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Comparative analysis of Jacobi's and game theory models for wealth creation from solid wastes in Enugu State, Nigeria

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Abstract

The study is aimed at carrying out comparative analysis of Jacobi's iteration Optimization and Game theory models for wealth creation from Solid Wastes generation in Enugu. The objectives are to quantify solid wastes volume and characteristics generated, develop the optimization modeling cost for cost effective solid wastes management and planning, apply Jacobi's iteration model and compare the results with Game theory models to optimize wealth creation in Enugu. There is progressive increase on solid wastes generation due to urbanization and industrialization, unhygienic disposal and associated problems that pose serious threat to public health and environment. Lack of public enlightenment on environmental control measures, protection of environment and human lives, and other pollution menace made the current Solid waste planning and management unsustainable. There is lack of capacity on the part of Enugu State Waste Management Authority (ESWAMA) to live up to the challenges. The methodology involve the use of Jacobi's iteration Optimization and Game theory optimization models to optimize the Solid Wastes generated in Enugu in order to create wealth for the inhabitants of Enugu. The results show that Jacobi's iteration optimization model objective function maximized profit at Z = N21,072,853.00 (N21.07million) per ton per day based on fifty years projection from 2006 census projection of the population of Enugu municipal. The benefit of cost recovery for optimal solution using Jacobi's iteration was N7.67billion per ton per annum with total revenue of N18.409trillion. The result obtained from Game theory optimization model was N18.284trillion which compares favourably with Jacobi's iteration with difference of N125.00billion. The work concludes that proper management of solid waste generated can create wealth through source reduction, re-use, recycling, recovery with treatment and disposal. Effective legislation based on the outcome of these optimization techniques will market solid waste as essential commodity, which will in turn improve the general sanitation of Enugu urban. Appropriate measures must be put in place on the implementation for the economic benefit of the inhabitants.

Keywords: Game theory; Jacobi's iteration; Optimization; Solid waste planning and management; Wealth creation

1. Introduction

The analysis of Jacobi's iteration and Game theory optimization for wealth creation in the planning and management of municipal Solid wastes in Enugu Nigeria is imperative due to;

- Progressive increase in Solid waste generation rates as a result of urbanization and industrialization with corresponding increase in disposal costs, environmental pollution, health hazards, limited landfill, composting and unkept dump sites that deface the environment.

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- There are insufficient dumpsters and dump sites, and substantial qualities of municipal solid wastes are disposed off unhygienic in open dumps which create problems to public health and environment.
- There are delays in collection of solid wastes; inadequate plant, machinery and equipment to face the challenges and lack of reliable data.
- Lack of public enlightenment to educate the inhabitants on environmental control measures to protect environment, human lives and reduce pollution menace for effective waste management.
- Lack of solid waste treatment plants and waste recycling plants which would assist in creating wealth and employment.
- Inconsistent government policies by successive governments on Solid waste planning and management have adversely affected the development of a master plan for solid waste in Enugu.

Solid waste generation planning and management has been an issue of concern to successive government in Enugu both at state and local government areas. Various strategies to tackling the problem enunciated by successive governments are inconsistent with the global standards. Solid waste problem in Enugu contributes to the contamination of the streams, river, land and the atmosphere. Waste disposal operations are becoming increasingly sophisticated with specialist companies and facilities, leaving the Enugu State with great responsibility to rise up to the challenges. Aramabi (1998) states that the possession of corrected and adequate information on the rate of generation and composition of wastes generated will make it easy to propose and implement an effective method of management. Therefore, the generation rate and composition of the wastes generated in Enugu must be first identified in order to use the best management options to manage the waste generated.

Sincero and Sincere (2006) opined that Solid waste survey and characterization are special tools in bringing to light the generation rate and composition of solid waste. It is too common to have solid waste disposed of in Enugu without attempting to explore the wealth creation options of solid waste management. Management methods such as recovery, recycling, and reuse are very important tools for creating wealth from waste. Oyinlola (1999) stated that it is not essentially every composition of solid waste that can be further utilized as resources for wealth creation. Therefore, solid waste components in Enugu must be well classified to make the compositions readily differentiable to intended stakeholders.

Enugu State Waste Management Authority (ESWAMA) charged with management of solid waste in Enugu urban is still grappling with the challenges of articulating an effective and efficient solid waste management programme. Most of the inhabitants in the area do not know what is expected of them as regards solid waste management. Out of the need to get rid of the solid waste from their domains and relieve themselves of its nuisance, some of the inhabitants dump their refuse behind their houses and indiscriminately in nearby open dumps. Some of the solid waste is dumped in the water channels, gullies, river side and any available spaces. In most cases, the refuse accumulates, encroaching on roads and streets. Other inhabitants opt for open burning as their main method of reducing the volume of solid waste leading to air pollution in Enugu. In fact, solid waste collection and disposal is one of the highest environmental problems because, there is no effective existing solid waste management disposal technology for the area.

The integrated Solid Wastes Management (ISWM) option refers to the selection and use of appropriate management programs, technologies, and techniques to achieve particular waste management goals and objectives. The U.S. Environmental Protection Agency (EPA) states that ISWM is composed of waste source reduction, recycling, waste combustion, and landfills. These activities can be done in either an interactive or hierarchical way (Clarke and Meantay, 2006).

It is important to stress that better solid waste management programs are urgently needed in some countries. Only about half of the waste generated in cities and one-quarter of what is produced in rural areas is collected. Internationally, the World Bank (1984) warns that global waste could increase by 70% by 2050 in a business-as-usual scenario, if ongoing efforts to improve the waste management system are an important part of preserving a healthy human and ecological future. It is against this back drop that the study articulated the use of Jacobis iteration and Game theory optimization models as a basis for wealth creation from solid waste generated in Enugu state.

2. Literature Review

The literature review was based on concept of Jacobi's iteration model and linear programming techniques of Game theory optimization model.

2.1. Jacobi's Method

The method is named after Carl Gustav Jacob Jacobi. The Jacobi's method is an iterative algorithm for determining the solutions of a strictly diagonally dominant system of linear equations in numerical linear algebra. It consists of solving for each diagonal element before an approximate value is plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization as explained by the following procedure;

Description

Let, $A_X = b$ be a square system of *n* linear equations, where: …….. (1)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_1 \end{bmatrix}, \ldots\ldots\ldots\ldots\ldots (2)$$

Then A can be decomposed into a diagonal component D, and the remainder R:

$$A = D + R \text{ where } D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}, \text{and } R = \begin{bmatrix} 0 & a_{22} & \cdots & a_{1n} \\ a_{11} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix}, \ldots\ldots (3)$$

The solution is then obtained iteratively via

$$x^{(k+1)} = D^{-1}\left(b - Rx^{(k)}\right), \ldots\ldots\ldots\ldots\ldots (4)$$

where $x^{(k)}$ is the kth approximation or iteration of x and $x^{(k+1)}$ is the next or k + 1 iteration of x. The element-based formula is thus:

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j \neq i} a_{ij} x_j^{(k)}\right), i = 1,2, \ldots, n. \ldots\ldots\ldots\ldots (5)$$

The computation of $x_i^{(k+1)}$ requires each element in $\mathbf{x}^{(k)}$ except itself. Unlike the Gauss–Seidel method, we can't overwrite $x_i^{(k)}$ with $x_i^{(k+1)}$, as that value will be needed by the rest of the computation. The minimum amount of storage is two vectors of size n.

Convergence: The standard convergence condition (for any iterative method) is when the spectral radius of the iteration matrix is less than 1:

$$\rho(D^{-1}R) < 1. \quad \ldots\ldots\ldots\ldots \quad (6)$$

A sufficient (but not necessary) condition for the method to converge is that the matrix A is strictly or irreducibly diagonally dominant. Strict row diagonal dominance means that for each row, the absolute value of the diagonal term is greater than the sum of absolute values of other terms:

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}| \ldots\ldots\ldots\ldots\ldots \quad (7)$$

The Jacobi method sometimes converges even if these conditions are not satisfied.

Note that the Jacobi method does not converge for every symmetric positive-definite matrix. For example

$$A = \begin{pmatrix} 29 & 2 & 1 \\ 2 & 6 & 1 \\ 1 & 1 & \frac{1}{5} \end{pmatrix} \to D^{-1}R = \begin{pmatrix} 0 & \frac{2}{29} & \frac{1}{29} \\ \frac{1}{3} & 0 & \frac{1}{6} \\ 5 & 5 & 0 \end{pmatrix} \Rightarrow \rho(D^{-1}R) \approx 1.0661 \quad \ldots\ldots\ldots \quad (8)$$

Example: A linear system of the form Ax = b with initial estimate $x^{(0)}$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}, b = \begin{bmatrix} 11 \\ 13 \end{bmatrix} and x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We use the equation, $x^{(k+1)} = D^{-1}\left(b - Rx^{(k)}\right)$, described above, to estimate x. First, we rewrite the equation in a more convenient form D^{-1} (b − R $x^{(k)}$) = T $x^{(k)}$ + C

Where T= $D^{-1}R$ *and* C = D^{-1} b. Note that $R = L + U$ where L and U are the strictly lower and upper parts of A. From the known values

$$D^{-1} = \begin{bmatrix} 1/2 & 1 \\ 0 & 1/7 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} and\ U = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

$$we\ determine\ T = -D^{-1}(L + U)\ as$$

$$T = \begin{bmatrix} 1/2 & 1 \\ 0 & 1/7 \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 \\ -5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix}$$

Further, C is found as

$$C = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/7 \end{bmatrix} \begin{bmatrix} 11 \\ 13 \end{bmatrix} = \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix}.$$

With T and C calculated, we estimate as $x^{(1)}$ and T$x^{(0)} + C$:

$$x^{(1)} = \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix} = \begin{bmatrix} 5.0 \\ 8/7 \end{bmatrix} \approx \begin{bmatrix} 5 \\ 1.143 \end{bmatrix}.$$

The next iteration yields

$$x^{(2)} = \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix} \begin{bmatrix} 5.0 \\ 8/7 \end{bmatrix} = \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix} = \begin{bmatrix} 69/14 \\ -12/7 \end{bmatrix} \approx \begin{bmatrix} 4.929 \\ -1.714 \end{bmatrix}$$

This process is repeated until convergence (i.e., until $\|Ax^{(n)} - b\|$ is small).

The solution after 25 iterations is $x = \begin{bmatrix} 7.111 \\ -3.222 \end{bmatrix}$

Another Example: Suppose we are given the following linear system:

$$10x_1 - x_2 + 2x_3 = 6\ ,$$

$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$

$$2x_1 - x_2 + 10x_3 - x_4 = -11\ ,$$

$$3x_2 - x_3 + 8x_4 = 15$$

If we choose (0, 0, 0, 0) as the initial approximation, then the first approximate solution is given by

$$x_1 = (6 + 0 - (2 * 0))/10 = 0.6,$$

$$x_2 = (25 + 0 + 0 - (3 * 0))/11 = 25/11 = 2.2727,$$

$$x_3 = (-11 - (2*0) + 0 + 0)/10 = -1.1,$$

$$x_4 = (15 - (3*0) + 0)/8 = 1.875$$

Using the approximations obtained, the iterative procedure is repeated until the desired accuracy has been reached. The following are the approximated solutions after five iterations as shown in Table 1.

Table 1 Approximated Solution after five iterations in Jacobi's Method

x_1	x_2	x_3	x_4
0.6	2.27272	-1.1	1.875
1.04727	1.7159	-0.80522	0.88522
0.93263	2.05330	-1.0493	1.13088
1.01519	1.95369	-0.9681	0.97384
0.98899	2.0114	-1.0102	1.02135

The exact solution of the system is (1, 2, -1, 1)

Weighted Jacobi method: The weighted Jacobi iteration uses a parameter$\omega$to compute the iteration as

$$X^{(k-1)} = \omega D^{-1}\left(b - Rx^{(k)}\right) + (1-\omega)x^{(k)} \dots\dots\dots\dots\dots\dots \quad (9)$$

With ω = 2 / 3 being the usual choice.

Convergence in the Symmetric Positive Definite Case: In case that the system matrix A is of symmetric positive-definite type one can show convergence.

Let $C = C_\omega = I - \omega D^{-1} A$ be the iteration matrix. Then, convergence is guaranteed for

$$\rho(C_w) < 1 \Leftrightarrow 0 < \omega < \frac{2}{\lambda_{max}(D^{-1}A)}, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (10)$$

whereλ_{max} is the maximal eigenvalue.

The spectral radius can be minimized for a particular choice of $\omega = \omega_{opt}$ as follows

$$\min_{\omega\ \omega} \rho(C_w) = \left(C_{\omega_{opt}}\right) = 1 - \frac{2}{k(D^{-1}A)+1} \; for \; \omega_{opt} := \frac{2}{\lambda_{max}(D^{-1}A)+\lambda_{max}(D^{-1}A)}$$

Where, $k$is the matrix condition number.

2.2. Linear programming method Game theory model

There is some relationship between Game theory and linear programming. Two-person zero-sum games can also be solved by linear programming technique. It has an additional advantage of being able to solve mixed strategy games of larger dimension payroll matrix. To illustrate the transformation of a game problem to a Linear programming problem, consider a payroll matrix of m × n size. Let a_{ij} be the element in the ith row and jth column of game payroll matrix, and letting p_i be the probabilities of m strategies (I = 1, 2, …, m) for player A. Then the expected gains for player A for each of B's strategies will be

$$\sum_{i=1}^{n} p_i a_{ij}, j = 1, 2, \dots n \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

The aim of player A is to select asset of strategies with probability pi(I = 1, 2, …, m) on any play of game such that he can maximize his minimum expected gains. To obtain values of probability pi, the value of the game to player A for all strategies by player B must be at least equal to V. thus to maximize the minimum expected gains, it is necessary that

$$\left.\begin{array}{l} a_{11}p_1 + a_{12}p_2 + \dots a_{m1}p_m \geq V \\ a_{12}p_1 + a_{22}p_2 + \dots a_{m2}p_m \geq V \\ \vdots \\ a_{1n}p_1 + a_{2n}p_2 + \dots a_{mn}p_m \geq V \end{array}\right\} \quad (12)$$

where $p_1 + p_2 + \dots + p_m = 1; \; p_i \geq 0$ for all i.

Dividing both sides of the m inequalities and equation by V, the division is valid as long as V > 0. In case V < 0, the direction of the inequality constraints must be reserved. But if V = 0, division would be meaningless. In this case, a constant can be added to all entries of the matrix ensuring that the value of the game (V) for the revised matrix becomes more than zero. After optimal solution is obtained, the true value of the game is obtained by subtracting the same constant value. Let $\frac{p_i}{V} = x_i, (\geq 0)$. Then we have

$$\left.\begin{array}{l} a_{11}\frac{p_1}{V} + a_{21}\frac{p_2}{V} + \dots a_{m1}\frac{p_m}{V} \geq 1 \\ a_{12}\frac{p_1}{V} + a_{22}\frac{p_2}{V} + \dots a_{m2}\frac{p_m}{V} \geq 1 \\ \vdots \\ a_{1n}\frac{p_1}{V} + a_{2n}\frac{p_2}{V} + \dots a_{mn}\frac{p_m}{V} \geq 1 \end{array}\right\} \quad (13)$$

where

$$\frac{p_1}{V} + \frac{p_2}{V} + \dots + \frac{p_m}{V} = 1.$$

Since the objective of player A is to maximize the value of the game, V which is equivalent to minimizing $\frac{1}{V}$, the resulting linear programming problem can be stated as

$$Minimize\ Z_p \left(= \frac{1}{V}\right) = x_1 + x_2 + \dots + x_n$$

Subject to the constraints:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots a_{m1}x_m \geq 1 \\ a_{12}x_1 + a_{22}x_2 + \dots a_{m2}x_m \geq 1 \\ \vdots \\ a_{1n}x_1 + a_{2n}x_2 + \dots a_{mn}x_m \geq 1 \\ x_i = \frac{p_i}{V} \geq 0; i = 1, 2, \dots, m \end{array}\right\} \quad \dots\dots\dots\dots\dots (14)$$

Similarly, player B has a similar problem with the inequalities of the constraints reversed, i.e. minimize the expected loss. Since minimizing of V is equivalent to maximizing $\frac{1}{V}$, therefore, the resulting linear programming problem can be stated as:

$$\text{Maximize } Z_q \left(= \frac{1}{V}\right) = y_1 + y_2 + \dots + y_n$$

Subject to the constraints.............

$$\left.\begin{array}{c} a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n \leq 1 \\ a_{12}y_1 + a_{22}y_2 + \ldots + a_{2b}y_n \leq 1 \\ . \\ \ldots\ldots\ldots\ldots\ldots \\ . \\ a_{m1}y_1 + a_{m2}y_2 + \ldots + a_{mn}y_n \leq 1 \\ \\ y_1, y_2, \ldots, y_n \geq 0 \\ \\ y_j = \frac{q_j}{V} \geq 0; j = 1, 2, \ldots, \mathrm{n} \end{array}\right\} \quad (15)$$

It may be noted that the linear programming problem of player B is the dual of linear programming problem of player A and vice versa. Therefore, solution of the dual problem can be obtained from the primal simplex table. Since for both players $Z_p = Z_q$, the expected gain to player A in the game will be exactly equal to expected loss to player B.

It should be noted that linear programming technique requires all variables to be non-negative and therefore to obtain a non-negative of value V of the game, the data of the problem, i.e. aij = 1 the payoff table should all be non-negative. If there are some negative elements in the payoff table, a constant to every element in the payoff table must be added so as to make the smallest element zero; the solution to this new game will give an optimal mixed strategy for the original game. The value of the original game then equals the value of the new game minus the constant.

2.3. Solid Waste Management Trends in Nigeria

Ebikapade and Jim (2016); investigated the current trend of solid waste management and affirm that the challenges inhibiting the attainment of sustainable solid waste management, is a major concern in Nigeria. However, they identified inadequate environmental policies and Legislations, low level of environmental awareness, poor funding and inappropriate technology; corruption and unplanned development were some of the challenges facing solid waste management in the country. They opined that for waste management to work, various aspects of Government services such as engineering, urban planning, Geography, economics, public health and law among others must be brought into waste management system. They contended that effective waste management system could be achieved when there is development of a clear policies as well as adequate enforcement to serve as a major driver towards sustainability in waste management.

Ogwueleka (2009) stated that “municipal solid waste management has emerged as one of the greatest challenges facing environmental protection agencies in developing countries especially Nigeria. Solid waste management is characterized by inefficient collection methods, insufficient coverage of the collection system and improper disposal. The waste density ranged from 280 to 370Kg/m^3 and the waste generation ranged from 0.44 to 0.66Kg/capita/day”. He identified constraints faced by environmental agencies to include lack of institutional arrangement, insufficient financial resources, absence of bye-laws and standards, inflexible work schedules, insufficient information on quantity and composition of waste, and in appropriate technology; and suggested study of institutional, political, social, financial, economic and technical aspects of municipal solid waste management in order to achieve sustainable and effective solid waste management.

The Federal Government of Nigeria has promulgated various laws and regulations to safeguard the environment. These include Federal Environmental Protection Agency (FEPA), which was created under the FEPA Act. Pursuant to the FEPA Act, each state and local government in the country set up its own environmental protection body for the protection and improvement within its jurisdiction. Municipal solid waste management is a major responsibility of state and local government environmental agencies. The agencies are charged with the responsibility of handling, employing and disposing of solid waste generated. The state agencies generate fund from subvention from state governments and internally generated revenue through sanitary levy and stringent regulations with heavy penalties for offenders of illegal dumping and littering of refuse along streets (Ogwueleka, 2003).

Municipal Solid Waste (MSW): is defined to include refuse from households, non-hazardous solid waste from industrial, commercial and institutional establishments (including hospitals), market waste, yard waste, and street sweepings. Municipal Solid Waste Management (MSWM) refers to collection, transfer, treatment, recycling, resources recovery and disposal of solid waste in urban areas. The goals of municipal solid waste management are to promote the quality of urban environment, generate employment and income, and protect environmental health and support the efficiency and productivity of the economy.

Ogwueleka (2009) observed that the volume of solid waste being generated continues to increase at a faster rate than the ability of the agencies to improve on the financial and technical resources needed to parallel this growth. The quantity of solid waste generated in urban areas in industrialized countries is higher than in developing countries; still municipal solid waste management remains inadequate in our built urban environment.

The distinction is in the areas of composition, density, political and economic framework, waste amount, access to waste for collection, awareness and attitude. The wastes are heavier, wetter and more corrosive in developing cities than developed cities. In developing countries, local authorities spend 77-95% of their revenue on collection and the balance on disposal (Ogwueleka, 2003), but can only collect almost 50-70% of municipal solid waste (MSW). In the past, the focus has been on the technical aspects of different means of collection and disposal (World Bank, 1992), but recently, attention has been on enhancing institutional arrangement to service delivery, with a special emphasis on privatization (Cointreau, 1994). Nigeria is presently experimenting with the privatization in this sector. The Federal government has instituted National Integrated Municipal Solid Waste Management Intervention Programme (NIMSWMIP) in seven (7) cities in Nigeria. The seven cities are Maiduguri, Kano, Kaduna, Onitsha, Uyo, Ota, and Lagos. Lagos state government established municipal solid waste management policy to encompass private sector participation in waste collection and transfer to designated Landfill sites.

Chukwuemeka, Ugwu and Igwegbe (2012) stated that the environment of man lies at the mercy of both natural disaster and negligence on the part of man in the course of controlling the gifts of nature. The later, takes the form of dumping solid/industrial waste in an uncompromising, desert encroachment, erosion, depletion of ozone layer, depletion of natural resources, pollution of land, rivers, seas, the air and generally the environment.

Kofoworola (2007) stated that recycling activities have increased all over the world during the last 10-15 years. The amount of resource separated from waste fractions being collected has also increased accordingly. However, the market has not been prepared to receive such vast amount of secondary raw materials, resulting in a large deviation between supply and demand. This situation leads to a low price for secondary raw materials, and in certain instance, even negative prices. At the same time, the collection and transportation costs related to all the waste fractions that are being sourced separately, have increased rapidly (Kofoworola, 2007). It is clear that the capacity of the end market should be investigated before any major recovery and recycling initiative is implemented.

Recycling Programmed Implementation Risks: In any recycling programmed development, many decisions regarding financial planning and management of the programme involves risks that must be recognized and properly allocated to the programmed participants (Clarke and Philips, 1999). The risk of most concern to communities and private firms in recycling programmes is monetary loss. Thus, the allocation of risk is the assignment of monetary loss, if it occurs, to a specific party prior to the occurrence of the loss. It must be noted that monetary loss does not refer to a net programmed loss but rather a loss exceeding that budgeted and funded for using responsible assumptions.

3. Results and Discussion

3.1. Analysis of Total Solid Waste Generated in Enugu Urban Using Forecast for the Period Generated

The total waste generated from the four (4) locations as at January, 2020 = 2,400,000 kg/day= 2,400 tons/day.

The population projection based on 2006 Census of 900319 persons generates

$$\frac{2{,}400{,}000\text{kg/day}}{900319\text{ persons}} = 2.666\text{kg/person/day}$$

With the projected population of 1,584,403 in 2056 based on next 50 years from 2006 Census adjusted for each of the four locations we have 1055894 kg/day as stated above. The total waste generated as per projected population of 2056 is

$$\frac{1584403}{900319} \times 2{,}400{,}000\text{kg/day} = 4223577.6\ kg/day\ \frac{4223577.6\ kg}{4} = 1055894\ kg/day$$

We use this to derive the Table 2 below for the quantity of options/day/tons.

Table 2 Quantity of Options/Day/Ton

Group	E-waste (X)	Plastics (X_2)	Ceramics (X_3)	Metal (X_4)	Total Available Wastes
A	$\frac{1055894}{864} \times \frac{100.2}{1000}$ = 122.46 tons	$\frac{1055894}{864} \times \frac{88.5}{1000}$ = 108.16 tons	$\frac{1055894}{864} \times \frac{1}{1000}$ = 2 tons	$\frac{1055894}{864} \times \frac{33.5}{1000}$ = 40.94 tons	$\frac{1055894}{864} \times \frac{93.4}{1000}$ = 1142 tons
B	$\frac{1055894}{864} \times \frac{31.6}{1000}$ = 38.62 tons	$\frac{1055894}{864} \times \frac{142.7}{1000}$ = 174.4tons	$\frac{1055894}{864} \times \frac{142.5}{1000}$ = 94.4 tons	$\frac{1055894}{864} \times \frac{29.2}{1000}$ = 35.68 tons	$\frac{1055894}{864} \times 1080$ = 1320 tons
C	$\frac{1055894}{864} \times \frac{90.6}{1000}$ = 111 tons	$\frac{190\ 128}{864} \times \frac{87.6}{1000}$ = 107.2 tons	$\frac{190\ 128}{864} \times \frac{96.4}{1000}$ = 117.8 tons	$\frac{190\ 128}{864} \times \frac{48.5}{1000}$ = 59.2 tons	$\frac{190\ 128}{864} \times \frac{1130}{1000}$ = 1381 tons
D	$\frac{1055894}{864} \times \frac{122.2}{1000}$ = 149.4 tons	$\frac{1055894}{864} \times \frac{40.1}{1000}$ = 49 tons	$\frac{1055894}{864} \times \frac{89.3}{1000}$ = 109.2 tons	$\frac{1055894}{864} \times \frac{142.5}{1000}$ = 174.2 tons	$\frac{1055894}{864} \times \frac{1200}{1000}$ = 1467 tons

Table 2 above shows the relationship with maximum available wastes and the various sample waste at location A, B, C, and D. Extracted from information in Appendix 2. These figures are derived based on the population projection for 2056 and will be used to formulate the constraints equation in the linear programming model.

3.2. Cost of the Recycling Options and Formulation of Linear Programming Model

Table 3 Cost of the Recycling Options

S/N	Material type/Option	Cost/kg (₦)	Cost per ton (₦ ton)
1	e-waste = 2350/kg×1.5 (X_1) = ₦3525	$\frac{1055894}{864} \times 3525$ = ₦ 4308349.6	4308
2	Plastics =77×1.5 (X_2) = ₦116	$\frac{1055894}{864} \times 116$ = ₦ 141778.32	141.8
3	Ceramics = 880×1.5 (X_3) = ₦1320	$\frac{1055894}{864} \times 1320$ = ₦ 1613339.44	1613.3
4	Metals = 846×1.5 (X_4) = ₦1269	$\frac{1055894}{864} \times 1269$ = ₦ 1551005.88	1551

The Cost per tons as Stated in Table 3 above will be used to formulate objective function for Profit maximization. Using the forgoing data, the optimization problem can be written in the form

Maximize, $Z = ₦4308\ X_1 + ₦142\ X_2 + ₦1613\ X_3 + ₦1551\ X_4$ (Objective Function)

Subject to:

$$\left.\begin{array}{ll} 122\ X_1 + 108\ X_2 + 2\ X_3 + 41\ X_4 \leq 1142 & \text{(i)} \\ 39\ X_1 + 174\ X_2 + 94\ X_3 + 36\ X_4 \leq 1320 & \text{(ii)} \\ 111\ X_1 + 107\ X_2 + 118 X_3 + 59\ X_4 \leq 1381 & \text{(iii)} \\ 149\ X_1 + 49\ X_2 + 109 X_3 + 174\ X_4 \leq 1467 & \text{(iv)} \\ X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0, \ - \ \text{Non-Negativity condition.} & \end{array}\right\} \text{System constraints}$$

Where;

- Z = Profit/day/ton/1,584,403 (Population)
- X_1= tons of optimal quantity of e-waste
- X_2 = tons of optimal quantity of Plastics
- X_3 = tons of optimal quantity of Ceramics
- X_4 = tons of optimal quantity of Metals.

This Linear Programming can be formulated using Jacobi's Method (Dass, 2000). We start with an approximation to the true solution and by applying the method repeatedly we get better and better approximation till accurate solution is achieved. This is referred to as "*Iterative Method or Indirect Methods"*. There are two iterative methods for solving the simultaneous equations;

- Jacobi's Method (Method of Simultaneous Correction)
- Gaus-Seidel Method (Method of Successive Correction).

However, the *Jacobi's iterative Method* is used for this analysis.

Note: Condition for using the iterative methods is that the coefficients in the leading diagonal are large compared to the other. If are not so, then on interchanging the equation, we can make the leading diagonal dominant diagonal.

3.3. The Jacobi's Iteration Optimization Model for Optimal Solution

To solve the Linear Programming Model using Jacobi's Iterative Method, the system constraints equation can be written as;

$$122\,X_1 + 108\,X_2 + 2\,X_3 + 41\,X_4 \leq 1142 \quad \ldots\ldots\ldots\ldots (16)$$

$$39\,X_1 + 174\,X_2 + 94\,X_3 + 36\,X_4 \leq 1320 \quad \ldots\ldots\ldots \quad (17)$$

$$111\,X_1 + 107\,X_2 + 118X_3 + 59\,X_4 \leq 1381 \quad \ldots\ldots\ldots\ldots (18)$$

$$149\,X_1 + 49\,X_2 + 109X_3 + 174\,X_4 \leq 1467 \quad \ldots\ldots\ldots\ldots (19)$$

After division of suitable constraints and transposition, the equations can be written as;

For, $122\,X_1 = 1142 - 108\,X_2 - 2X_3 - 41\,X_4$

Divide through by 122,

$$\Rightarrow X_1 = \frac{1142}{122} - \frac{108}{122}X_2 - \frac{2}{122}X_3 - \frac{41}{122}X_4 \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

For, $174\,X_2 = 1320 - 39\,X_1 - 94X_3 - 36\,X_4$

Divide through by 174,

$$\Rightarrow X_2 = \frac{1320}{174} - \frac{39}{174}X_1 - \frac{94}{174}X_3 - \frac{36}{174}X_4 \ldots\ldots \quad (21)$$

For, $118\,X_3 = 1381 - 111\,X_1 - 107X_2 - 59\,X_4$

Divide through by 118,

$$\ldots \Rightarrow X_3 = \frac{1381}{118} - \frac{111}{118}X_1 - \frac{107X_2}{118} - \frac{59}{118}X_4 \quad \ldots\ldots\ldots (22)$$

For, $174\,X_4 = 1467 - 149\,X_1 - 49X_2 - 109\,X_4$

$$\Rightarrow X_4 = \frac{1467}{174} - \frac{149}{174}X_1 - \frac{49X_2}{174} - \frac{109}{174}X_3 \quad \ldots\ldots\ldots (23)$$

Therefore,....... $X_1 = \frac{1142}{122} - \frac{108}{122}X_2 - \frac{2}{122}X_3 - \frac{41}{122}X_4$...... (24)

$$X_2 = \frac{1320}{174} - \frac{39}{174}X_1 - \frac{94}{174}X_3 - \frac{36}{174}X_4 \quad \text{........} \quad (25)$$

$$X_3 = \frac{1381}{118} - \frac{111}{118}X_1 - \frac{107}{118}X_2 - \frac{59}{118}X_4 \ldots\ldots \quad (26)$$

$$X_4 = \frac{1467}{174} - \frac{149}{174}X_1 - \frac{49}{174}X_2 - \frac{109}{174}X_3 \ldots\ldots\ldots(27)$$

OR, $X_1 = 9.36 - 0.89X_2 - 0.02X_3 - 0.34\,X_4$ ....(28)

$$X_2 = 7.59 - 0.22\,X_1 - 0.54X_3 - 0.21\,X_4 \quad \ldots\ldots\ldots\ldots(29)$$

$$X_3 = 11.70 - 0.94\,X_1 - 0.91X_2 - 0.5\,X_4 \ldots\ldots\ldots\ldots(30)$$

$$X_4 = 8.43 - 0.86\,X_1 - 0.28X_2 - 0.63X_3 \ldots\ldots\ldots\ldots\ (31)$$

First (1st) Iteration

When,

$$X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 0$$

Substituting the values in RHS of the above equations (i) to (iv)

We have,

$$X_1 = 9.36; X_2 = 7.59; X_3 = 11.70; X_4 = 8.43$$

These values are used for the second iteration.

Second (2nd) iteration

Substituting the new values of $X_1 = 9.36$; $X_2 = 7.59$; $X_3 = 11.70$; $X_4 = 8.43$ in equation (i) to (iv), we have,

$X_1 = 9.36 - 0.89(7.59) - 0.02(11.70) - 0.34\,(8.43) = \mathbf{-0.50}$

$X_2 = 7.59 - 0.22\,(9.36) - 0.54(11.70) - 0.21\,(8.43) = \mathbf{-2.56}$

$X_3 = 11.70 - 0.94\,(9.36) - 0.91(7.59) - 0.5\,(8.43) = \mathbf{-8.22}$

$X_4 = 8.43 - 0.86\,(9.36) - 0.28(7.59) - 0.63(11.70) = \mathbf{-9.12}$

Discussion on the result of the 2nd Iteration

In this 2nd iteration, the values of $X_1 = -0.50$; $X_2 = -2.56$; $X_3 = -8.22$; and $X_4 = -9.12$. These values are used for the third (3rd) iteration.

Third (3rd) Iteration

Again, substituting the new values of $X_1 = -0.50$; $X_2 = -2.56$; $X_3 = -8.22$; $X_4 = -9.12$ in equations (i) to (iv), we have,

$X_1 = 9.36 - 0.89(-2.56) - 0.02(-8.22) - 0.34\,(-9.12) = \mathbf{14.90}$

$X_2 = 7.59 - 0.22\,(-0.50) - 0.54(-8.22) - 0.21\,(-9.12) = \mathbf{14.05}$

$X_3 = 11.70 - 0.94\,(-0.50) - 0.91(-2.56) - 0.5\,(-9.12) = \mathbf{19.06}$

$X_4 = 8.43 - 0.86\,(-0.50) - 0.28(-2.56) - 0.63(-8.22) = \mathbf{14.76}$

Discussion on the results of iteration 3

The result of the 3rd iteration shows that the values of the variables are; X_1 = 14.90, X_2= 14.05, X_3 = 19.06, and X_4= 14.76 in the right hand side (RHS) of equations (i) to (iv). These values are used for the 4th iteration.

Fourth (4th) Iteration

Substituting the new values of X_1 = 14.90, X_2= 14.05, X_3 = 19.06, and X_4= 14.76 in equations (i) to (iv), we obtain,

$$X_1 = 9.36 - 0.89(14.05) - 0.02(19.06) - 0.34\,(14.76) = \mathbf{-8.54}$$

$$X_2 = 7.59 - 0.22\,(14.90) - 0.54(19.06) - 0.21\,(14.76) = \mathbf{-9.08}$$

$$X_3 = 11.70 - 0.94\,(14.90) - 0.91(14.05) - 0.5\,(14.76) = \mathbf{-22.47}$$

$$X_4 = 8.43 - 0.86\,(14.90) - 0.28(14.05) - 0.63(19.06) = \mathbf{-20.33}$$

Discussion on the results of fourth(4th) iteration

The result of the fourth (4th) iteration shows that the values of the variables are; X_1 = –8.54, X_2= –9.08, X_3 = –22.47, X_4= –20.33. These values are used for the 5th iteration.

Fifth (5th) Iteration

Substituting the new values of X_1 = –8.54, X_2= –9.08, X_3 = –22.47, and X_4= –20.33 in the RHS of equations (i) to (iv), we obtain the following;

$$X_1 = 9.36 - 0.89(-9.08) - 0.02(-22.47) - 0.34\,(-20.33) = \mathbf{24.80}$$

$$X_2 = 7.59 - 0.22\,(-8.54) - 0.54(-22.47) - 0.21\,(-20.33) = \mathbf{25.87}$$

$$X_3 = 11.70 - 0.94\,(-8.54) - 0.91(-9.08) - 0.5\,(-20.33) = \mathbf{38.15}$$

$$X_4 = 8.43 - 0.86\,(-8.54) - 0.28(-9.08) - 0.63(-22.47) = \mathbf{32.47}$$

Discussion on the results of fifth (5th) iteration

The results of the fifth iteration show that the new values of the variables are; X_1 = 24.80, X_2= 25.87, X_3 = 38.15, and X_4= 32.47. These values are used for the sixth (6th) iteration.

Sixth (6th) Iteration

Substituting the new values of X_1 = 24.80, X_2= 25.87, X_3 = 38.15, and X_4= 32.47 in the RHS of equations (i) to (iv), we obtain the following;

$$X_1 = 9.36 - 0.89(25.87) - 0.02(38.15) - 0.34\,(32.47) = -25.47$$

$$X_2 = 7.59 - 0.22\,(24.80) - 0.54(38.15) - 0.21\,(32.47) = -25.29\ldots\ldots$$

$$X_3 = 11.70 - 0.94\,(24.80) - 0.91(25.87) - 0.5\,(32.47) = -51.39$$

$$X_4 = 8.43 - 0.86\,(24.80) - 0.28(25.87) - 0.63(38.15) = -44.18$$

Discussion on the results of sixth(6th) iteration

The result of the sixth (6th) iteration shows that the values of the variables are; X_1 = –25.47, X_2= –25.29, X_3 = –51.39, and X_4= –44.18. These values are used for the seventh (7th) iteration.

Seventh (7th) Iteration

Substituting the new values of $X_1 = -25.47$, $X_2 = -25.29$, $X_3 = -51.39$, and $X_4 = -44.18$ in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(-25.29) - 0.02(-51.39) - 0.34\,(-44.18) = 47.92$$

$$X_2 = 7.59 - 0.22\,(-25.47) - 0.54(-51.39) - 0.21\,(-44.18) = 50.22\ldots\ldots$$

$$X_3 = 11.70 - 0.94\,(-25.47) - 0.91(-25.29) - 0.5\,(-44.18) = 80.75$$

$$X_4 = 8.43 - 0.86\,(-25.47) - 0.28(-25.29) - 0.63(-51.39) = 69.79$$

Discussion on the results of seventh (7th) iteration

The result of the seventh (7th) iteration shows that the values of the variables are; $X_1 = 47.92$, $X_2 = 50.22$, $X_3 = 80.75$, and $X_4 = 69.79$. These values are used for the eighth (8th) iteration.

Eighth (8th) Iteration

Substituting the new values of $X_1 = 47.92$, $X_2 = 50.22$, $X_3 = 80.75$, and $X_4 = 69.79$ in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(50.22) - 0.02(80.75) - 0.34\,(69.79) = -60.68$$

$$X_2 = 7.59 - 0.22\,(47.92) - 0.54(80.75) - 0.21\,(69.79) = -61.21$$

$$X_3 = 11.70 - 0.94\,(47.92) - 0.91(50.22) - 0.5\,(69.79) = -113.94$$

$$X_4 = 8.43 - 0.86\,(47.92) - 0.28(50.22) - 0.63(80.75) = -97.72$$

Discussion on the results of Eighth (8th) iteration

The result of the eighth (8th) iteration shows that the values of the variables are; $X_1 = -60.68$, $X_2 = -61.21$, $X_3 = -113.94$, and $X_4 = -97.72$. These values are used for the 9th iteration.

Ninth (9th) Iteration

Substituting the new values of $X_1 = -60.68$, $X_2 = -61.21$, $X_3 = -113.94$, and $X_4 = -97.72$ in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(-61.21) - 0.02(-113.94) - 0.34\,(-97.72) = 99.34$$

$$X_2 = 7.59 - 0.22\,(-60.68) - 0.54(-113.94) - 0.21\,(-97.72) = 102.99\ldots\ldots$$

$$X_3 = 11.70 - 0.94\,(-60.68) - 0.91(-61.21) - 0.5\,(-97.72) = 173.30$$

$$X_4 = 8.43 - 0.86\,(-60.68) - 0.28(-61.21) - 0.63(-113.94) = 149.54$$

Discussion on the results of ninth (9th) iteration

The result of the ninth iteration shows that the new values of the variables are; $X_1 = 99.34$, $X_2 = 102.99$, $X_3 = 173.30$, and $X_4 = 149.54$. These values are used for the tenth (10th) iteration.

Tenth (10th) Iteration

Substituting the new values of $X_1 = 99.34$, $X_2 = 102.99$, $X_3 = 173.30$, and $X_4 = 149.54$ in the RHS of equations (i) to (iv), we have the following;

$X_1 = 9.36 - 0.89(102.99) - 0.02(173.30) - 0.34\ (149.54) = -136.61$

$X_2 = 7.59 - 0.22\ (99.34) - 0.54(173.30) - 0.21\ (149.54) = -139.25$

$X_3 = 11.70 - 0.94\ (99.34) - 0.91(102.99) - 0.5\ (149.54) = -250.17$

$X_4 = 8.43 - 0.86\ (99.34) - 0.28(102.99) - 0.63(173.30) = -215.01$

Discussion on the results of Tenth (10th) iteration

The result of the tenth (10th) iteration shows that the values of the variables are; X_1 = –136.61, X_2= –139.25, X_3 = –250.17, and X_4= –215.01. These values are used for the eleventh (11th) iteration.

Eleventh (11th) Iteration

Substituting the new values of X_1 = –136.61, X_2= –139.25, X_3 = –250.17, and X_4= –215.01 in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(-139.25) - 0.02(-250.17) - 0.34\ (-215.01) = 211.40$

$X_2 = 7.59 - 0.22\ (-136.61) - 0.54(-250.17) - 0.21\ (-215.01) = 217.89\ldots\ldots$

$X_3 = 11.70 - 0.94\ (-136.61) - 0.91(-139.25) - 0.5\ (-215.01) = 374.34$

$X_4 = 8.43 - 0.86\ (-136.61) - 0.28(-139.25) - 0.63(-250.17) = 322.51$

Discussion on the results of Eleventh (11th) iteration

The result of the eleventh (11th) iteration shows that the values of the variables are; X_1 = 211.40, X_2= 217.89, X_3 = 374.34, and X_4= 322.51. These values are used for the twelfth (12th) iteration.

Twelfth (12th) Iteration

Substituting the new values of X_1 = 211.40, X_2= 217.89, X_3 = 374.34, and X_4= 322.51 in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(217.89) - 0.02(374.34) - 0.34\ (322.51) = -301.70$

$X_2 = 7.59 - 0.22\ (211.40) - 0.54(374.34) - 0.21\ (322.51) = -308.79$

$X_3 = 11.70 - 0.94\ (211.40) - 0.91(217.89) - 0.5\ (322.51) = -546.55$

$X_4 = 8.43 - 0.86\ (211.40) - 0.28(217.89) - 0.63(374.34) = -470.22$

Discussion on the results of Twelfth (12th) iteration

The result of the twelfth (12th) iteration shows that the values of the variables are; X_1 = –301.70, X_2= –308.79, X_3 = –546.55, and X_4= –470.22. These values are used for the thirteenth (13th) iteration.

Thirteenth (13th) iteration

Substituting the new values of X_1 = –301.70, X_2= –308.79, X_3 = –546.55, and X_4= –470.22 in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(-308.79) - 0.02(-546.55) - 0.34\ (-470.22) = 454.99$

$X_2 = 7.59 - 0.22\ (-301.70) - 0.54(-546.55) - 0.21\ (-470.22) = 467.85\ldots\ldots$

$X_3 = 11.70 - 0.94\ (-301.70) - 0.91(-308.79) - 0.5\ (-470.22) = 811.41$

$X_4 = 8.43 - 0.86\ (-301.70) - 0.28(-308.79) - 0.63(-546.55) = 740.92$

Discussion on the results of Thirteenth (13th) iteration

The result of the Thirteenth (13th) iteration shows that the values of the variables are; X_1 = 454.99, X_2= 467.85, X_3 = 811.46, and X_4= 740.92. These values are used for the fourteenth (14th) iteration.

Fourteenth (14th) iteration

Substituting the new values of X_1 = 454.99, X_2= 467.85, X_3 = 811.46, and X_4= 740.92 in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(467.85) - 0.02(811.41) - 0.34\ (740.92) = -675.17$

$X_2 = 7.59 - 0.22\ (454.99) - 0.54(811.41) - 0.21\ (740.92) = -686.26$

$X_3 = 11.70 - 0.94\ (454.99) - 0.91(467.85) - 0.5\ (740.92) = -1212.19$

$X_4 = 8.43 - 0.86\ (454.99) - 0.28(467.85) - 0.63(811.41) = -1025.05$

Discussion on the results of Fourteenth (14th) iteration

The result of the fourteenth (14th) iteration shows that the values of the variables are; X_1 = –675.17, X_2= –686.26, X_3 = –1212.19, and X_4= –1025.05. These values are used for the fifteenth (15th) iteration.

Fifteenth (15th) iteration

Substituting the new values of X_1 = –675.17, X_2= –686.26, X_3 = –1212.19, and X_4= –1025.05 in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(-686.26) - 0.02(-1212.19) - 0.34\ (-1025.05) = 992.89$

$X_2 = 7.59 - 0.22\ (-675.17) - 0.54(-1212.19) - 0.21\ (-1025.05) = 1025.97$

$X_3 = 11.70 - 0.94\ (-675.17) - 0.91(-686.26) - 0.5\ (-1025.05) = 1783.38$

$X_4 = 8.43 - 0.86\ (-675.17) - 0.28(-686.26) - 0.63(-1212.19) = 1544.91$

Discussion on the results of Fifteenth (15th) iteration

The result of the Fifteenth (15th) iteration shows that the values of the variables are; X_1 = 992.89, X_2= 1025.97, X_3 = 1783.38, and X_4= 1544.91. These values are used for the sixteenth (16th) iteration.

Sixteenth (16th) iteration

Substituting the new values of X_1 = 992.89, X_2= 1025.97, X_3 = 1783.38, and X_4= 1544.91in the RHS of equations (i) to (iv), we have the following result;

$X_1 = 9.36 - 0.89(1025.97) - 0.02(1783.38) - 0.34\ (1544.91) = -1464.69$

$X_2 = 7.59 - 0.22\ (992.89) - 0.54(1783.38) - 0.21\ (1544.91) = -1498.30$

$X_3 = 11.70 - 0.94\ (992.89) - 0.91(1025.97) - 0.5\ (1544.91) = -2627.70$

$X_4 = 8.43 - 0.86\ (992.89) - 0.28(1025.97) - 0.63(1783.38) = -2256.25$

Discussion of results in Sixteenth (16th) iteration

The result of the Sixteenth (16th) iteration shows that the values of the variables are; $X_1 = -1464.69$, $X_2= -1498.30$, $X_3 = -2627.70$, and $X_4= -2256.26$. These values will be used for the seventeenth (17th) iteration. It is worthy to note that values cannot converge on negative values hence, the need for the next iteration.

Seventeenth (17th) iteration

Substituting the new values of $X_1 = -1464.69$, $X_2= -1498.30$, $X_3 = -2627.70$, and $X_4= -2256.26$ in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(-1498.30) - 0.02(-2627.70) - 0.34(-2256.26) = 2162.03$$

$$X_2 = 7.59 - 0.22(-1464.69) - 0.54(-2627.70) - 0.21(-2256.26) = 2222.09$$

$$X_3 = 11.70 - 0.94(-1464.69) - 0.91(-1498.30) - 0.5(-2256.26) = 3880.09$$

$$X_4 = 8.43 - 0.86(-1464.69) - 0.28(-1498.30) - 0.63(-2627.70) = 3343.04$$

Discussion on the results of Seventeenth (17th) iteration

The result of the Seventeenth (17th) iteration shows that the values of the variables are; $X_1 = 2162.03$, $X_2= 2222.09$, $X_3 = 3880.09$, and $X_4= 3343.04$. The iteration shows that it has reached a convergence point where the respective values of X_1, X_2, X_3, X_4have attained the optimal solution. Therefore, the optimal solutions for the variables are actual values which are; $X_1 = 2162$, $X_2= 2222$, $X_3 = 3880$, and $X_4= 3343$.

Table 4 Summary of Results of Jacobi's Iteration Model

S/N	Iterations	0	1	2	3	4	5	6	7	8	9	10
1	X_4=9.36–0.89X_2 –0.02X_3 – 0.34X_4	0	9.36	– 0.50	14.90	– 8.54	24.80	– 25.47	47.92	– 60.68	99.34	– 136.61
2	X_2=7.59–0.22X_1 –0.34X_3 – 0.21X_4	0	7.59	– 2.56	14.05	– 9.08	25.87	– 25.29	50.22	– 61.21	102.99	– 139.25
3	X_3=11.70–0.94X_1–0.91X_2– 0.50X_4	0	11.70	– 8.22	19.06	– 22.47	38.15	– 51.39	80.75	- 113.94	173.30	– 250.17
4	X_4=8.43–0.86X_1 –0.28X_2 – 0.63X_3	0	8.43	– 9.12	14.76	– 20.33	32.47	– 44.18	69.79	– 97.72	149.54	– 215.01

S/N	Iterations Continued	11	12	13	14	15	16	17
1	X_4=9.36–0.89X_2 –0.02X_3 –0.34X_4	211.40	– 301.70	454.99	– 675.17	992.89	– 1464.69	2162.03
2	X_2=7.59–0.22X_1 –0.34X_3 –0.21X_4	217.89	– 308.79	467.85	– 686.26	1025.97	– 1498.30	2222.09
3	X_3=11.70–0.94X_1–0.91X_2–0.50X_4	374.34	– 546.55	811.41	– 113.94	1212.19	– 2627.70	3880.09
4	X_4=8.43–0.86X_1 –0.28X_2 –0.63X_3	322.51	– 470.22	740.92	– 97.72	1025.05	– 2256.26	3343.04

These values of X_1, X_2, X_3, and X4are now substituted in the objective function.

Maximize $Z = 4308X_1 + 142X_2 + 1613 X_3 + 1551X_4$

Therefore, the optimal solution is

Z = 4308× 2162 + 142× 2222 + 1613× 3880 + 1551× 3343.

= ₦ 21,072,853 per ton per day,

= ₦ 21.07 million per ton per day,

In a week, the waste generated will yield a revenue of ₦ 21,072,853.00 × 7 days = ₦147,509,971.00 = ₦147.51 million per ton per week

Then, the Annual cost will be

₦147,509,971.00 × 52 weeks = ₦7, 670,518,492 per ton per annum

₦7.67 billion per ton per annum

However, The Total Revenue generated are as follows:

For a day:

Total revenue = ₦ 21,072,853.00 × 2400 tons =₦ 50,574,847,200.00

=₦ 50.575 billion daily

For a week:

Total revenue = ₦ 147,509,971.00 × 2400 tons = ₦ 354,023,930,400.00

=₦ 354.024 billion weekly

In a year:

Total revenue = ₦7, 670,518,492 × 2400 tons = ₦18,409,244,380,000.00

= ₦ 18.409 trillion annually.

3.4. Comparison of Jacobi's Optimization Model to Game Theory Model

In modeling Jacobi's Model into Game theory model, we have to formulate the matrix for the computation of waste generation in Enugu urban. The difference from the characterized wastes was classified as organic wastes. The total waste from the option separated were deducted from the total waste generated from the projected period 2056 to determine the total organic waste generated at each location. Also some quantity of wastes generated in some wastes location at Enugu East and Enugu North were separated to create the fifth location referred to as Enugu East and the former Enugu East were renamed Enugu East Central in order to have a 5 × 5 matrix for the purpose of determining the Game theory model. This resulted to the information in Table 5 with the same quantity of tons of waste generated per day in the city.

Table 5 Quantity of waste options generated at various locations in Enugu urban

Waste locations	Tons of available wastes generated					
	E-waste (x_1)	Plastics (x_2)	Ceramics (x_3)	Metals (x_4)	Organic wastes (x_5)	Total wastes
Enugu South (A_1)	122	108	3	41	869	1142
Enugu East Central (A_2)	31	142	82	28	752	1035
Enugu North East (A_3)	111	107	118	59	986	1381
Enugu North Central (A_4)	138	41	60	139	779	1157

Enugu East (A_5)	19	40	61	43	432	595
Total	421	438	323	310	3818	5310

The total revenue generated of N50.575 billion per day were used to determine the benefits of each of the tons of waste generated using pro-rata adjustment as shown below.

First (1st) row:

$$(i).\frac{122}{1142} \times 50.575 = 5.40\ billion$$
$$(ii).\frac{108}{1142} \times 50.575 = 4.78\ billion$$
$$(iii).\frac{2}{1142} \times 50.575 = 0.09\ billion$$
$$(iv).\frac{41}{1142} \times 50.575 = 1.82\ billion$$
$$(v).\frac{869}{1142} \times 50.575 = 38.48\ billion$$

Second (2nd) row:

$$(i).\frac{31}{1035} \times 50.575 = 1.51\ billion$$
$$(ii).\frac{142}{1035} \times 50.575 = 6.94\ billion$$
$$(iii).\frac{82}{1035} \times 50.575 = 4.01\ billion$$
$$(iv).\frac{28}{1035} \times 50.575 = 1.37\ billion$$
$$(v).\frac{752}{1035} \times 50.575 = 36.75\ billion$$

Third (3rd) row:

$$(i).\frac{111}{1381} \times 50.575 = 4.07\ billion$$
$$(ii).\frac{107}{1381} \times 50.575 = 3.92\ billion$$
$$(iii).\frac{118}{1381} \times 50.575 = 4.32\ billion$$
$$(iv).\frac{59}{1381} \times 50.575 = 2.16\ billion$$
$$(v).\frac{986}{1381} \times 50.575 = 36.11\ billion$$

Fourth (4th) row:

$$(i).\frac{138}{1157} \times 50.575 = 6.03\ billion$$
$$(ii).\frac{41}{1157} \times 50.575 = 1.79\ billion$$
$$(iii).\frac{60}{1157} \times 50.575 = 2.62\ billion$$
$$(iv).\frac{139}{1157} \times 50.575 = 6.08\ billion$$
$$(v).\frac{779}{1157} \times 50.575 = 34.05\ billion$$

Fifth (5th) row:

$$(i).\frac{19}{595} \times 50.575 = 1.62\ billion$$
$$(ii).\frac{40}{595} \times 50.575 = 3.40\ billion$$
$$(iii).\frac{61}{595} \times 50.575 = 5.19\ billion$$
$$(iv).\frac{43}{595} \times 50.575 = 3.66\ billion$$
$$(v).\frac{432}{595} \times 50.575 = 36.72\ billion$$

These costs are summarized in Table 6.

Table 6 Summary of costs/benefits of various waste types generated

Player A	**Player B**					**Minimum**
	B_1 (X_1)	**B_2 (X_2)**	**B_3 (X_3)**	**B_4 (X_4)**	**B_5 (X_5)**	
A_1	5.40	4.78	0.09	1.82	38.48	0.09
A_2	--1.51	6.94	4.01	1.37	36.75	1.37
A_3	4.07	3.92	4.32	2.16	36.11	2.16
A_4	6.03	1.79	2.62	6.08	34.05	1.79
A_5	1.62	3.40	5.19	3.66	36.72	1.62
Maximum	6.03	6.94	5.19	6.08	38.48	

Minimum = 2.16

Maximum = 5.19

Since there is no saddle point, the value of the game will be calculated through iteration. Also no row or column is completely dominated by the other.

So, we apply the linear programming of game theory.

Let the value of the game = V and q_1, q_2, q_3, q_4, q_5 be the probabilities of selecting the strategies B_1, B_2, B_3, B_4, B_5 respectively.

The objectives/benefits is to maximize value of the wastes which will be generated from Table 6.

$$\left\{\begin{matrix} 5.40q_1 + 4.78q_2 + 0.09q_3 + 1.82q_4 + 38.48q_5 \leq V \\ 1.51q_1 + 6.94q_2 + 4.01q_3 + 1.37q_4 + 36.75q_5 \leq V \\ 4.07q_1 + 3.92q_2 + 4.32q_3 + 2.16q_4 + 36.11q_5 \leq V \\ 6.03q_1 + 1.79q_2 + 2.62q_3 + 6.08q_4 + 34.05q_5 \leq V \\ 1.62q_1 + 3.4q_2 + 5.19q_3 + 3.66q_4 + 36.72q_5 \leq V \\ q_1 + q_2 + q_3 + q_4 + q_5 = 1\ (Probabilitycondition) \end{matrix}\right\} \dots\dots\dots \quad (32)$$

Divide Equations 32 through by V, we have;

$$\left\{\begin{matrix} 5.40q_1/V + 4.78q_2/V + 0.09q_3/V + 1.82q_4/V + 38.48q_5/V \leq 1 \\ 1.51q_1/V + 6.94q_2/V + 4.01q_3/V + 1.37q_4/V + 36.75q_5/V \leq 1 \\ 4.07q_1/V + 3.92q_2/V + 4.32q_3/V + 2.16q_4/V + 36.11q_5/V \leq 1 \\ 6.03q_1/V + 1.79q_2/V + 2.62q_3/V + 6.08q_4/V + 34.05q_5/V \leq 1 \\ 1.62q_1/V + 3.4q_2/V + 5.19q_3/V + 3.66q_4/V + 36.72q_5/V \leq 1 \\ q_1 */V + q_2/V + q_3/V + q_4/V + q_5/V = 1 \end{matrix}\right\} \dots \quad (33)$$

$$\frac{letq_1}{V} = \frac{x_1,q_2}{V} = x_2, \frac{q_3}{V} = x_3, \frac{q_4}{V} = x_4 and, \frac{q_5}{V} = x_5 \quad (34)$$

The values in equations 33 and 34 are converted into a linear programming problem as;

$$MaximizeZ_p = \left(\frac{1}{V}\right) = x_1 + x_2 + x_3 + x_4 + x_5$$

Subject to:

$$\left.\begin{matrix} 5.4x_1 + 4.78x_2 + 0.09x_3 + 1.82x_4 + 38.48x_5 \leq 1 \\ 1.51x_1 + 6.94x_2 + 4.01x_3 + 1.37x_4 + 36.75x_5 \leq 1 \\ 4.07x_1 + 3.92x_2 + 4.32x_3 + 2.16x_4 + 36.11x_5 \leq 1 \ldots\ldots \\ 6.03x_1 + 1.97x_2 + 2.62x_3 + 6.08x_4 + 34.05x_5 \leq 1 \\ 1.62x_1 + 3.40x_2 + 5.19x_3 + 3.66x_4 + 36.72x_5 \leq 1 \end{matrix}\right\} \quad (35)$$

Since we have more than two variables, the simplex method of linear programming is used to solve the problem by introducing slack variables to convert the inequalities to equations which becomes;

$$MaximizeZ_q = \left(\frac{1}{V}\right) = x_1 + x_2 + x_3 + x_4 + x_5 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5$$

Subject to the following constraints;

$$5.4x_1 + 4.78x_2 + 0.09x + 1.82x_4 + 38.48x_5 + S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 = 1$$

$$1.51x_1 + 6.94x_2 + 4.01x_3 + 1.37x_4 + 36.75x_5 + 0S_1 + S_2 + 0S_3 + 0S_4 + 0S_5 = 1$$

$$4.07x_1 + 3.92x_2 + 4.32x_3 + 2.16x_4 + 36.11x_5 + 0S_1 + 0S_2 + S_3 + 0S_4 + 0S_5 = 1 \quad (36)$$

$$6.03x_1 + 1.97x_2 + 2.62x_3 + 6.08x_4 + 34.05x_5 + 0S_1 + 0S_2 + 0S_3 + S_4 + 0S_5 = 1$$

$$1.62x_1 + 3.40x_2 + 5.19x_3 + 3.66x_4 + 36.72x_5 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + S_5 = 1$$
$$x_1, x_2, x_3, x_4, x_5, S_1, S_2, S_3, S_4, S_5 \geq 0$$

The equations formulated are used to solve the Simplex method by forming the initial Simplex table.

Table 7 Initial Simplex Table

.......Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	Amount	
Basis	C_j	1	1	1	1	**1**	0	0	0	0	0		Trade ratio
S_1	0	**5.40**	**4.78**	**0.09**	**1.82**	**38.48**	**1**	**0**	**0**	**0**	**0**	**1**	$\frac{1}{38.48} = 0.026$ ←
S_2	0	1.51	6.94	4.01	1.37	**36.75**	0	1	0	0	0	1	$\frac{1}{36.75} = 0.027$
S_3	0	4.07	3.92	4.32	2.16	**36.11**	0	0	1	0	0	1	$\frac{1}{36.11} = 0.028$
S_4	0	6.03	1.79	2.62	6.08	**34.05**	0	0	0	1	0	1	$\frac{1}{34.05} = 0.029$
S_5	0	1.62	3.40	5.19	3.66	**36.72**	0	0	0	0	1	1	$\frac{1}{36.72} = 0.027$
Z_j		0	0	0	0	**0**	0	0	0	0	0	0	
$C_j - Z_j$		1	1	1	1	**1**	0	0	0	0	0		

↑ **.......** Bring in

Table 8 Second (2nd) Simplex table

Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_5	1	0.14	0.12	**0.002**	0.05	1	0.026	0	0	0	0	0.026	$\frac{0.026}{0.002} = 13$
S_2	0	-3.64	6.94	**4.01**	1.37	0	-0.96	1	0	0	0	0.045	$\frac{0.045}{3.94} = 0.011$
S_3	0	-0.99	3.92	**4.32**	2.16	0	-0.94	0	1	0	0	0.061	$\frac{0.061}{4.25} = 0.014$
S_4	0	1.26	1.79	**2.62**	6.08	0	-0.89	0	0	1	0	0.115	$\frac{0.115}{2.55} = 0.045$
S_5	**0**	**-3.52**	**3.40**	**5.19**	3.66	**0**	**-0.95**	**0**	**0**	**0**	**1**	**0.045**	$\frac{0.045}{4.24} = 0.011$ ***take out***
Z_j		0.14	0.12	0.002	0.05	1	0.026	0	0	0	0	0.026	
$C_j - Z_j$		0.36	0.88	0.998	0.95	0	-0.026	0	0	0	0		

Bring in

Computing of values for S_2 Row	Computing of values for S_3 Row
$x_1 = 1.51 - 0.14 \times 36.75 = -3.64$	$x_1 = 4.07 - 0.14 \times 36.11 = -3.64$
$x_2 = 6.94 - 0.12 \times 36.75 = 2.53$	$x_2 = 3.92 - 0.12 \times 36.11 = 2.53$
$x_3 = 4.01 - 0.002 \times 36.75 = 3.94$	$x_3 = 4.32 - 0.002 \times 36.11 = 3.94$
$x_4 = 1.37 - 0.05 \times 36.75 = 0.47$	$x_4 = 2.16 - 0.05 \times 36.11 = 0.47$
$x_5 = 6.94 - 1 \times 36.75 = 0$	$x_5 = 36.11 - 1 \times 36.11 = 0$
$S_1 = 0 - 0.026 \times 36.75 = -0.96$	$S_1 = 0 - 0.026 \times 36.11 = -0.94$
$S_2 = 1 - 0 \times 36.75 = 1$	$S_2 = 0 - 0 \times 36.11 = 0$
$S_3 = 0 - 0 \times 36.75 = 0$	$S_3 = 1 - 0 \times 36.11 = 1$
$S_4 = 0 - 0 \times 36.75 = 0$	$S_4 = 0 - 0 \times 36.11 = 0$
$S_5 = 0 - 0 \times 36.75 = 0$	$S_5 = 0 - 0 \times 36.11 = 0$
Amount = $S_2 = 1 - 0.026 \times 36.75 = 0.045$	Amount = $S_3 = 1 - 0.026 \times 36.11 = 0.061$
Computing of values for S_4 Row	Computing of values for S_5 Row
$x_1 = 6.03 - 0.14 \times 34.05 = 1.26$	$x_1 = 1.62 - 0.14 \times 36.72 = -3.52$
$x_2 = 1.97 - 0.12 \times 34.05 = -2.12$	$x_2 = 3.40 - 0.12 \times 36.72 = -1.01$
$x_3 = 2.62 - 0.002 \times 34.05 = 2.55$	$x_3 = 5.19 - 0.002 \times 36.72 = 4.24$
$x_4 = 6.08 - 0.05 \times 34.05 = 4.38$	$x_4 = 3.66 - 0.05 \times 36.72 = 1.82$
$x_5 = 34.05 - 1 \times 34.05 = 0$	$x_5 = 36.72 - 1 \times 36.72 = 0$
$S_1 = 0 - 0.026 \times 34.05 = -0.89$	$S_1 = 0 - 0.026 \times 36.72 = -0.95$
$S_2 = 0 - 0 \times 34.05 = 0$	$S_2 = 0 - 0 \times 36.72 = 0$
$S_3 = 0 - 0 \times 34.05 = 0$	$S_3 = 0 - 0 \times 36.72 = 0$
$S_4 = 1 - 0 \times 34.05 = 1$	$S_4 = 0 - 0 \times 36.72 = 0$
$S_5 = 0 - 0 \times 34.05 = 0$	$S_5 = 1 - 0 \times 36.72 = 1$
Amount = $S_4 = 1 - 0.026 \times 34.05 = 0.115$	Amount = $S_5 = 1 - 0.026 \times 36.72 = 0.045$

Table 9 Third (3rd) Simplex table

Variables		**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**		
Basis	**C_j**	**1**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**Amount**	**Trade ratio**
x_3	1	**-0.83**	-0.23	1	0.43	0	-0.22	0	0	0	0.24	0.011	$\frac{0.011}{-0.83} = 0.013$
x_5	1	**0.14**	0.12	0	0.05	1	-0.026	0	0	0	-0.005	0.026	$\frac{0.026}{0.14} = 0.186$
S_2	0	**-0.37**	3.44	0	-1.22	0	-0.09	1	0	0	-0.95	0.002	$\frac{0.002}{0.37} = 0.0054$
S_3	0	**2.54**	**0.57**	**0**	**-1.48**	**0**	**-0.005**	**0**	**1**	**0**	**-1.02**	**0.014**	$\frac{0.014}{2.54} = 0.0055$ ←
S_4	0	3.38	-1.53	0	3.28	0	-0.33	0	0	1	-0.61	0.087	$\frac{0.087}{3.38} = 0.0257$
Z_j		-0.69	-0.11	1	0.48	1	-0.19	0	0	0	0.24	0.037	
$C_j - Z_j$		1.69	1.11	0	0.52	0	0.19	0	0	0	-0.24		

↑ Bring in

Computing of values for X_5 Row	Computing of values for S_2 Row
x_1 = 0.14 – (-0.83 × 0.002) = 1.14 x_2 = 0.12 – (-0.23 × 0.002) = 0.12 x_3 = 0.002 – 1 × 0.002 = 0 x_4 = 0.05 – 0.43 × 0.002 = 0.05 x_5 = 1 – 0 × 0.002 = 1 S_1 = – 0.026 – (-0.22 × 0.002) = - 0.026 S_2 = 0 – 0 × 0.002 = 0 S_3 = 0 – 0 × 0.002 = 0 S_4 = 0 – 0 × 0.002 = 0 S_5 = 0 – 0.24 × 0.002 = -0.0005 Amount = 0.026 – 0.011× 0.002 = 0.026	x_1 = -3.64 – (-0.83 × 3.94) = -0.37 x_2 = 0.12 – (-0.23 × 3.94) = 3.44 x_3 = 3.94 – 1 × 3.94 = 0 x_4 = 0.47 – 0.43 × 3.94 = -1.22 x_5 = 0 – 0 × 3.94 = 0 S_1 = – 0.96 – (-0.22 × 3.94) = - 0.09 S_2 = 1 – 0 × 3.94 = 1 S_3 = 0 – 0 × 3.94 = 0 S_4 = 0 – 0 × 3.94 = 0 S_5 = 0 – 0.24 × 3.94 = -0.95 Amount = 0.045 – 0.011× 3.94 = 0.002
Computing of values for S_3 Row x_1 = -0.99 – (-0.83 × 4.25) = 2.54 x_2 = 0.41 – (-0.23 × 4.25) = 0.57 x_3 = 4.25 – 1 × 4.25 = 0 x_4 = 0.35 – 0.43 × 4.25 = -1.48 x_5 = 0 – 0 × 4.25 = 0 S_1 = – 0.94 – (-0.22 × 4.25) = - 0.05 S_2 = 0 – 0 × 4.25 = 0 S_3 = 1 – 0 × 4.25 = 1 S_4 = 0 – 0 × 4.25 = 0 S_5 = 0 – 0.24 × 4.25 = -1.02 Amount = 0.061 – 0.011× 4.25 = 0.014	Computing of values for S_4 Row x_1 = 1.26 – (-0.83 × 2.55) = 3.38 x_2 = -2.12 – (-0.23 × 2.55) = -1.53 x_3 = 2.55 – 1 × 2.55 = 0 x_4 = 4.38 – 0.43 × 2.55 = 3.28 x_5 = 0 – 0 × 2.55 = 0 S_1 = – 0.89 – (-0.22 × 2.55) = - 0.33 S_2 = 0 – 0 × 2.55 = 0 S_3 = 0 – 0 × 2.55 = 0 S_4 = 1 – 0 × 2.55 = 1 S_5 = 0 – 0.24 × 2.55 = -0.61 Amount = 0.061 – 0.011× 2.55 = 0.087

Table 10 Fourth (4th) Simplex table

Variables		**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**		
Basis	C_j	1	1	1	**1**	1	0	0	0	0	0	Amount	Trade ratio
x_1	1	1	0.22	0	**-0.58**	0	-0.02	0	0.39	0	-0.40	0.0055	$\frac{0.0055}{-0.58} = 0.009$
x_3	1	0	-0.05	1	**0.05**	0	-0.22	0	0.32	0	-0.09	0.016	$\frac{0.016}{0.05} = 0.32$
x_5	1	0	0.09	0	**0.13**	1	0.026	1	-0.05	0	0.06	0.025	$\frac{0.025}{0.13} = 0.192$
S_3	0	0	3.52	0	**-1.43**	0	-0.09	0	0.14	0	-1.10	0.004	$\frac{0.004}{-1.43} = 0.0028$
S_4	0	**0**	**-2.27**	**0**	**5.24**	**0**	**-0.32**	**0**	**-1.32**	**1**	**0.74**	**0.068**	$\frac{0.068}{5.24} = 0.013$ take out ←
Z_j		1	0.26	1	-0.4	1	-0.20	0	0.66	0	-0.53	0.047	
$C_j - Z_j$		0	0.74	0	1.4	0	0.20	0	-0.66	0	0.53		

↑ Bring in

Computing of values for X_3 Row	Computing of values for S_2 Row
x_1 = -0.83 – (1 × – 0.83) = 0	x_1 = -0.37 – (1 × – 0.37) = 0
x_2 = -0.23 – 0.22 × –0.83 = -0.047	x_2 = 3.44 – 0.22 × –0.37 = 3.52
x_3 = 1–0 × –0.83 = 1	x_3 = 0–0 × –0.37 = 0
x_4 = –0.43– (–0.58× –0.83) = 0.051	x_4 = –1.22 – (–0.58× –0.37) = – 1.43
x_5 = 0 – 0 × –0.83 = 0	x_5 = 0 – 0 × –0.37 = 0
S_1 = – 0.22 – (-0.002 × –0.83) = - 0.22	S_1 = – 0.09 – (– 0.002 × –0.37) = – 0.091
S_2 = 1 – 0 × –0.83 = 0	S_2 = 1 – 0 × –0.37 = 1
S_3 = 0 – 0.39 × –0.83 = 0.32	S_3 = 0 – 0.39 × –0.37 = 0.14
S_4 = 0 – 0 × –0.83 = 0	S_4 = 0 – 0 × –0.37 = 0
S_5 = 0 – 0.24 –(–0.40 ×–0.83) = -0.09	S_5 = 0 – 0.95 –(–0.40 ×–0.37) = -1.10
Amount = 0.011 – 0.0055× –0.83 = 0.016.	Amount = 0.002 – 0.0055× –0.37 = 0.004.

Computing of values for X_5 Row	Computing of values for S_4 Row
x_1 = 0.14 – 1 × – 0.14 = 0	x_1 = 3.38 – 1 × – 3.38 = 0
x_2 = 0.12 – 0.22 × 0.14 = 0.09	x_2 = –1.53 – 0.22 × 3.38 = –2.27
x_3 = 0 – 0 × 0.14 = 0	x_3 = 0 – 0 × 3.38 = 0
x_4 = 0.05 – (–0.58× 0.14) = 0.13	x_4 = 3.28 – (–0.58× 3.38) = 5.24
x_5 = 1 – 0 × 0.14 = 1	x_5 = 0 – 0 × 3.38 = 0
S_1 = 0.026 – (– 0.002 × 0.14) = – 0.026	S_1 = – 0.33 – (– 0.002 × 3.38) = – 0.32
S_2 = 0 – 0 × 0.14 = 0	S_2 = 0 – 0 × 3.38 = 0
S_3 = 0 – 0.39 × 0.14 = 0.05	S_3 = 0 – 0.39 × 3.38 = –1.32
S_4 = 0 – 0 × 0.14 = 0	S_4 = 1 – 0 × 3.38 = 1
S_5 = – 0.0005 –(–0.40 × 0.14) = 0.06	S_5 = – 0.61 –(–0.40 × 3.38) = 0.74
Amount = 0.026 – 0.0055× 0.14 = 0.025.	Amount = 0.087 – 0.0055× 3.38 = 0.068.

Table 11 Fifth (5th) Simplex table

Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_4	1	1	**-0.43**	0	1	0	-0.06	0	-0.25	0.19	0.14	0.013	$\frac{0.013}{-0.43} = -0.009$
x_1	1	0	**-0.03**	0	0	0	0.03	0	0.25	0.11	-0.32	0.013	$\frac{0.013}{-0.03} = -0.455$
x_3	1	0	**-0.03**	1	0	0	-0.22	0	0.33	-0.01	-0.1	0.015	$\frac{0.015}{-0.03} = -0.5$
x_5	1	0	**0.15**	0	0	1	0.03	0	0.02	-0.03	0.04	0.023	$\frac{0.023}{0.15} = 0.153$
S_2	0	**0**	**2.91**	**0**	**0**	**0**	**-0.18**	**1**	**-0.22**	**0.27**	**-0.90**	**0.023**	$\frac{0.023}{2.91} = 0.0079$ take out
Z_j		1	-0.34	1	1	1	-0.22	0	0.35	0.26	-0.24	0.064	
$C_j - Z_j$		0	1.34	0	0	0	0.22	0	-0.35	-0.26	0.24		

Bring in

Computing of values for X_1 Row	Computing of values for X_5 Row
$x_1 = 1 - 0 \times -0.58 = 1$	$x_1 = 0 - 0 \times 0.13 = 0$
$x_2 = -0.22 - (-0.43 \times -0.58) = 0.03$	$x_2 = 0.09 - (-0.43 \times 0.13) = 0.15$
$x_3 = 0 - 0 \times -0.58 = 0$	$x_3 = 0 - 0 \times 0.13 = 0$
$x_4 = 0.58 - 1 \times -0.58 = 0$	$x_4 = 0.13 - 1 \times 0.13 = 0$
$x_5 = 0 - 0 \times -0.58 = 0$	$x_5 = 1 - 0 \times 0.13 = 1$
$S_1 = -0.002 - 0.06 \times -0.58 = 0.03$	$S_1 = 0.026 -(-0.06 \times 0.13) = 0.03$
$S_2 = 0 - 0 \times -0.58 = 0$	$S_2 = 0 - 0 \times 0.13 = 0$
$S_3 = 0.39 - (-0.25 \times -0.58) = 0.25$	$S_3 = -0.05 - (-0.25 \times 0.13) = 0.02$
$S_4 = 0 - 0.19 \times -0.58 = 0.11$	$S_4 = 0 - 0.19 \times 0.13 = -0.025$
$S_5 = -0.40 - 0.14 \times -0.58 = -0.32$	$S_5 = -0.06 - 0.14 \times 0.13 = 0.042$
Amount = $0.0055 - 0.013 \times -0.58 = 0.013$.	Amount = $0.025 - 0.013 \times 0.13 = 0.023$.
Computing of values for X_3 Row	Computing of values for S_2 Row
$x_1 = 0 - 0 \times 0.05 = 0$	$x_1 = 0 - 0 \times -1.43 = 0$
$x_2 = -0.05 - (-0.43 \times 0.05) = -0.03$	$x_2 = 3.52 - (-0.43 \times -1.43) = 2.91$
$x_3 = 1 - 0 \times 0.05 = 1$	$x_3 = 0 - 0 \times -1.43 = 0$
$x_4 = 0.05 - 1 \times 0.05 = 0$	$x_4 = 1.43 - 1 \times -1.43 = 0$
$x_5 = 0 - 0 \times 0.05 = 0$	$x_5 = 0 - 0 \times -1.43 = 0$
$S_1 = -0.22 -(-0.06 \times 0.05) = -0.22$	$S_1 = -0.09 -(-0.06 \times -1.43) = 0.18$
$S_2 = 0 - 0 \times 0.05 = 0$	$S_2 = 1 - 0 \times -1.43 = 1$
$S_3 = 0.32 - (-0.25 \times 0.05) = 0.33$	$S_3 = 0.14 - (-0.25 \times -1.43) = -0.22$
$S_4 = 0 - 0.19 \times 0.05 = -0.010$	$S_4 = 0 - 0.19 \times -1.43 = -0.25$
$S_5 = -0.00 - 0.14 \times 0.05 = -0.10$	$S_5 = -1.10 - 0.14 \times -1.43 = 0.042$
Amount = $0.016 - 0.013 \times 0.05 = 0.015$.	Amount = $0.004 - 0.013 \times -1.43 = 0.023$.

Table 12 Sixth (6th) Simplex Table (Optimal Solution)

Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount
x_2	1	0	1	0	0	0	-0.06	0.34	-0.08	0.09	-0.31	**0.008**
x_4	1	1	0	0	1	0	-0.03	0.15	0.22	0.23	0.07	**0.0164**
x_1	1	0	0	0	0	0	0.03	0.01	0.25	0.11	-0.33	**0.0132**
x_3	1	0	0	1	0	0	-0.22	0.01	0.33	0.01	0.33	**0.0152**
x_5	1	0	0	0	0	1	0.04	-0.05	0.03	-0.04	-0.09	**0.0218**
Z_j		1	1	1	1	1	-0.24	0.46	0.75	0.4	-0.92	**0.0746**
$C_j - Z_j$		0	0	0	0	0	0.24	-0.46	-0.75	-0.4	0.92	

Computing of values for X_4 Row $x_1 = 1 - 0 \times - 0.43 = 1$ $x_2 = -0.43 - 1 \times -0.43 = 0$ $x_3 = 0 - 0 \times -0.43 = 0$ $x_4 = 1 - 0 \times -0.43 = 1$ $x_5 = 0 - 0 \times -0.43 = 0$ $S_1 = - 0.006 - 0.06 \times -0.43 = 0.03$ $S_2 = 0 - 0.34 \times -0.43 = 0.15$ $S_3 = - 0.25 - (- 0.25 \times -0.43) = 0.22$ $S_4 = 0.19 - 0.09 \times -0.43 = 0.23$ $S_5 = - 0.14 -(-0.31 \times -0.43 = 0.007$ Amount= $0.013 - 0.008 \times -0.43 = 0.0164$.	Computing of values for X_1 Row $x_1 = 0 - 0 \times - 0.03 = 0$ $x_2 = -0.03 - 1 \times -0.03 = 0$ $x_3 = 0 - 0 \times -0.03 = 0$ $x_4 = 0 - 0 \times -0.03 = 0$ $x_5 = 0 - 0 \times -0.03 = 0$ $S_1 = - 0.03 - (-0.06 \times -0.03) = 0.028$ $S_2 = 0 - 0.34 \times -0.03 = 0.01$ $S_3 = 0.25 - (- 0.08 \times -0.03) = 0.25$ $S_4 = 0.11 - 0.09 \times -0.03 = 0.11$ $S_5 = - 0.32 - (-0.31 \times -0.03) = -0.33$ Amount= $0.013 - 0.003 \times -0.03 = 0.0132$.
Computing of values for X_3 Row $x_1 = 0 - 0 \times - 0.03 = 0$ $x_2 = -0.03 \times 1 \times - 0.03 = 0$ $x_3 = 1 - 0 \times -0.03 = 1$ $x_4 = 0 - 0 \times -0.03 = 0$ $x_5 = 0 - 0 \times -0.03 = 0$ $S_1 = -0.22 -(- 0.06 \times 0.13) = -0.22$ $S_2 = 0 - 0.34 \times -0.03 = 0.01$ $S_3 = -0.33 - (- 0.08 \times -0.03) = 0.33$ $S_4 = -0.01 - 0.09 \times -0.03 = - 0.007$ $S_5 = - 0.1 -(- 0.31 \times -0.03) = -0.38$ Amount = $0.015-0.008 \times -0.03 = 0.0152$.	Computing of values for X_5 Row $x_1 = 0 - 0 \times - 0.15 = 0$ $x_2 = 0.15 - 1 \times 0.15 = 0$ $x_3 = 0 - 0 \times 0.15 = 0$ $x_4 = 0 - 0 \times 0.15 = 0$ $x_5 = 1 - 0 \times 0.15 = 1$ $S_1 = - 0.03 - (-0.06 \times 0.15) = 0.04$ $S_2 = 0 - 0.34 \times 0.15 = - 0.05$ $S_3 = 0.02 - (- 0.08 \times 0.15) = 0.03$ $S_4 = -0.03 - 0.09 \times 0.15 = 0.04$ $S_5 = 0.04 -(-0.31 \times 0.15) = 0.09$ Amount= $0.023 - 0.008 \times 0.15 = 0.0218$.

The optimal solution from the game model simplex method of linear programming in Table 12 shows that $x_1 = 0.0132$, $x_2 = 0.008$, $x_3 = 0.0152$, $x_4 = 0.0164$ and $x_5 = 0.0218$. The expected value of the game obtained from the relation $Z_q = \frac{1}{V} = 0.0746$.

$$Therefore, V = \frac{1}{0.0746.} = 13.404826 \approx 13.405$$

Converting these solution values back into original variables, we have

$$From, x_n = \frac{q_n}{V} \Rightarrow q_n = x_n \times V; substituting the values,$$

$$q_1 = x_1 \times V = 0.0132 \times 13.405 = 0.176946 \approx 0.18$$

$$q_2 = x_2 \times V = 0.008 \times 13.405 = 0.107240 \approx 0.11$$

$$q_3 = x_3 \times V = 0.0152 \times 13.405 = 0.203756 \approx 0.20$$

$$q_4 = x_4 \times V = 0.0164 \times 13.405 = 0.219842 \approx 0.22$$

$$q_5 = x_5 \times V = 0.0218 \times 13.405 = 0.292229 \approx 0.29$$

Total = 1.000013 ≈ 1.00.

The probability of selecting the various categories of wastes generated.

- tons of optimal quantity of e-waste (q_1) = 0.18
- tons of optimal quantity of plastics (q_2) = 0.11
- tons of optimal quantity of ceramics (q_3) = 0.20
- tons of optimal quantity of metals (q_4) = 0.22
- tons of optimal quantity of organic wastes (q_5) = 0.29.

Table 13 Weighted Average tons of Waste Generated

Categories of wastes	Total generated in tons	Probability	Weighted average in tons	Total value in tons
e-waste (x_1)	421	0.18	421 × 0.18	75.78
Plastics (x_2)	438	0.11	438 × 0.11	48.18
Ceramics (x_3)	323	0.20	323 × 0.20	64.60
Metal (x_4)	310	0.22	310 × 0.22	68.20
Organic waste (x_5)	3818	0.29	3818 × 0.29	1107.22
Total	5310	1.00		1363.93

Therefore the financial benefit under the worst condition will be 1363.93 × 13.405 = ₦18284.15 billion or ₦18.284 trillion.

Alternatively, the probabilities can be applied to value of the game and the cost multiplied by the quantity of each tons of waste generated to get the total benefit, we have;

Table 14 Total Revenue Generated in Billions of Naira

Types of wastes (B_1)	Prob. (B_2)	Cost in billions (B_3)	Total weight in tons (B_4)	Total revenue generated in billions ($B_3 \times B_4$)
e-waste (x_1)	0.18	0.18 × 13.405 = ₦2.413	421	1015.873
Plastics (x_2)	0.11	0.11 × 13.405 = ₦1.475	438	646.05
Ceramics (X_3)	0.20	0.20 × 13.405 = ₦2.681	323	865.963
Metal (X_4)	0.22	0.22 × 13.405 = ₦2.949	310	94.19
Organic waste (x_5)	0.29	0.29 × 13.405 = ₦3.881	3818	14, 840.566
		Total value	=	₦18, 282.642

The Total revenue is the same which amounts to ₦18, 284 trillion.

The Total revenue generated compares favourably with that generated from Jacobi's Method of ₦18, 409 trillion.

4. Conclusion and Recommendation

The unsatisfactory management of municipal solid waste generated in Enugu need adequate solution strategies to mitigate the situation. The combined option of integrated solid waste management and system approach should be used for the assessment of the competing options.

- The result from the Jacobi's Iteration optimization model has revealed that with waste generation capacity of 2400 metric tons per day, Enugu urban can generate revenue of ₦21, 072, 853. 00 per ton per day. This will amount to ₦147, 509, 791.00 per ton per week or ₦7, 670, 518, 492.00per ton per annum. The total revenue that will be generated per 2400 tons per day will be ₦50,574, 847, 200.00 i.e. ₦50.575 billion daily with 2056 projected population. This amounts to ₦ 354, 023, 930, 400.00 i.e. ₦354.024 billion weekly and ₦18, 409, 244, 380, 000.00 i.e. ₦18.409 trillion per annum. This will reduce drastically the quantity of waste generated in Enugu urban because it will be on demand and a scarce commodity.
- The results from Game Optimization model also show total revenue of ₦18.282 trillion per annum. This amount compares favourably ₦18.409 obtained from Jacobi's iteration optimization model.
- Waste recycling will be profitable venture because it will help to grow the economy of Enugu urban. It will help to create employment among the youths and increase the standard of living of the people as shown in the result of Jacobi's iteration and Game theory optimization.
- Effective legislation should be enacted to encourage source reduction for manufacturing companies to use less hazardous materials to package their products thereby reducing waste and encourage recycling of packages for manufactured products.
- The integrated solid waste management system will be articulated to solve the problem of solid waste handling through;
- The procurement of more compaction vehicles already in use by ESWAMA for compaction of solid waste generated during collection at designated dump sites.
- Methane gas from waste generation obtained from recovery products in landfills produced by degradable wastes is useful in estimating the recovery value of methane from landfill emission. Methane production from landfills is estimated on chemical composition of solid wastes from Enugu urban. This product if harnessed will be another source of revenue.
- The biogas recovery value: 1m^3 of biogas can generate 1.2Kw/hr. of electricity or 200m^3 of biogas can generate 1.25 x 180 = 225 Kw/hr. of electricity. This will generate a lot of revenue from biodegradable wastes for the government.
- The work also recommends a holistic arrangement of public enlightment, the media, involvement, participation and cooperation of local communities and government for waste management and environment to educate the inhabitants' n how to handle waste to achieve optimum benefits.
- Good access roads should be provided by the government in order to have access to all the residential, commercial and industrial areas for ease of collection.
- Public private partnership agreement between private operators (investors) and government agencies must be encouraged for proper handling and disposal of solid waste in Enugu State.

There should be laws and regulations to monitor implementation, strict compliance and appropriate penalty for defaulters.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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