World Journal of Advanced Research and Reviews
eISSN: 2581-9615 CODEN (USA): WJARAI
Cross Ref DOI: 10.30574/wjarr
Journal homepage: https://wjarr.com/

(RESEARCH ARTICLE)

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Mathematics Performance and Polya's Method in Problem Solving

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World Journal of Advanced Research and Reviews, 2024, 23(03), 2156–2162

Publication history: Received on 01 July 2024; revised on 15 September 2024; accepted on 18 September 2024

Article DOI: https://doi.org/10.30574/wjarr.2024.23.3.2873

Abstract

The perpetual challenge of uncertain problem-solving skills among local education administrators is underscored by data from a public science and technology high school despite being a performing public science high school, students have been still struggling to enhance their critical thinking skills, posing an unceasing issue. The purpose of this study is to determine the impact of Polya's four-step method on the Grade 9 students' performance in mathematics. Utilizing a one-group pretest-posttest design, the researcher aimed to determine which aspects of Polya's method improved students' mathematics performance. The data analysis indicated a significant improvement in students' problem-solving performance using Polya's four-step method from pre-test to post-test. This study reaffirms that Polya's method helps students develop a structured way of thinking and solving problems, which is fundamental for their mathematical development.

Keywords: Mathematics Performance; Polya's Four-Step Method; Problem-Solving Skills

1. Introduction

The Department of Education in the Philippines acknowledges the value of helping students acquire critical thinking abilities for the twenty-first century. In order to solve issues and come to wise judgments, learners need to be able to analyze, evaluate, and apply knowledge as the world grows increasingly linked and complicated. The Department of Education has put in place a number of projects, programs, and regulations aimed at improving students' critical thinking abilities in order to accomplish this goal. These include giving teachers' chances for professional development, encouraging inquiry-based learning, and incorporating critical thinking into the K–12 curriculum (DepEd Order 021, s. 2019). Some educators would even go beyond and above to find additional tactics that will aid in the development of these abilities in their students.

In the 2022 Programme for International Student Assessment (PISA) report, almost no learners in the Philippines were top performers in mathematics, meaning they achieved Level 5 or 6 in the PISA mathematics test (OECD average: 9%). This means that our students in these levels could hardly model challenging circumstances mathematically, and had difficulty in choosing, comparing, and assessing effective problem-solving solutions for coping with them. This PISA result implies the need to develop strategies to enhance critical thinking in problem solving.

Abazov [1] states that problem-solving skill is fairly simple: it is the capacity to identify the essence of a problem, deconstruct (break it down), and build an effective set of measures to solve associated challenges. An educational journal by Kailani et al. [2], also states that students' education should include the development of critical thinking abilities, and educational institutions have the duty to foster and assess these abilities through the teaching and learning process. Hence, strategies should be underway to develop critical thinking skills.

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One of the popular methods of solving problems in mathematics is Polya's 4-step method. This includes: 1.) identifying the problem, 2.) devising a plan, 3.) carrying out the plan, and 4.) looking back. According to Polya, taking the time to reflect and look back on what you've done can help you learn a lot about what worked and what didn't. This allows you to foresee which strategy to take to address future difficulties.

Consequently, the first step in strengthening a learner's critical thinking skills through mathematical performance is to learn how to harness his or her problem-solving skills by viewing every scenario as an identifiable challenge. Learning problem-solving skills also entails being able to divide an issue into little pieces, or smaller and more manageable bits, by identifying the problem's major components.

Finding potential solutions is a tough step in the problem-solving process because, on the surface, it looks that most of the work has already been done and the end goal is near. The final phase in the problem-solving process is to create a step-by-step execution plan and then execute effectively and decisively. This is also a crucial skill since it makes little difference how well students recognize the problem, define its aspects, and investigate alternative remedies; it all boils down to the capacity to take specific measures to carry out the action plan.

The uncertainty of performance in problem-solving skills in the region is still a challenge to the local education administrators. The data of this science high school on learners' performance indicates that mathematical skills of Grade 8 students from the previous school year (2022–2023), 38% of the 245 students were classified as struggling learners. The school, though being one of the performing public science high schools in the region, students still face the problem in enhancing their critical thinking. It is therefore, in this study that the mathematics performance using Polya's four-step method in problem solving among Grade nine (9) students was conceptualized.

2. Methodology

2.1. Research Design

This study used a one-group pretest-posttest design of conducting research in an objective and controlled manner to optimize precision and reach findings regarding a hypothesis statement. Analyzing the impact of an independent variable or factor on a dependent variable is usually the goal.

In this study, the researcher used one-group pretest-posttest design which aims to determine the mathematics performance and critical thinking skills using Polya's four-step method in problem solving among grade nine (9) students at Pilot Provincial Science High School and Technology officially enrolled in the school year 2023 – 2024 as revealed by their pre-test and post test results.

2.2. The Respondents

The respondents of the study were composed of the officially enrolled forty-seven (47) Grade nine section Gulam learners. Complete enumeration method is the research sampling technique used to identify the number of respondents.

2.3. Research Instruments

The researcher used an adapted module from (DepEd) Department of Education's learning modules and modified it with Polya's four-step methods of problem- solving technique. The researcher also used a survey questionnaire adapted and modified from "Catch 21st Century Skills" [3].The said survey questionnaire is composed of thirteen (13) questions that describe the critical thinking skills of a student based on the ability to evaluate, identify, understand mathematical inconsistencies, effectively answer mathematics problems and make informed opinions, separate what's important and what's inapplicable information, construct strong fine substantiation- grounded arguments, view fine situations from different perspectives, ask hard questions to challenge compliance and hypothetical, interpret what fine information really means, dissect fine ideas and argument, reach conclusions grounded on substantiation, and assess whether people got the data right, identify misconceptions and gaps in their own logic.

A pre-test / post-test questionnaire consists of seven (7) items, divided into (4) four parts of Polya's four-step method in problem solving mainly – (A.) understanding the problem, (B.) devising a plan, (C.) carrying out the plan, and (D.) looking back.

The instrument – test questionnaire was validated by the district's Mathematics coordinator, graduates of the same course and panels to assess and ascertain the relevance and comprehensiveness of the items before administration. The mean of 3.57 which is classified as "Strongly Agree", shows that instrument is valid and is ready to be administered.

Cronbach's alpha was utilized to assess the instrument's reliability by comparing the amount of shared variation, or covariance, among the items in an instrument to the total variance. The concept is that if the instrument is dependable, the items should have a high degree of covariance in relation to the variance. The pilot study obtained a questionnaire reliability test of 0.990 which indicates that the instrument is "very reliable". In addition, the alpha value for the test questionnaire showed 0.914 which means that the internal consistency was excellent.

A rubric considering Polya's four-step process for problem solving which involves understanding the problem, devising a plan, carrying out the plan and looking back was described as follows:

Range of Scores	**Description**	**Interpretation**
5.61 – 7.00	Very High	Outstanding
4.21 – 5.60	High	Very Satisfactory
2.81 – 4.20	Average	Satisfactory
1.41 – 2.80	Low	Unsatisfactory
0 – 1.40	Very Low	Did Not Meet Expectation

To clearly show the measures on the level of agreement on the critical thinking skills of the students on the conduct of lesson integration on Polya's four-step method in problem solving. The survey questionnaire made use of a four Likert scale of Strongly Agree (SA), Agree (A), Disagree (D), and Strongly Disagree (SD) to determine the level of agreement described as follows:

Scale	**Description**
4	Strongly Agree
3	Agree
2	Disagree
1	Strongly Disagree

To interpret the weighted average of the critical thinking skills in mathematics performance, the following scale was used:

Scale	Description	Interpretation
3.50-4.00	Strongly Agree	The statements are completely favorable.
2.50-3.49	Agree	The statements are almost favorable.
1.50-2.49	Disagree	The statements are somehow unfavorable.
1.00-1.49	Strongly Disagree	The statements are completely unfavorable.

2.4. Data Gathering Procedure

After the instrument was validated and undergone reliability test – for the survey questionnaire, the researchers sought permission to the office of the Schools Division Superintendent and the principal of the school to conduct the study. A letter of permission indicating the purpose of the survey was secured.

The instrument was distributed to the respondents and was given thirty (30) minutes to answer the survey questionnaire regarding critical thinking skills, one (1) hour to answer the pre-test questionnaire regarding mathematical problem-solving skills. A three-week lesson integration on Polya's four-step method in problem solving was conducted. The researcher then gave the post-test of the same content of questionnaire after the experiment has

conducted. The data gathered from this research was tallied and computed for interpretation according to the responses. Along with primary data, the researcher made use of secondary sources to support survey results.

2.5. Statistical Treatment

The researcher made use of descriptive statistics such as mean and standard deviation to examine the students' pre-test and post-test performance in problem solving using Polya's four-step method and the critical thinking skills of the students before and after the lesson integration based on Polya's four-step method of problem solving.

A two-tailed test was also used to see if there was a significant difference between the students' pre-test and post-test performance in problem solving using Polya's four-step method and if there was a considerable change in students' critical thinking skills before and after the lesson integration based on Polya's four-step method of problem solving. The level of significance was set at 0.05.

3. Results and discussion

Table 1 and Table 2 determine the pre-test and posttest results of the respondents as they are engaged in the Polya's four-step method in problem-solving.

Table 1 Pre-test performance of the students in problem solving using Polya's four-step method.

Polya's Four-Step Method	Mean	Description	Interpretation
Understanding the Problem	1.98	Low	Unsatisfactory
Devising a Plan	1.95	Low	Unsatisfactory
Carrying Out the Plan	4.98	High	Very Satisfactory
Looking Back	1.91	Low	Unsatisfactory
Overall Mean	2.68	Low	Unsatisfactory

Table 1 presents the pre-test mean scores performance of the students in Polya's four step in problem solving. As indicated in Table 1, *Carrying Out the Plan* got the highest mean of 4.98 which indicates a "Very Satisfactory" interpretation result. The results shows that students are able to carry out the plan adequately.

"*Understanding the Problem"*, "*Devising a Plan",* and "*Looking Back"* were all "Unsatisfactory" as shown by their mean of 1.98, 1.95, and 1.91 respectively. This shows that students are in need for developing their ability to comprehend the problem, construct outlines and look back on their gathered answers or solution.

In general, the pre-test performance of the students was identified as "Unsatisfactory" as revealed by the mean score in the over-all mean 2.68 with an overall description of low and interpreted as unsatisfactory. This means that prior to the lesson integration, students performed poorly in their problem-solving skills. It is important to note that they are already good in solving worded problems yet find it difficult to follow step-by-step process in problem-solving.

This aligns with the findings of Lee [4] that the current mathematics educational system focuses only on techniques, formulas, and processes, disregarding the importance of comprehension, presentation, and reasoning. Thus, it turns out that some learners tend to be only good in using formulas that they do not understand. Simatupang [5] also found out that students with limited knowledge on Polya's method can still complete the comprehending stage of the problem by writing down what they know and are requested to do, but they will not be able to describe it in full words. Due to their incapacity to associate facts, students are unable to create a plan. Another study by Bradshaw and Hazell [6], states that students' inability to competitiveness that need for an exact solution hindered their ability to thoroughly investigate the subject at hand, leading them to miss important details needed to solve the problem. Many students have developed a fascination for problem solving due to its independence and unstructured nature, which teachers should support.

Table 2 The post-test performance of the students in problem solving using Polya's four-step method.

Polya's Four-Step Method	Mean	Description	Interpretation
Understanding the Problem	6.02	Very High	Outstanding
Devising a Plan	5.77	Very High	Outstanding
Carrying Out the Plan	5.66	Very High	Outstanding
Looking Back	5.00	High	Very Satisfactory
Overall Mean	5.61	Very High	Outstanding

Table 2 presents the post-test mean scores performance of the students in Polya's four steps in problem solving. As shown in table 2, "*Understanding the problem*" obtained the highest mean score of 6.02 which is interpreted as outstanding followed by "*Devising a Plan"* that gained mean score of 5.77, and "*Carrying Out the Plan"* that gained mean score of 5.66 and of the same interpretation as outstanding. The results showed that after the lesson integration on Polya's four-step in problem solving, the students are now able to identify each step.

"*Looking Back"* obtained the mean scores of 5.00 described as high and interpreted as very satisfactory. The results also showed that step 4 or "looking back" was also performed greatly as part of Polya's four-step method in problem solving as the lesson integration.

The overall post-test performance of the students was interpreted outstanding as revealed by the over-all mean score 5.61 described as very high and interpreted as outstanding. This means that after the lesson intervention, the students performed greatly in their problem-solving skills. Consequently, the respondents are able to determine what is asked and given, what operation to be used and its number sentence as the steps 1 (Understanding the Problem), and 2 (Devising a Plan), showed the clear and concise answer resulting to having very high and outstanding scores. Although step 3 (Carrying Out the Plan) was already pertinent to every respondent, it also shows that there is a significant improvement in the results of their performance after the lesson integration signifying very high and outstanding results. Clear improvements were also revealed in step 4 (Looking Back), resulting into increasingly high score and very satisfactory result as shown in the table.

Parallel to the study conducted by Obiano and Parangat [7], it is revealed that Polya's method is effective in increasing problem-solving abilities in terms of understanding the problem, planning, solving, and looking back. The results are also parallel to the study conducted by Yapatang and Polyiem [8] where the study yielded positive results in terms of learners' ability to solve mathematical problems effectively by using Polya's problem solving strategy. Learners also demonstrated higher learning achievement in surface area and volume on the posttest, and they expressed satisfaction with the lesson plans utilized. Another study conducted Simatupang [5] found out that learners with proper knowledge on Polya's strategy could complete the steps of comprehending the issue, coming up with a strategy, carrying it out, and look back.

Table 3 Significant Difference on the Pre-test and post-test performance of the students in problem solving using Polya's four-step method in problem solving.

Variable	Mean	Mean Diff.	*Standard Deviation*	*t*-value	*p*-value	Interpretation
Pre-test	10.72	11.73	0.93	28.834	0.000*	Significant
Post-test	22.45		1.31			

*Significant at 0.05 level

In Table 3. the result of *t*-value, 28.834 with its *p*-value 0.000* revealed that $p < 0.05$, significant at 5% level, is strong evidence that this table is deduced to be statistically significant, implying that the null hypothesis should be rejected. Thus, the hypothesis stating that there is no significant difference between the pre-test and post-test performance of the students in problem solving using Polya's four-step method in problem solving is rejected. It implies that the intervention on improving the performance of students in problem solving with the use of Polya's method helps improve the performance of students.

The result is supported by Olatide et al. [9] that students perform better in a variety of mathematics topics when they are introduced to Polya's problem-solving method. Better comprehension of mathematical ideas, a stronger capacity for handling challenging issues, and improved critical thinking abilities could all be examples of this learning progress. Another study conducted by Yapatang & Polyiem [8] yielded positive results: students' ability to solve mathematical problems was effectively developed by the learning mathematics lesson using Polya's problem-solving process and applied cooperative learning; students also demonstrated higher learning achievement in surface area and volume on the posttest, and they expressed satisfaction with the lesson plans utilized. Iringan [11 study examined students' exposure to Polya's approach activities, which enhance the development of critical thinking and problem-solving skills. The findings show that students receive a "great extent" of instruction that facilitates the acquisition of desirable mathematics skills, notably in terms of content, techniques, assessment, and instructional tools.

Some educational studies continue to emphasize the relevance of Polya's method. Reys and Reys (2012) in *Teaching Mathematics: A Problem-Solving Approach* discussed how present teaching methods benefit from Polya's principles. They highlight that Polya's method remains a valuable tool for encouraging problem-solving skills in students, demonstrating its continuing effectiveness in modern instruction. Further supporting Polya's approach, Artigue [12] investigates the role of didactical design in mathematics education, noting that Polya's problem-solving strategies are essential to developing effective teaching practices. Artigue suggests that integrating these methods into the curriculum helps students become more skilled at undertaking unfamiliar problems, thereby developing their overall problem-solving competency.

Mason, Burton, and Stacey [13] also provide the analysis in *Thinking Mathematically*, where they investigate various problem-solving strategies and confirm the utility of Polya's method in enhancing students' mathematical skills. Their analysis proves that Polya's method helps students develop a structured way of thinking and solving problems, which is fundamental for their mathematical development.

4. Conclusion

The performance of the students before the lesson integration of Polya's four-step method revealed "Unsatisfactory" and "Outstanding" after the integration of Polya's four-step method. Results revealed that there is a significant difference on the pre-test and post-test results on the students' performance before and after the lesson integration of Polya's four-step method in problem solving.

This study showed a significant improvement in student performance and critical thinking skills after integrating Polya's four-step method into problem-solving lessons. Students did not do well at first, but after the lesson integration, they did remarkably well. Students' critical thinking abilities were well-founded, but they still needed to work on building arguments based on evidence, which became much better after the lesson integration. Additionally, this study demonstrated a significant difference between the pre- and post-test results on mathematics performance, demonstrating the efficacy of the approach. Students eventually found the learning process pleasurable and fulfilling as they improved their problem-solving abilities, despite their mixed experiences throughout the study. Hence, the use of Polya's four - step method into problem solving is effective. Extensive research, encompassing both theoretical and empirical studies, validates the continued application and adaptation of Polya's method in educational environments.

Compliance with ethical standards

Disclosure of Conflict of interest

There is no conflict of interest in this study.

Statement of ethical approval

This study followed an ethical approval.

Statement of informed consent

Informed consent was obtained from all individual participants included in this study.

Funding source

This study did not receive any specific grant from any funding agency.

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