the rate $\kappa_2$.
- Those with second dose vaccination that receives covid-19 booster are represented by the proportion, $v$. They are migrated to the recovered class since they can not get infected for a long while [41, 49].

**Table 1** Parameters and interpretation

| Parameters | Values |
|---|---|
| $\Lambda$ | Rate of recruitment of humans |
| $a_1$ | Proportion of recruited humans that receive first dose of vaccination |
| $a_2$ | Proportion of recruited humans that receive second dose of vaccination |
| $\delta_N$ | Natural mortality rate |
| $\delta_I$ | Disease-induced death rate of undetected symptomatic infectious humans |
| $\delta_H$ | Disease-induced death rate of hospitalized detected infectious humans |
| $\delta_M$ | Disease-induced death rate of self-medicated humans |
| $\mu$ | Quarantined humans who do not develop symptoms and are not infected that progressed to susceptible class again |
| $\alpha$ | Exposed humans that are quarantined (via contact tracing) |
| $\beta$ | Effective contact rate |
| $\varepsilon_A$ | Recovery rate of undetected asymptomatic infectious humans due to strong immune system |
| $\varepsilon_I$ | Recovery rate of undetected symptomatic infectious humans due to strong immune system |
| $\varepsilon_M$ | Recovery rate of humans under self-medication |

| | |
|---|---|
| $\nu$ | Rate at which humans receive vaccine boosters |
| $\kappa_1$ | Rate at which susceptible humans receive first vaccination |
| $\kappa_2$ | Rate at which humans with first vaccination receive second vaccination |
| $\psi$ | Modification parameter that accounts for a reduced transmission from $V_1$ |
| $\sigma$ | Progression rate from exposed state to infectious state |
| $\gamma$ | Recovery rate of detected and hospitalized infectious humans due to treatment |
| $k$ | Fraction of new infectious humans that is asymptomatic |
| $\eta$ | Progression rate from quarantined class to hospitalize detected infectious humans |
| $\omega$ | Detection rate (via testing) for the undetected asymptomatic infectious class |
| $q$ | Transition rate from undetected symptomatic infectious class to $I_H$ |
| $\tau_1$ | Rate of compliance to social distancing |
| $\tau_2$ | Fraction of undetected symptomatic infectious humans that adhered strictly to COVID-19 safety protocols and avoided self-medication |
| $\phi$ | Sensitization rate on the danger of self-medication |
| $\theta$ | Progression rate from $M$ class to $I_H$ class due to severity of COVID-19 in humans under self-medication |
| $\rho_1$ | Rate of compliance to wearing of Face mask |
| $\rho_2$ | Rate of compliance to the use of hand sanitizer |
| $c_1$ | Modification parameter that accounts for a reduced transmission from A class |
| $c_2$ | Modification parameter that accounts for increased transmission from M class |

Our model (1) is an extension of models that have been formulated towards gaining insights into how the novel coronavirus disease is transmitted from one person to another [10] our work presents a major preventive strategy. We introduced two vaccination compartments accounting for humans with first and second dose vaccination. These was not in-cooperated in the existing literature [10] , in their work they did not capture these compartments.

2. Demographic features such as natural birth and death rates are incorporated into the model of [10] as can be seen in most recent works such as [40, 41, 45,47] . In [10] these features were neglected considering that covid-19 was still knew and might not have been influenced much by natural and death rate but after five years of the epidemic, it is quite appropriate to include these features as demonstrated by many works in literature. We incorporated it in the present work. We reformulated our model (1) by introducing the preventive strategy to adequately capture the dynamics of the transmission and preventive measures to curb the transmission and help Nigerian health policy makers to put under control its spread represented by parameters, $V_1$ and $V_2$ where $\Lambda(1-\alpha)$ stands for recruited humans that receive first dose of vaccination.

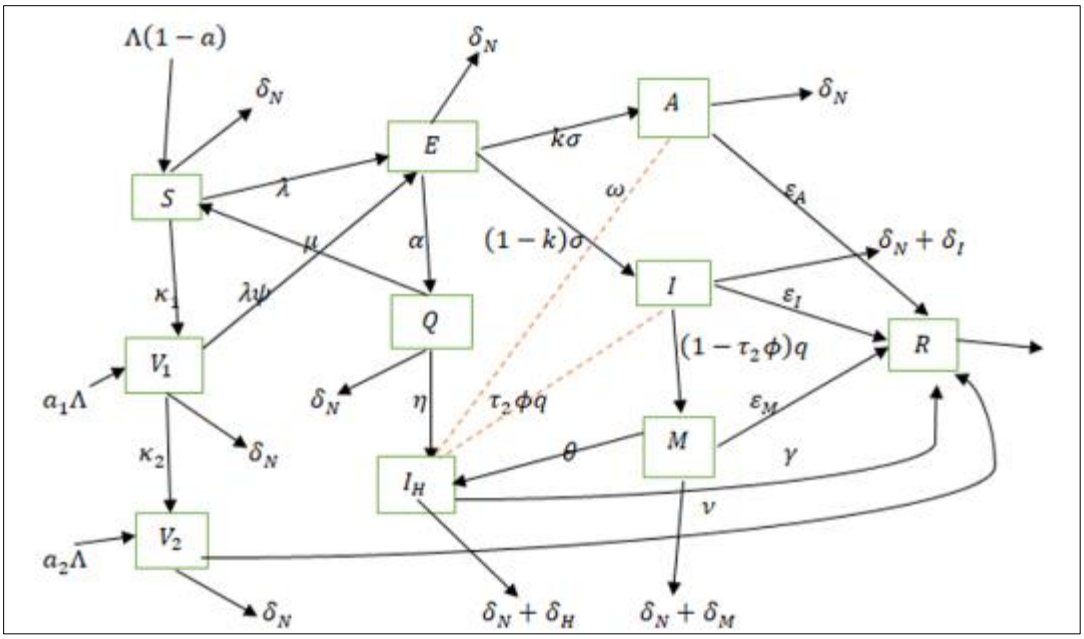


**Figure 1** Disease transmission flow diagram

Thus, the model becomes;

$$\frac{dS}{dt} = \Lambda(1-a) - (\lambda + \delta_N + \kappa_1)S + \mu Q,$$

$$\frac{dV_1}{dt} = a_1\Lambda + \kappa_1 S - (\lambda\psi + \kappa_2 + \delta_N)V_1,$$

$$\frac{dV_2}{dt} = a_2\Lambda + \kappa_2 V_1 - (\delta_N + \nu)V_2,$$

$$\frac{dE}{dt} = \lambda(S + \psi V_1) - (\alpha + \sigma + \delta_N)E,$$

$$\frac{dQ}{dt} = \alpha E - (\eta + \mu + \delta_N)Q,$$

$$\frac{dA}{dt} = k\sigma E - (\omega + \varepsilon_A + \delta_N)A, \qquad \text{.....(1)}$$

$$\frac{dI}{dt} = (1-k)\sigma E - (q + \delta_N + \delta_I + \varepsilon_I)I,$$

$$\frac{dM}{dt} = (1-\tau_2\phi)qI - (\theta + \delta_N + \delta_M + \varepsilon_M)M,$$

$$\frac{dI_H}{dt} = \eta Q + \omega A + \tau_2\phi qI + \theta M - (\gamma + \delta_N + \delta_H)I_H,$$

$$\frac{dR}{dt} = \varepsilon_A A + \varepsilon_I I + \varepsilon_M M + \gamma I_H + \nu V_2 - \delta_N R$$

Where

$$a = a_1 + a_2 \text{ and } \lambda = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(c_1A + I + c_2M)}{N}.$$

The total human population is given by:

$$N = S + V_1 + V_2 + E + Q + A + I + M + I_H + R.$$

## 3. Results and discussion

Let $||.||$ be the maximum norm in $\Gamma \in \mathcal{R}_+^{10}$ taking to be the banach domain for continuous functions where

$$\big||Y(t)|\big| = \sum \big||Y|\big|_\infty.$$

Let

$$\big||S|\big| \le k_1, \big||V_1|\big| \le k_2, \big||V_2|\big| \le k_3, \big||E|\big| \le k_4, \big||Q|\big| \le k_5, \big||A|\big| \le k_6, \big||I|\big| \le k_7, \big||M|\big| \le k_8, \big||I_H|\big| \le k_9, \big||R|\big| \le k_{10} \; and \; 0 \le w_{\{i=1,\dots,10\}} < 1.$$

From the system (1), we will have that for any $S_1$ and $S_2 \in \Gamma$, then

$$\begin{aligned}\big||f(t,S_1) - f(t,S_2)|\big| &= \big||\Lambda(1-a) - (\lambda + \delta_N + \kappa_1)S_1 + \mu Q + \pi R - (\Lambda(1-c) - (\lambda + \delta_N + \kappa_1)S_2 + \mu Q + \pi R)|\big| \\ &= \big|(\lambda + \delta_N + \kappa_1)(S_1 - S_2)|\big| \le (\lambda + \delta_N + \kappa_1)\big||S_1 - S_2|\big| \\ &\le w_1||S_1 - S_2||\end{aligned}$$

The Lipschitz continuity in $S$ is established with $w_1$ as the Lipschitz constant. Similarly, we can establish the Lipschitz continuity in other state variables as follows;

$$\begin{aligned}\big||f(t,V_{11}) - f(t,V_{12})|\big| &= \|a_1\Lambda + \kappa_1 S - (\lambda\psi + \kappa_2 + \delta_N)V_{11} - (a_1\Lambda + \kappa_1 S - (\lambda\psi + \kappa_2 + \delta_N)V_{12})\| \\ &= \big|(\lambda\psi + \kappa_2 + \delta_N)(V_{11} - V_{12})|\big| \le (\lambda\psi + \kappa_2 + \delta_N)\big||V_{11} - V_{12}|\big| \le w_2\|V_{11} - V_{12}\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,V_{21}) - f(t,V_{22})|\big| &= \|a_2\Lambda + \kappa_2 V_1 - (\delta_N + \nu)V_{21} - (a_2\Lambda + \kappa_2 V_1 - (\delta_N + \nu)V_{21})\| = \big|(\delta_N + \nu)(V_{21} - V_{22})|\big| \\ &\le (\delta_N + \nu)\big||V_{21} - V_{22}|\big| \le w_3\|V_{21} - V_{22}\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,E_1) - f(t,E_2)|\big| &= ||\lambda(S + \psi V_1) - (\alpha + \sigma + \delta_N)E_1 - (\lambda(S + \psi V_1) - \\ &(\alpha + \sigma + \delta_N)E_2)|| = \|(\alpha + \sigma + \delta_N)(E_1 - E_2)\| \le (\alpha + \sigma + \delta_N)\|E_1 - E_2\| \\ &\le w_4\|E_1 - E_2\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,Q_1) - f(t,Q_2)|\big| &= \|\alpha E - (\eta + \mu + \delta_N)Q_1 - (\alpha E - (\eta + \mu + \delta_N)Q_2)\| \\ &= \|(\eta + \mu + \delta_N)(Q_1 - Q_2)\| \le (\eta + \mu + \delta_N)\|(Q_1 - Q_2)\| \le w_5\|(Q_1 - Q_2)\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,A_1) - f(t,A_2)|\big| &= \|k\sigma E - (\omega + \varepsilon_A + \delta_N)A_1 - (k\sigma E - (\omega + \varepsilon_A + \delta_N)A_1)\| \\ &= \|(\omega + \varepsilon_A + \delta_N)(A_1 - A_2)\| \le (\omega + \varepsilon_A + \delta_N)\|(A_1 - A_2)\| \le w_6\|(A_1 - A_2)\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,I_1) - f(t,I_2)|\big| &= \big\|(1-k)\sigma E - (q + \delta_N + \delta_I + \varepsilon_I)I_1 - \big((1-k)\sigma E - (q + \delta_N + \delta_I + \varepsilon_I)I_2\big)\big\| \\ &= \|(q + \delta_N + \delta_I + \varepsilon_I)(I_1 - I_2)\| \le (q + \delta_N + \delta_I + \varepsilon_I)\|(I_1 - I_2)\| \le w_7\|(I_1 - I_2)\|.\end{aligned}$$

$$\begin{aligned}\big||f(t,M_1) - f(t,M_2)|\big| &= ||(1 - \tau_2\phi)qI - (\theta + \delta_N + \delta_M + \varepsilon_M)M_1 - ((1 - \tau_2\phi)qI \\ &-(\theta + \delta_N + \delta_M + \varepsilon_M)M_2|| = ||(\theta + \delta_N + \delta_M + \varepsilon_M)(M_1 - M_2)|| \\ &\le (\theta + \delta_N + \delta_M + \varepsilon_M)\|M_1 - M_2\| \le w_8\|M_1 - M_2\|.\end{aligned}$$

$$\big||f(t,I_{H1}) - f(t,I_{H2})|\big| = ||\eta Q + \omega A + \tau_2\phi qI + \theta M - (\gamma + \delta_N + \delta_H)I_{H1} - (\eta Q + \omega A$$

$$+\tau_2\phi qI + \theta M - (\gamma + \delta_N + \delta_H)I_{H2})|| = \|(\gamma + \delta_N + \delta_H)(I_{H1} - I_{H2})\|$$

$$\leq (\gamma + \delta_N + \delta_H)\|I_{H1} - I_{H2}\| \leq w_9\|I_{H1} - I_{H2}\|.$$

$$\left||f(t,R_1) - f(t,R_2)\right|| = ||\varepsilon_A A + \varepsilon_I I + \varepsilon_M M + \gamma I_H + \nu V_2 - (\delta_N + \pi)R_1 - (\varepsilon_A A + \varepsilon_I I$$

$$+\varepsilon_M M + \gamma I_H + \nu V_2 - (\delta_N + \pi)R_2)|| = \|(\delta_N + \pi)(R_1 - R_2)\| \leq (\delta_N + \pi)\|R_1 - R_2\|$$

$$w_{10}\|R_1 - R_2\|.$$

Where

$$w_1 = \lambda + \delta_N + \kappa_1, w_2 = \lambda\psi + \kappa_2 + \delta_N, w_3 = \delta_N + \nu, w_4 = \alpha + \sigma + \delta_N, w_5 = \eta + \mu + \delta_N, w_6 = \omega + \varepsilon_A + \delta_N, w_7 = q + \delta_N + \delta_I + \varepsilon_I, w_8 = \theta + \delta_N + \delta_M + \varepsilon_M, w_9 = \gamma + \delta_N + \delta_H, w_{10} = \delta_N + \pi.$$

Lipschitz continuity has been established for all the state solutions. Also, all the $w_i's$ are guaranteed to be less than one since they represent fractional outflow from the compartments and their sum must be less than one [1, 3,24]. Hence, by Banach fixed point theorem, the solution to the system exists and is unique.

### 3.1. Disease-Free Equilibrium of The System

The disease-free equilibrium of the model system is the steady state solution to (1) when there is no covid-19 infection in the population. That is, it is the solution to (1) when $E = Q = A = I = M = I_H = 0$ [4, 5, 6, 8, 9, 32,40]. Let the disease-free equilibrium be denoted $\mathcal{E}^0$, then $\mathcal{E}_0$ has the form $\mathcal{E}^0 = (S^0, V_1^0, V_2^0, E^0, Q^0, A^0, I^0, M^0, I_H^0, R^0)$. The nonzero components $S^0, V_1^0, V_2^0$ and $R^0$ of $\mathcal{E}_0$ are calculated as follows:

$$\Lambda(1-a) - (\lambda + k_1 + \delta_N)S^0 + \mu Q^0 = 0 \quad \Rightarrow \quad S^0 = \frac{\Lambda(1-a)}{k_1+\delta_N}.$$

$$a_1\Lambda + k_1 S^0 - (\lambda\psi + k_2 + \delta_N)V_1^0 = 0 \quad \Rightarrow \quad V_1^0 = \frac{a_1\Lambda + k_1 S^0}{k_2+\delta_N} = \frac{a_1\Lambda(k_1+\delta_N)+k_1\Lambda(1-a)}{(k_2+\delta_N)(k_1+\delta_N)}.$$

$$\alpha E^0 - (\eta + \mu + \delta_N)Q^0 = 0 \quad \Rightarrow \quad Q^0 = \frac{\alpha E^0}{(\eta+\mu+\delta_N)} = 0 \quad \text{since } E = 0.$$

$$\epsilon_A A^0 + \epsilon_I I^0 + \epsilon_M M^0 + \gamma I_H^0 + \nu V_2^0 - \delta_N R^0 = 0 \quad \Rightarrow \quad R^0 = \frac{\nu V_2^0}{\delta_N}$$

$$\Rightarrow \quad R^0 = \frac{\nu a_2\Lambda(k_2+\delta_N)(k_1+\delta_N)+\nu k_2 a_1\Lambda(k_1+\delta_N)+\nu k_1 k_2\Lambda(1-a)}{\delta_N(k_1+\delta_N)(k_2+\delta_N)(\nu+\delta_N)}.$$

We have $\mathcal{E}^0 = (S^0, V_1^0, V_2^0, E^0, Q^0, A^0, I^0, M^0, I_H^0, R^0) = (S^0, V_1^0, V_2^0, 0,0,0,0,0,0, R^0)$ where,

$$S^0 = \frac{\Lambda(1-a)}{k_1 + \delta_N}$$

$$V_1^0 = V_2^0 = \frac{a_2\Lambda(k_2 + \delta_N)(k_1 + \delta_N) + k_2 a_1 \wedge (k_1 + \delta_N) + k_1 k_2 \wedge (1-a)}{(k_1 + \delta_N)(k_2 + \delta_N)(\nu + \delta_N)}$$

$$R^0 = \frac{\nu a_2\Lambda(k_2 + \delta_N)(k_1 + \delta_N) + \nu k_2 a_1 \wedge (k_1 + \delta_N) + \nu k_1 k_2 \wedge (1-a)}{\delta_N(k_1 + \delta_N)(k_2 + \delta_N)(\nu + \delta_N)} \ldots\ldots\ldots(3)$$

The disease-free equilibrium is useful in determining the basic reproduction number of the disease.

### 3.2. The Basic Reproduction Number

The relevant equations in the calculation of the NGM are the subsystem for the compartments where there can be disease infections. Therefore, the subsystem for the calculation of the basic reproduction number is therefore

$$\frac{dE}{dt} = \lambda(S + \psi V_1) - (\alpha + \sigma + \delta_N)E,$$

$$\frac{dQ}{dt} = \alpha E - (\eta + \mu + \delta_N)Q,$$

$$\frac{dA}{dt} = k\sigma E - (\omega + \epsilon_A + \delta_N)A, \ldots\ldots(4)$$

$$\frac{dI}{dt} = (1-k)\sigma E - (q + \delta_I + \epsilon_I + \delta_N)I,$$

$$\frac{dM}{dt} = (1-\tau_2\phi)qI - (\theta + \epsilon_M + \delta_M + \delta_N)M,$$

$$\frac{dI_H}{dt} = \eta Q + \omega A + \tau_2\phi qI + \theta M - (\gamma + \delta_N + \delta_H)I_H.$$

Therefore, from the subsystem, we have that

$$\mathcal{F}_i(x) = \begin{pmatrix} \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(c_1A + I + c_2M)(S+\psi V_1)}{N} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\mathcal{V}_i(x) = \begin{pmatrix} (\alpha + \sigma + \delta_N)E \\ -\alpha E + (\eta + \mu + \delta_N)Q \\ -k\sigma E + (\omega + \epsilon_A + \delta_N)A \\ -(1-k)\sigma E + (q + \delta_I + \epsilon_I + \delta_N)I \\ -(1-\tau_2\phi)qI + (\theta + \epsilon_M + \delta_M + \delta_N)M \\ -\eta Q - \omega A - \tau_2\phi qI - \theta M + (\gamma + \delta_N + \delta_H)I_H \end{pmatrix}$$

Hence, the matrices $F$ and $V$ are given by

$$F = \begin{pmatrix} 0 & 0 & F_1 & F_2 & F_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Where

$$F_1 = \frac{c_1\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^0}(S^0 + \psi V^0),\ F_2 = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^0}(S^0 + \psi V^0),$$

$$F_3 = \frac{c_2\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^0}(S^0 + \psi V^0), \text{ with } N^0 = \frac{\Lambda}{\delta_H}$$

and

$$V = \begin{pmatrix} \alpha + \sigma + \delta_N & 0 & 0 & 0 & 0 & 0 \\ -\alpha & \eta + \mu + \delta_N & 0 & 0 & 0 & 0 \\ -k\sigma & 0 & \omega + \epsilon_A + \delta_N & 0 & 0 & 0 \\ -(1-k)\sigma & 0 & 0 & q + \delta_I + \epsilon_I + \delta_N & 0 & 0 \\ 0 & 0 & 0 & -(1-\tau_2\phi)q & \theta + \epsilon_M + \delta_M + \delta_N & 0 \\ 0 & -\eta & -\omega & -\tau_2\phi q & -\theta & \gamma + \delta_N + \delta_H \end{pmatrix}.$$

The inverse of the matrix, $V$ is nonnegative and is given by

$$V^{-1} = \begin{pmatrix} \frac{1}{B_1} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sigma}{B_1 B_5} & \frac{1}{B_5} & 0 & 0 & 0 & 0 \\ \frac{k\sigma}{B_1 B_2} & 0 & \frac{1}{B_2} & 0 & 0 & 0 \\ \frac{(1-k)\sigma}{B_1 B_3} & 0 & 0 & \frac{1}{B_3} & 0 & 0 \\ \frac{(1-k)(1-\tau_2\phi)\sigma q}{B_1 B_3 B_4} & 0 & 0 & \frac{(1-\tau_2\phi)q}{B_3 B_4} & \frac{1}{B_4} & 0 \\ B_7 & \frac{\eta}{B_5 B_6} & \frac{\omega}{B_2 B_6} & \frac{\tau_2\phi q B_4 + \theta(1-\tau_2\phi)q}{B_3 B_4 B_6} & \frac{\theta}{B_4 B_6} & \frac{1}{B_6} \end{pmatrix}$$

*where* $B_1 = \alpha + \sigma + \delta_N, B_2 = \omega + \epsilon_A + \delta_N, B_3 = q + \delta_I + \epsilon_I + \delta_N, B_4 = \theta + \epsilon_M + \delta_M + \delta_N$ , $B_5 = \eta + \mu + \delta_N$ , $B_6 = \gamma + \delta_N + \delta_H$ and

$$B_7 = \frac{\eta\alpha B_2 B_3 B_4 + \omega k\sigma B_3 B_4 B_5 + B_2 B_4 B_5 (1-k)\sigma\tau_2\phi q + \theta B_2 B_5 (1-k)(1-\tau_2\phi)\sigma q}{B_1 B_2 B_3 B_4 B_5 B_6}.$$

Then, the next-generation matrix (NGM) becomes

$$FV^{-1} = \begin{pmatrix} \frac{F_1 k\sigma}{B_1 B_2} + \frac{F_2(1-k)\sigma}{B_1 B_3} + \frac{F_3(1-k)(1-\tau_2\phi)\sigma q}{B_1 B_3 B_4} & 0 & \frac{F_1}{B_2} & \frac{F_2}{B_3} + \frac{F_3(1-\tau_2\phi)q}{B_3 B_4} & \frac{F_3}{B_4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{.....(6)}$$

The eigenvalues of the matrix $FV^{-1}$ are $\left(0,0,0,0,0, \frac{F_1 k\sigma}{B_1 B_2} + \frac{F_2(1-k)\sigma}{B_1 B_3} + \frac{F_3(1-k)(1-\tau_2\phi)\sigma q}{B_1 B_3 B_4}\right)$.

The only non-zero eigenvalue of the next generation matrix,

$$FV^{-1} \text{ is } \frac{F_1 k\sigma}{B_1 B_2} + \frac{F_2(1-k)\sigma}{B_1 B_3} + \frac{F_3(1-k)(1-\tau_2\phi)\sigma q}{B_1 B_3 B_4}.$$

Therefore, the control reproduction number of COVID-19 in this model is therefore given by

$$\mathcal{R}_c = \frac{F_1 k\sigma}{B_1 B_2} + \frac{F_2(1-k)\sigma}{B_1 B_3} + \frac{F_3(1-k)(1-\tau_2\phi)\sigma q}{B_1 B_3 B_4} \text{ .....(7)}$$

This can be rewritten as

$$\mathcal{R}_c = \mathcal{R}_c^A + \mathcal{R}_c^I + \mathcal{R}_c^M \text{ ..... (8)}$$

where

$$\mathcal{R}_c^A = \frac{c_1\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(S^0 + \psi V_1^0)k\sigma}{N^0(\alpha+\sigma+\delta_N)(\omega+\epsilon_A+\delta_N)},$$

$$\mathcal{R}_c^I = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(S^0 + \psi V_1^0)(1-k)\sigma}{N^0(\alpha+\sigma+\delta_N)(q+\delta_I+\epsilon_I+\delta_N)},$$

$$\mathcal{R}_c^M = \frac{c_2\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(S^0 + \psi V_1^0)(1-k)(1-\tau_2\phi)\sigma q}{N^0(\alpha+\sigma+\delta_N)(q+\delta_I+\epsilon_I+\delta_N)(\theta+\epsilon_M+\delta_M+\delta_N)}, \text{ .... (9)}$$

$\mathcal{R}_c$ as calculated, is the average number of persons that can be infected with COVID-19 by index case of the disease throughout his infectious lifetime when placed in a purely susceptible population with vaccination. The reproduction number $\mathcal{R}_c$ must be reduced below one in order to ensure that the disease dies out. In this case, we must have $1 > \mathcal{R}_c < \mathcal{R}_0$, where $\mathcal{R}_0$ is the basic reproduction number of COVID-19.

### 3.3. Global Stability of the Disease-free equilibrium for the covid-19 model

We shall use the method described in [16] , to investigate the global asymptotic stability of the disease-free equilibrium point. Hence, we have the system written as

$$\frac{dX_1}{dt} = F(X_1, X_2) \text{ ..... } (10)$$

$$\frac{dX_2}{dt} = G(X_1, X_2), G(X_1, 0)$$

The two conditions (C1) and (C2) listed below must be satisfied to guarantee global asymptotic stability of $\mathcal{E}^0$ when $\mathcal{R}_c < 1$. [11, 18, 19,20, 21]

(C1): For $\frac{dX_1}{dt} = F(X_1, 0)$, $\mathcal{E}_1^0$ is globally asymptotically stable

(C2): $G(X_1, X_2)$ can be written as $G(X_1, X_2) = BX_2 - \breve{G}(X_1, X_2), \breve{G}(X_1, X_2) \geq 0$ where $B$ is the Jacobian matrix of $G(X_1, X_2)$, evaluated at $\mathcal{E}^0$. If the model system satisfies the above two conditions, then the following theorem holds:

[16]**.** The disease-free equilibrium point $\mathcal{E}^0 = \left(\mathcal{E}_1^0, \widetilde{\mathbf{0}}\right)$ is globally asymptotically stable provided $\mathcal{R}_c < 1$ and the conditions (C1) and (C2) are satisfied.

Following the method described above, the system $\frac{dX_1}{dt} = F(X_1, 0)$ at the disease-free equilibrium is given by

$$\frac{dS}{dt} = \Lambda(1 - a) - (k_1 + \delta_N)S,$$

$$\frac{dV_1}{dt} = a_1\Lambda + k_1 S - (k_2 + \delta_N)V_1,$$

$$\frac{dV_2}{dt} = a_2\Lambda + k_2 V_1 - (\nu + \delta_N)V_2, \text{ ..... } (11)$$

$$\frac{dR}{dt} = \nu V_2 - \delta_N R,$$

with the disease-free equilibrium $\mathcal{E}_1^0 = (S^0, V_1^0, V_2^0, R^0)$. On the other hand, the subsystem for

the infected individuals become

$$\frac{dE}{dt} = \lambda(S + \psi V_1) - (\alpha + \sigma + \delta_N)E,$$

$$\frac{dQ}{dt} = \alpha E - (\eta + \mu + \delta_N)Q,$$

$$\frac{dA}{dt} = k\sigma E - (\omega + \epsilon_A + \delta_N)A, \text{ .....} (12)$$

$$\frac{dI}{dt} = (1 - k)\sigma E - (q + \delta_I + \epsilon_I + \delta_N)I,$$

$$\frac{dM}{dt} = (1 - \tau_2\phi)qI - (\theta + \epsilon_M + \delta_M + \delta_N)M,$$

$$\frac{dI_H}{dt} = \eta Q + \omega A + \tau_2\phi qI + \theta M - (\gamma + \delta_H + \delta_N)I_H,$$

The system (11) can be rearranged to get

$$\frac{dS}{dt} + (k_1 + \delta_N)S = \Lambda(1-a),$$

$$\frac{dV_1}{dt} + (k_2 + \delta_N)V_1 = a_1\Lambda + k_1 S,$$

$$\frac{dV_2}{dt} + (\nu + \delta_N)V_2 = a_2\Lambda + k_2 V_1, \ldots\ldots (13)$$

$$\frac{dR}{dt} + \delta_N R = \nu V_2,$$

which can be solved by use of integrating factor to get

$$S(t) = \frac{\Lambda(1-a)}{k_1+\delta_N} + A_1 e^{-\alpha_2 t},\ V_1(t) = \frac{a_1\Lambda}{k_2+\delta_N} + A_2 e^{-\alpha_3 t} + \frac{k_1 A_1}{\alpha_3-\alpha_2} e^{-\alpha_2 t}$$

$$V_2(t) = \frac{a_2\Lambda(k_2+\delta_N) + k_2 a_1 \wedge}{(k_2+\delta_N)(\nu+\delta_N)} + A_3 e^{-\alpha_4 t} + \frac{k_2 A_2}{\alpha_4-\alpha_3} e^{-\alpha_3 t} + \frac{k_1 k_2 A_1}{(\alpha_3-\alpha_2)(\alpha_4-\alpha_2)} e^{-\alpha_2 t}$$

$$R(t) = \frac{\nu a_2\Lambda(k_2+\delta_N)(k_1+\delta_N) + \nu k_2 a_1 \wedge (k_1+\delta_N) + \nu k_1 k_2 \wedge (1-a)}{\delta_N(k_1+\delta_N)(k_2+\delta_N)(\nu+\delta_N)} + A_4 e^{-\delta_N t}$$

$$+ \frac{\nu k_1 k_2 A_1}{(\alpha_3-\alpha_2)(\alpha_4-\alpha_2)(\delta_N-\alpha_2)} e^{-\alpha_2 t} + \frac{\nu\, k_2 A_2}{(\alpha_4-\alpha_3)(\delta_N-\alpha_3)} e^{-\alpha_3 t} + \frac{\nu A_3}{(\delta_N-\alpha_4)} e^{-\alpha_4 t}$$

where $A_i, i = 1,2,3,4$ are constants and $\alpha_1 = \eta + \mu + \delta_N, \alpha_2 = k_1 + \delta_N, \alpha_3 = k_2 + \delta_N, \alpha_4 = \nu + \delta_N, \alpha_5 = \gamma + \delta_H + \delta_N$. As $t \to \infty$, $\big(S(t), V_1(t), V_2(t), , R(t)\big) \to (S^0, V_1^0, V_2^0, R^0)$. Hence, $\mathcal{E}_1^0$ is globally asymptotically stable. For the second condition (C2), the Jacobian matrix $B$ is same as $F - V, X_2 = (E, Q, A, I, M, I_H)^T$,

$$G(X_1, X_2) = \begin{pmatrix} \lambda(S + \psi V_1) - \Phi_4 E \\ \alpha E - \Phi_5 Q \\ k\sigma E - \Phi_6 A \\ (1-k)\sigma E - \Phi_7 I \\ (1-\tau_2\phi)qI - \Phi_8 M \\ \eta Q + \omega A + \tau_2\phi qI + \theta M - \Phi_9 I_H \end{pmatrix}$$

and

$$B = \begin{pmatrix} -\Phi_4 & 0 & F_1 & F_2 & F_3 & 0 \\ \alpha & -\Phi_5 & 0 & 0 & 0 & 0 \\ k\sigma & 0 & -\Phi_6 & 0 & 0 & 0 \\ (1-k)\sigma & 0 & 0 & -\Phi_7 & 0 & 0 \\ 0 & 0 & 0 & (1-\tau_2\phi)q & -\Phi_8 & 0 \\ 0 & \eta & \omega & \tau_2\phi q & \theta & -\Phi_9 \end{pmatrix}.$$

Hence,

$$\breve{G}(X_1, X_2) = BX_2 - G(X_1, X_2)$$

$$= \begin{pmatrix} -\Phi_4 & 0 & F_1 & F_2 & F_3 & 0 \\ \alpha & -\Phi_5 & 0 & 0 & 0 & 0 \\ k\sigma & 0 & -\Phi_6 & 0 & 0 & 0 \\ (1-k)\sigma & 0 & 0 & -\Phi_7 & 0 & 0 \\ 0 & 0 & 0 & (1-\tau_2\phi)q & -\Phi_8 & 0 \\ 0 & \eta & \omega & \tau_2\phi q & \theta & -\Phi_9 \end{pmatrix} \begin{pmatrix} E \\ Q \\ A \\ I \\ M \\ I_H \end{pmatrix} - \begin{pmatrix} \lambda(S + \psi V_1) - \Phi_4 E \\ \alpha E - \Phi_5 Q \\ k\sigma E - \Phi_6 A \\ (1-k)\sigma E - \Phi_7 I \\ (1-\tau_2\phi)qI - \Phi_8 M \\ \eta Q + \omega A + \tau_2\phi qI + \theta M - \Phi_9 I_H \end{pmatrix}$$

$$= \begin{pmatrix} -\Phi_4 E + F_1 A + F_2 I + F_{3M} - \lambda(S + \psi V_1) + \Phi_4 E \\ \alpha E - \Phi_5 Q - \alpha E + \Phi_5 Q \\ k\sigma E - \Phi_6 A - k\sigma E + \Phi_6 A \\ (1-k)\sigma E - \Phi_7 I - (1-k)\sigma E + \Phi_7 I \\ (1-\tau_2\phi)qI - \Phi_8 M - (1-\tau_2\phi)qI + \Phi_8 M \\ \eta Q + \omega A + \tau_2\phi qI + \theta M - \Phi_9 I_H - \eta Q - \omega A - \tau_2\phi qI - \theta M + \Phi_9 I_H \end{pmatrix}$$

$$= \begin{pmatrix} \beta(1-\rho_1)(1-\rho_2)(1-\tau_1)(c_1 A + I + c_2 M)\left(\frac{S^0}{N^0} - \frac{S}{N} + \psi\left(\frac{V_1^0}{N^0} - \frac{V_1}{N}\right)\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Since from (10), $N^0 > S^0, N > S, N > N^0, S < S^0, N^0 > V_1^0, N > V_1, V_1 < V_1^0$, we have that $\breve{G}(X_1, X_2) \geq 0$. Hence, the second condition (C2) is satisfied. Therefore, the disease-free equilibrium is globally asymptotically stable. The global asymptotical stability of the disease-free equilibrium shows that in the system, covid-19 will persist in the system at a stable rate and keeping the control reproduction number less than one is enough to eradicate the disease from the system.

### 3.4. Approximate Analytical Solution for the covid-19 model

*3.4.1. Differential Transform Method*

The differential transform method (DTM) is a semi-analytical method for obtaining approximate solutions of both linear and non-linear equations. The method was used to solve linear and nonlinear initial value problems in electrical circuit analysis by [41`,50, 52]. It was also used to solve linear and nonlinear Ordinary differential equations (ODEs) by [6]. The differential transform of a function $f(x)$ is defined as follows by [51].

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{\{x=0\}}, \quad \text{.....(14)}$$

where $f(x)$ is the original function and $F(k)$ is the transformed function. The differential inverse transform of $F(k)$ is given by

$$y(x) = \sum_{k=0}^{\infty} F(k)x^k, \quad \text{.....(15)}$$

Substituting (14) into (15) gives

$$f(k) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=0}, \quad \text{.....(16)}$$

Equation (16) shows that differential transform derived its concept from Taylor series expansion. The standard operations by differential transform method as given by [6, 51, 53] are listed in Table 2 below.

**Table 2** Standard Operations of DTM

| Original function | Transformed function |
|---|---|
| $f(x) = g(x) \pm h(x)$ | $F = G(k) \pm H(k)$ |
| $f(x) = \alpha g(x)$ | $F = \alpha G(k)$ |
| $f(x) = \frac{dg(x)}{dx}$ | $F(k) = (k+1)G(k+1)$ |
| $f(x) = \frac{d^2 g(x)}{dx^2}$ | $F(k) = (k+1)(k+2)G(k+2)$ |
| $f(x) = \frac{d^m g(x)}{dx^m}$ | $F(k) = (k+1)(k+2)\cdots(k+m)G(k+m)$ |
| $f(x) = 1$ | $F(k) = \delta(k)$ |
| $f(x) = x$ | $F(k) = \delta(k-1)$ |
| $f(x) = x^m$ | $Y(k) = \begin{cases} \delta(\mathrm{k}-\mathrm{m}) = 1, \text{if k} = \mathrm{m} \\ \delta(\mathrm{k}-1) = 0, \text{if k} \neq \mathrm{m} \end{cases}$ |
| $f(x) = g(x)h(x)$ | $Y(k) = \sum_{m=0}^{k=0} H(m)g(k-m)$ |
| $f(x) = e^{\lambda x}$ | $Y(k) = \frac{\lambda^k}{k!}$ |
| $f(x) = (Hx)^m$ | $Y(k) = \frac{m(m-1)\cdots\ m-k+1}{k!}$ |

Using the DTM method described and initial data from Table 2, we will proceed to obtain the approximate analytical solution for the model (1).

**Table 3** Parameters Values and Sources used in model

| Parameters | Values | Source |
|---|---|---|
| $\Lambda$ | 100 | Assumed |
| $a$ | 0.2 | Assumed |
| $a_1$ | 0.125 | Assumed |
| $a_2$ | 0.075 | Assumed |
| $\delta_N$ | 0.00004 | [23] |
| $\delta_I$ | 0.015 | [26] |
| $\delta_M$ | 0.21 | [10] |
| $\mu$ | 0.025 | [10] |
| $\alpha$ | 0.1429 | (Estimated[10] |
| $\beta$ | 0.4 | (Fitted) [10] |
| $\varepsilon_A$ | 0.1429 | [16, 44, 50] |
| $\varepsilon_I$ | 0.1429 | [17, 44,50] |
| $\varepsilon_M$ | 0.1429 | [44,50] |
| $\nu$ | 0.005 | Assumed |

| | | |
|---|---|---|
| $\kappa_1$ | 0.1582 | [43] |
| $\kappa_2$ | 0.05 | Assumed |
| $\psi$ | 0.05 | Assumed |
| $\sigma$ | 0.1923 | [33] |
| $\gamma$ | 0.0667 | [31] |
| $k$ | 0.5 | [31] |
| $\eta$ | 0.514 | (Fitted)[10] |
| $\omega$ | $2.2719 \times 10^{-11}$ | (Fitted)[10] |
| $q$ | 0.04 | (Fitted) [10] |
| $\tau_1$ | 0.2 | [39] |
| $\tau_2$ | 0.0135 | [39] |
| $\phi$ | 0.01 | [10] |
| $\theta$ | 0.164 | (Fitted)[10] |
| $\rho_1$ | 0.1 | World Bank (2021) |
| $\rho_2$ | 0.2 | [39] |
| $c_1$ | 0.5 | [31] |
| $c_2$ | 0.4341 | [10] |

From the standard operations of DTM in Table 2, we will have

$$(k+1)S^{(k+1)} = \Lambda(1-a) - \lambda_1\left[c_1\sum_{m=0}^{k}A^{(\mathrm{k-m})} + \sum_{m=0}^{k}I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k}M^{(\mathrm{k-m})}\right]S^m$$

$$-(\delta_N + \kappa_1)S^k + \mu Q^k,$$

$$(k+1)V_1^{(k+1)} = a_1\Lambda + \kappa_1 S^k - \lambda_1\psi\left[c_1\sum_{m=0}^{k}A^{(\mathrm{k-m})} + \sum_{m=0}^{k}I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k}M^{(\mathrm{k-m})}\right]V_1^m - (\kappa_2 + \delta_N)V_1^k,$$

$$(k+1)V_2^{(k+1)} = a_2\Lambda + \kappa_2 V_1^k - (\delta_N + \nu)V_2^k,$$

$$(k+1)E^k = \lambda_1\left[c_1\sum_{m=0}^{k}A^{(\mathrm{k-m})} + \sum_{m=0}^{k}I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k}M^{(\mathrm{k-m})}\right]S^m$$

$$+\lambda_1\psi\left[c_1\sum_{m=0}^{k}A^{(\mathrm{k-m})} + \sum_{m=0}^{k}I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k}M^{(\mathrm{k-m})}\right]V_1^m - (\alpha + \sigma + \delta_N)E^k,$$

$$(k+1)Q^{(k+1)} = \alpha E^k - (\eta + \mu + \delta_N)Q^k,$$

$$(k+1)A^{(k+1)} = k\sigma E^k - (\omega + \varepsilon_A + \delta_N)A^k,$$

$$(k+1)I^{(k+1)} = (1-k)\sigma E^k - (q + \delta_N + \delta_I + \varepsilon_I)I^k,$$

$$(k+1)M^{(k+1)} = (1-\tau_2\phi)qI^k - (\theta + \delta_N + \delta_M + \varepsilon_M)M^k,$$

$$(k+1)I_H^{(k+1)} = \eta Q^k + \omega A^k + \tau_2 \phi q I^k + \theta M^k - (\gamma + \delta_N + \delta_H) I_H^k,$$

$$(k+1)R^{(k+1)} = \varepsilon_A A^k + \varepsilon_I I^k + \varepsilon_M M^k + \gamma I_H^k + \nu V_2^k - \delta_N R^k,$$

where $\lambda_1^k = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^k}$.

Making the state variables subject of the formula, we get

$$S^{(k+1)} = \frac{1}{(k+1)}\left[\Lambda(1-a) - \lambda_1^k\left[c_1\sum_{m=0}^{k} A^{(\mathrm{k-m})} + \sum_{m=0}^{k} I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k} M^{(\mathrm{k-m})}\right]S^m\right]$$

$$-\frac{1}{(k+1)}[(\delta_N + \kappa_1)S^k + \mu Q^k],$$

$$V_1^{(k+1)} = \frac{1}{(k+1)}\left[a_1\Lambda + \kappa_1 S^k - \lambda_1^k\psi\left[c_1\sum_{m=0}^{k} A^{(\mathrm{k-m})} + \sum_{m=0}^{k} I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k} M^{(\mathrm{k-m})}\right]V_1^m - (\kappa_2 + \delta_N)V_1^k\right],$$

$$V_2^{(k+1)} = \frac{1}{(k+1)}[a_2\Lambda + \kappa_2 V_1^k - (\delta_N + \nu)V_2^k],$$

$$E^{(k+1)} = \frac{\lambda_1^k}{(k+1)}\left[c_1\sum_{m=0}^{k} A^{(\mathrm{k-m})} + \sum_{m=0}^{k} I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k} M^{(\mathrm{k-m})}\right]S^m$$

$$+\frac{1}{(k+1)}\left[\lambda_1^k\psi\left[c_1\sum_{m=0}^{k} A^{(\mathrm{k-m})} + \sum_{m=0}^{k} I^{(\mathrm{k-m})} + c_2\sum_{m=0}^{k} M^{(\mathrm{k-m})}\right]V_1^m - (\alpha + \sigma + \delta_N)E^k\right],$$

$$Q^{(k+1)} = \frac{1}{(k+1)}[\alpha E^k - (\eta + \mu + \delta_N)Q^k],$$

$$A^{(k+1)} = \frac{1}{(k+1)}[k\sigma E^k - (\omega + \varepsilon_A + \delta_N)A^k],$$

$$I^{(k+1)} = \frac{1}{(k+1)}[(1-k)\sigma E^k - (q + \delta_N + \delta_I + \varepsilon_I)I^k],$$

$$M^{(k+1)} = \frac{1}{(k+1)}[(1-\tau_2\phi)qI^k - (\theta + \delta_N + \delta_M + \varepsilon_M)M^k],$$

$$I_H^{(k+1)} = \frac{1}{(k+1)}[\eta Q^k + \omega A^k + \tau_2\phi q I^k + \theta M^k - (\gamma + \delta_N + \delta_H)I_H^k],$$

$$R^{(k+1)} = \frac{1}{(k+1)}[\varepsilon_A A^k + \varepsilon_I I^k + \varepsilon_M M^k + \gamma I_H^k + \nu V_2^k - \delta_N R^k],$$

When $k = 0, m = 0$, we have

$$N^0 = S^0 + V_1^0 + V_2^0 + E^0 + Q^0 + A^0 + I^0 + M^0 + I_H^0 + R^0$$

$$= 206{,}000{,}000 + 15{,}000 + 5000 + 200{,}000 + 7{,}000 + 30{,}000$$

$$+150{,}000 + 50{,}000 + 1719 + 164{,}415 = 206{,}623{,}134.$$

$$\lambda_1^0 = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^0} = \frac{0.4(1-0.1)(1-0.2)(1-0.2)}{206{,}623{,}134} = \frac{0.4(0.9)0.8(0.8)}{206{,}623{,}134} = \frac{0.2304}{206{,}623{,}134} = 1.1\times10^{-9}$$

Therefore, the numerical solutions of each state variable will be given by;

$$S^1 = \Lambda(1-a) - \lambda_1^0[c_1A^0 + I^0 + c_2M^0]S^0 - (\delta_N + \kappa_1)S^0 + \mu Q^0$$

$$= 100(1-0.2) - (1.1\times10^{-9})[0.5(30000) + 150000 + 0.4341(50000)]206000000$$

$$-(0.00004 + 0.1582)206000000 + 0.025(7000)$$

$$= 80 + 175 - (1.1\times10^{-9})[15000 + 150000 + 21705]206000000 - 32597440$$

$$= 255 - 42307.353 - 32597440 = -32{,}639{,}492.353.$$

$$V_1^1 = a_1\Lambda + \kappa_1 S^0 - \lambda_1^0\psi[c_1A^0 + I^0 + c_2M^0]V_1^0 - (\kappa_2 + \delta_N)V_1^0 = 32{,}588{,}461.746.$$

$$V_2^1 = a_2\Lambda + \kappa_2 V_1^0 - (\delta_N + \nu)V_2^0 = 732.3.$$

$$E^1 = \lambda_1^0[c_1A^0 + I^0 + c_2M^0]S^0 + \lambda_1^0\psi[c_1A^0 + I^0 + c_2M^0]V_1^0 - (\alpha + \sigma + \delta_N)E^0 = -24{,}740.493.$$

$$Q^1 = \alpha E^0 - (\eta + \mu + \delta_N)Q^0 = 24{,}806.72.$$

$$A^1 = k\sigma E^0 - (\omega + \varepsilon_A + \delta_N)A^0 = 14{,}941.80$$

$$I^1 = (1-k)\sigma E^0 - (q + \delta_N + \delta_I + \varepsilon_I)I^0 = 10461.$$

$$M^1 = (1-\tau_2\phi)qI^0 - (\theta + \delta_N + \delta_M + \varepsilon_M)M^0 = -19847.81.$$

$$I_H^1 = \eta Q^0 + \omega A^0 + \tau_2\phi qI^0 + \theta M^0 - (\gamma + \delta_N + \delta_H)I_H^0 = 11{,}658.2989.$$

$$R^1 = \varepsilon_A A^0 + \varepsilon_I I^0 + \varepsilon_M M^0 + \gamma I_H^0 + \nu V_2^0 - \delta_N R^0 33{,}000.0807.$$

Thus, after the first iteration, the values of the state variables become;

$$S^1 = 33{,}000.0807.$$

For the second iteration, when $k = 1, m = 0{,}1$, we have $N^1 = S^1 + V_1^1 + V_2^1 + E^1 + Q^1 + A^1 + I^1 + M^1 + I_H^1 + R^1 = -20{,}809.193$.

$$\lambda_1^1 = \frac{\beta(1-\rho_1)(1-\rho_2)(1-\tau_1)}{N^0} = \frac{0.4(1-0.1)(1-0.2)(1-0.2)}{-20{,}809.193} = \frac{0.4(0.9)0.8(0.8)}{-20{,}940.7104}$$

$$= \frac{0.2304}{-20{,}940.7104} = -1.1002\times10^{-5}.$$

Thus, after the second iteration, the values of the state variables become;

$$S^2 = -1{,}421.9987.$$

For the second iteration, when $k = 2, m = 0{,}1{,}2$, we have

$$N^2 = S^2 + V_1^2 + V_2^2 + E^2 + Q^2 + A^2 + I^2 + M^2 + I_H^2 + R^2 = 3{,}352{,}047.8665.$$

Thus, after the third iteration, the values of the state variables become;

$$S^3 = -2{,}811{,}207.2036, V_1^3 = 38{,}303.0752, V_2^3 = -38{,}073.7983, E^3 = -4{,}814{,}131.851, Q^3 = 2{,}141{,}850.5735, A^3 = 1{,}442{,}269.7586, I^2 = 1{,}442{,}922.3264, M^3 = 15.2476, I_H^3 = 1{,}326.5180, R^3 = 411.7099.$$

Thus, the solution of the system given the initial values of the state variables and the parameters is; $S(t) = 206{,}000{,}000 - 32{,}639{,}492.353t + 49{,}257{,}668.6891t^2 - 2{,}811{,}207.2036t^3$

$$V_1(t) = 15{,}000 + 32{,}588{,}461.746t - 2{,}202{,}454.7803t^2 + 38{,}303.0752t^3$$

$$V_2(t) = 5000 + 732.3t + 814{,}713.4483t^2 - 38{,}073.7983t^3$$

$$E(t) = 200{,}000 - 24{,}740.493t + 44{,}997{,}260.7246t^2 - 4{,}814{,}131.851t^3$$

$$Q(t) = 7{,}000 + 24{,}806.72t + 8{,}453.6154t^2 + 2{,}141{,}850.5735t^3$$

$$A(t) = 30{,}000 + 14{,}941.80t - 2{,}257.2897t^2 + 1{,}442{,}269.7586t^3$$

$$I(t) = 150{,}000 - 10461t - 154.0741t^2 + 1{,}442{,}922.3264t^3$$

$$M(t) = 50{,}000 - 19{,}847.81t - 100.4083t^2 + 15.2476t^3$$

$$I_H(t) = 1719 + 11{,}658.2989t + 4{,}271.3037t^2 + 1{,}326.5180t^3$$

$$R(t) = 164{,}415 + 33{,}000.0807t - 1{,}421.9987t^2 + 411.7099t^3$$

*3.4.2. Comparison of the DTM solutions with RK-4 solutions*

The solutions obtained by the differential transform method (DTM) is compared with the solutions obtained by Runge-Kutta method of order 4 (RK-4) to check if the DTM solutions is consistent and convergent. The comparison is shown in Figures 2 – 9. The plots showed that both solutions are relatively comparable within some time interval. Though both the differential transform method and Runge-Kutta methods are approximation methods, the DTM is a semi-analytical method while the Runge-Kutta methods are numerical techniques

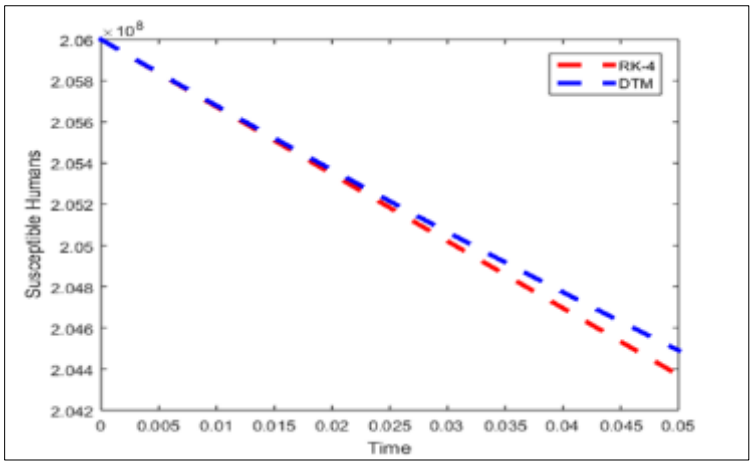


**Figure 2** Susceptible humans

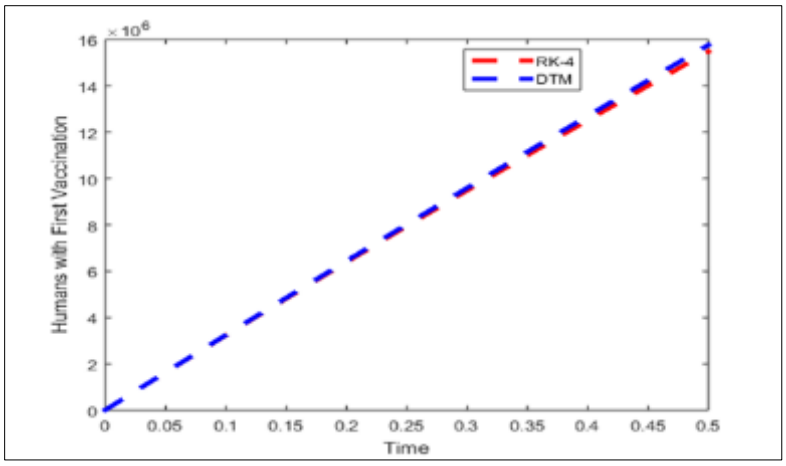


**Figure 3** Humans with first vaccination

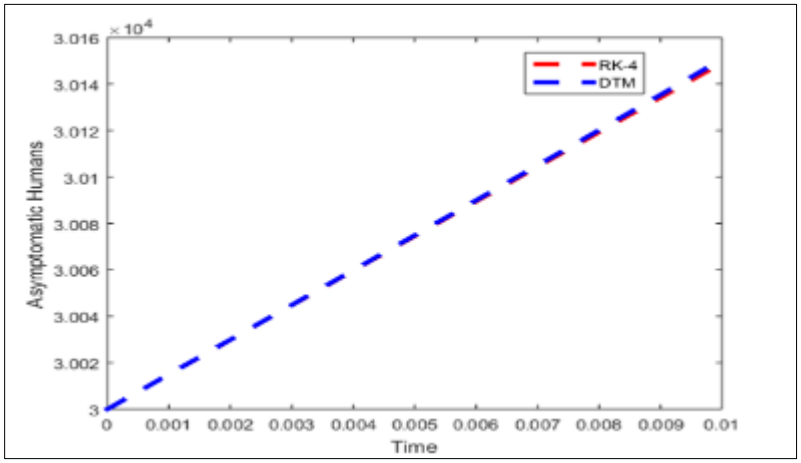


**Figure 4** Asymptomatic humans

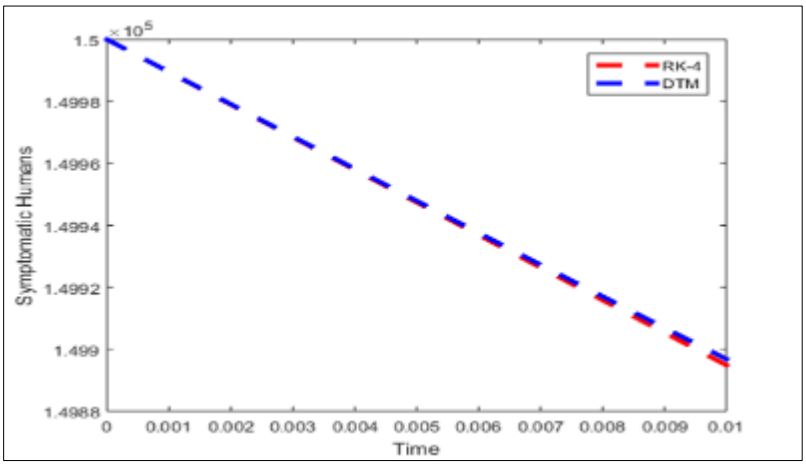


**Figure 5** Symptomatic humans

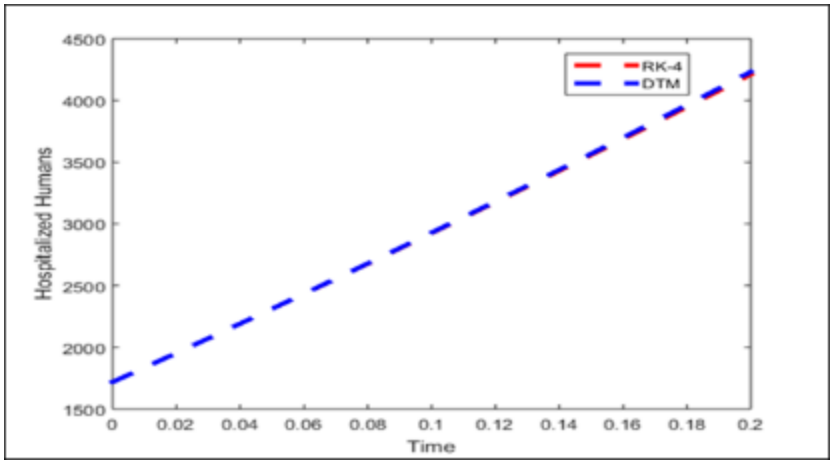


**Figure 6** Hospitalized humans

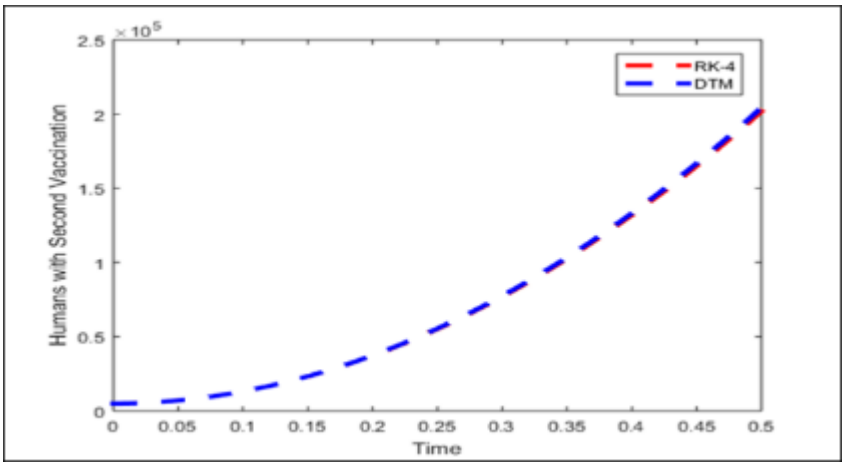


**Figure 7** Humans with second vaccination

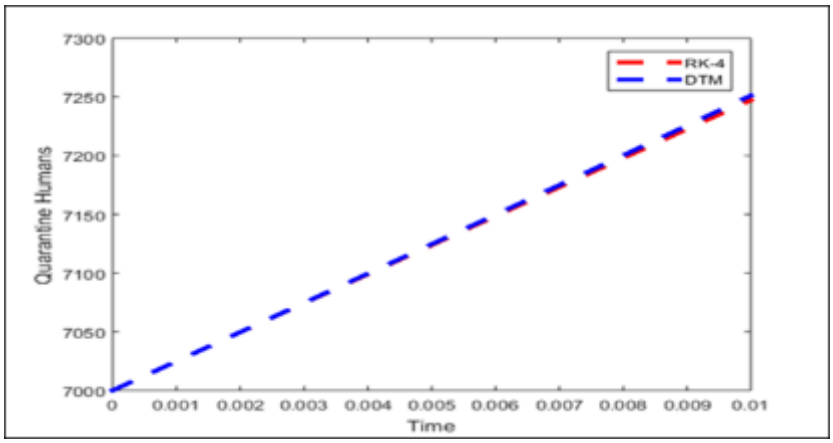


**Figure 8** Quarantine Humans

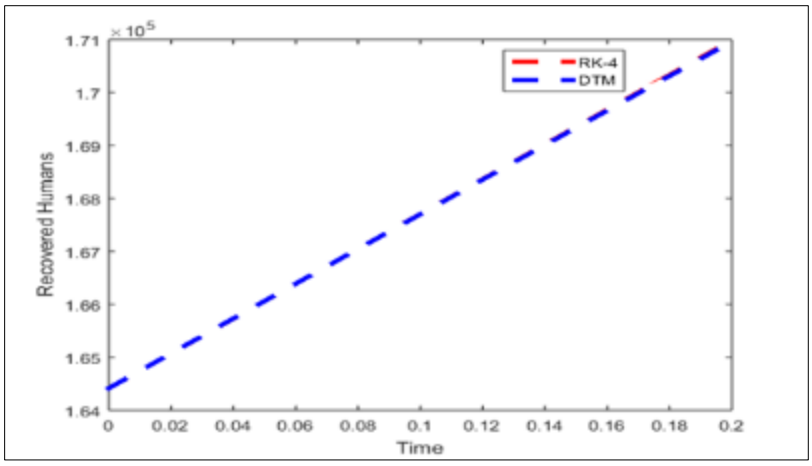


**Figure 9** Recovered humans

### 3.5. Sensitivity Analysis of the Parameters in the Control Reproduction Number

We use forward normalized sensitivity index method to measure the relative change in $\mathcal{R}_c$, to the relative change in the model parameter, $x$[14, 23,29]. This is defined as

$$\mathcal{S}_x^{\mathcal{R}_c} = \frac{\partial \mathcal{R}_c}{\partial x} \times \frac{x}{\mathcal{R}_c} \text{ ..... (17)}$$

**Table 4** Sensitivity indices of parameters contained in the reproduction number

| Parameter | Sensitivity Index | Parameter | Sensitivity Index |
|---|---|---|---|
| $c_1$ | +0.1445 | $\omega$ | -2.2955× $10^{-11}$ |
| $c_2$ | +0.0209 | $\epsilon_A$ | -2.2955× $10^{-11}$ |
| $\beta$ | +1 | $q$ | -0.7111 |
| $k$ | +0.7111 | $\delta_I$ | -0.0648 |
| $\sigma$ | +0.4264 | $\epsilon_I$ | -0.6176 |
| $a_1$ | +0.1058 | $\theta$ | -0.0069 |
| $\psi$ | +0.1198 | $\delta_M$ | -0.0088 |
| $\phi$ | 0 | $\epsilon_M$ | -0.0052 |
| $\rho_1$ | -0.1111 | $\tau_2$ | -0.0083 |
| $\rho_2$ | -0.25 | $k_1$ | -0.8780 |
| $\tau_1$ | -0.25 | $k_2$ | -0.1197 |
| $\alpha$ | -0.4263 | | |

The result of the sensitivity analysis shows that the parameters with positive sensitivity indices are $\beta, c_1, c_2, k, \sigma, \psi$ and $a_1$ These parameters with positive sensitivity indices are the parameters whose values must be reduced in order to stop the spread of corona virus disease.

The parameter with the highest sensitivity index is $\beta$, the effective contact rate between the susceptible class and the infectious classes. The sensitivity index shows that a reduction in the contact rate will lower the spread of the disease. This aligns with the use of the control measures represented by the parameters, $\rho_1, \rho_2$ and $\tau_1$. These parameters aim to reduce the rate of contact between the susceptible class and the infectious classes namely, $A, I, and\ M$. The negative sign of the sensitivity indices of these parameters is an indication that the spread of the disease can be reduced by increasing the value of these parameters. The importance of first dose of vaccination is seen in the sensitivity index with respect to the parameter, $a_1$, which represents the proportion of those recruited into the susceptible class that have received

first dose of the vaccination. The sensitivity index with respect to this parameter advises that more people recruited into the population should receive first dose of the vaccine to help reduce the rate of infection. The parameter, $\sigma$ represents the rate at which people infected with the disease becomes infectious. The positive sign of the sensitivity index with respect to this parameter indicates the need to reduce the rate of infectiousness of those infected with the disease. The main aim of receiving doses of corona virus disease vaccine is to reduce the probability of being infected with the virus. The parameter which represents this probability is $\psi$, with sensitivity index, $+0.1198$. This shows that reducing the probability of infection by increasing the efficacy of the vaccine will help reduce the rate at which people contract the disease. The need to increase the proportion, $k_1$ of the susceptible class that receive first dose of the vaccine, and the proportion, $k_2$ of those in $V_1$, that receive the second dose is seen in the sensitivity indices with respect to these parameters. Quarantining of those that are exposed to corona virus is vital in the management of the disease outbreak. This is observed in the sensitivity index with respect to $\alpha$, which indicates the need to increase the rate at which the exposed persons are quarantined. The rates of hospitalization $\eta, \omega, q$ and $\theta$ for those in the compartments $Q, A, I$ and $M$, respectively show negative sensitivity indices, which indicates that the rates at which infected persons get hospitalized to receive appropriate treatment should be increased for effective management of the disease outbreak

## 4. Conclusion

This paper discusses the ongoing public health risk that the COVID-19 pandemic poses and assesses how well different control measures with a special emphasis on vaccination, hospitalization and quarantine policies address its spread. The study emphasizes the significance of scientifically based public health treatments, given that COVID-19 is characterized by high transmissibility and considerable mortality, particularly among vulnerable populations. We use a deterministic mathematical model to Obtain the disease-free equilibrium state, Compute the control reproduction number, Obtain conditions for the local and global stability of the disease-free equilibrium state and carryout sensitivity analysis on the control reproduction number. The study does thorough mathematical analysis using programs like MATLAB and Maple.

## Compliance with ethical standards

### *Disclosure of conflict of interest*

We the authors declare no affiliations with or involvement in any organization or entity with any financial interest such as honoraria, educational grants, employment stock ownership, or other equity interest or non-financial interest such as personal or professional relationships, affiliations, knowledge or belief in the subject matter or materials discussed in this manuscript.

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### *Statement of ethical approval*

This study is based entirely on mathematical modeling and computational analysis of publicly available data. It does not involve any experiments on human participants or animals, nor does it collect or process any personal, sensitive, or identifiable information. Therefore, ethical approval was not required. All procedures performed in this study were in accordance with relevant institutional, national, and international ethical guidelines and regulations for computational research.

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