World Journal of Advanced Research and Reviews

eISSN: 2581-9615 CODEN (USA): WJARAI

Cross Ref DOI: 10.30574/wjarr

Journal homepage: https://wjarr.com/

(Research Article)

Marks vs. Percentile data of the JEE (Main): A study to assess the preparation level of the examinees

Sudipto Roy *

Department of Physics, St. Xavier's College, Kolkata, West Bengal, India.

World Journal of Advanced Research and Reviews, 2024, 23(02), 735–743

Publication history: Received on 25 June 2024; revised on 06 August 2024; accepted on 08 August 2024

Article DOI: https://doi.org/10.30574/wjarr.2024.23.2.2384

Abstract

In the present study, I have carried out a mathematical formulation to find a simple way to determine the mean and standard deviation of marks and also the most probable marks obtained by examinees in an examination like the Joint Entrance Examination - Main [or, JEE (Main)], using *Marks vs. Percentile* data. Observing the nature of variation of the percentile score as a function of marks, based on information obtained from examinees, I have chosen an empirical expression for the percentile score (in terms of marks) for this study. This expression represents the behavior of real data, with sufficient accuracy, for a certain combination of values of the constant parameters associated with the expression. A parameter among them has been identified as one representing the difficulty level of the question paper. Using my empirical expression for percentile score, I have derived an expression for the probability of scoring marks of a certain value. I have defined a *cumulative probability* here, which represents the probability of scoring marks equal to or more than a certain value. Findings of this study have been depicted graphically. The mathematical expressions, chosen or derived in this study, can be used for making predictions whose accuracy depends upon the largeness of the sample of *Marks vs. Percentile* data used for determination of constant parameters.

Keywords: JEE (Main); Marks-versus-Percentile; Average Preparation Level; Most Probable Marks; Standard Deviation of Marks; Cumulative Probability; NTA; Negative Marking

1. Introduction

The competitive examination, known as the Joint Entrance Examination - Main or JEE (Main), is conducted every year by National Testing Agency (NTA) in two sessions in India [1, 2]. The admission of a candidate to various engineering streams (for a *BE* or *BTech* degree), in many engineering institutions in India, depends upon one's performance in this examination. A few days after the examination is conducted, NTA releases a document containing answers to all questions (referred to as the *final answer key*) which enables an examinee to calculate the marks to be awarded. The JEE (Main) has a question paper of 300 marks, containing 75 *multiple-choice-type* questions with 4 marks each. For each wrong answer, 1 mark is deducted. Due to this negative marking, the total marks secured by a candidate can also be negative. Once the marks are calculated using the *final answer key*, the candidates need to estimate their percentile scores which would determine the chances of getting admission to the institutions of their choice and, in their preferred disciplines in those institutions. Percentile score of a candidate, as NTA calculates, is the percentage of examinees who have secured marks equal to and less than that of the candidate. Several online coaching agencies put up *Marks vs. Percentile* (*expected*) data on their websites which help the engineering aspirants to estimate the percentile scores based on marks. These datasheets are generally prepared on the basis of previous year's results (obtained mainly from the information received from their students), with probable modifications based on the difficulty level of current year's question paper of a certain session (January or April) relative to that of the last year. After the announcement of the online publication of results by the NTA, the examinees can individually check their results (marks, percentile score, all India rank) from the NTA website for the JEE (Main). The NTA itself does not publish a document containing the *Marks*

* Corresponding author: Sudipto Roy; Email: roy.sudipto@sxccal.edu

vs. Percentile data of all candidates who have taken the examination in a particular session of the year. For an analysis of results, one therefore has to depend upon the datasheets provided by the coaching agencies, some of whose links are given in the list of references at the end of this article [3-5].

In an article published earlier, I discussed the derivation of a mathematical formula which expresses the percentile score in terms of marks [6]. In the present study, I have attempted to find a simple way to determine the average preparation level of the examinees of the JEE (Main) by analyzing a set of *Marks vs. Percentile* data, obtained from the internet. Based on the nature of the dataset, I have chosen an empirical expression representing the percentile score as a function of marks. The constant parameters, associated with that function, have been determined by finding a unique combination of values for them for which the function generates values (i.e., percentile scores) closest to those in the dataset. Based on this function, I have formulated an expression for the probability, denoted by $f(m)$ here, of securing any marks (m) in the examination. It allows one to calculate the most probable marks and the corresponding probability. Based on $f(m)$, I have derived a cumulative probability $g(m)$ which is the probability of getting marks greater than or equal to m. Using $f(m)$, one can also calculate the probability of scoring marks within a specified range of values of the variable m (representing marks). Based on *Marks vs. Percentile* data, I have determined the expectation value and the standard deviation of marks obtained by the examinees, using the probability function $f(m)$. The findings of this study have been shown graphically.

2. Methodology

The definition of percentile score, followed by the NTA, is given below [6, 7].

$$percentile\ score\ of\ a\ candidate = 100 \times \left(\frac{\substack{number\ of\ examinees\ who\ have\ secured\ marks \\ equal\ to\ and\ less\ than\ that\ of\ the\ candidate}}{total\ number\ of\ examinees} \right) \quad (1)$$

It means that the *percentile score* of a candidate is the percentage of examinees who have scored equal and less marks in comparison to the marks scored by the candidate. Apart from being used for assessing one's performance in an examination, the *percentile* is widely used as a bibliometric measure to evaluate a publication in terms of how many more times it has been cited relative to other publications in the same field [8-10].

From a certain website, I obtained a *Marks vs. Percentile* record based on feedbacks received from examinees, regarding the JEE (Main) Session-1, held in January 2023 [11, 12]. It was compiled by Ms. Neha Agrawal, a teacher of mathematics who has a YouTube channel named "Neha Agrawal Mathematically Inclined" [13]. In an online lecture, she used these data to analyze the results of the JEE (Main) Session-1 [14]. The last column of that datasheet contains the average marks (i.e., the average of all shifts) for 20 different percentile scores (80 to 99.93) listed in the 1st column. Using these data, I have prepared Table-1 here, having percentile scores against twenty different marks.

Table 1 *Marks vs. Percentile* Data of January 2023 session of the JEE (Main)

Marks	Percentile	Marks	Percentile
61	80	127	96.5
69	85	133	97
84	90	140	97.5
89	91	148	98
94	92	159	98.5
100	93	174	99
106	94	195	99.5
113	95	199	99.65
117	95.5	223	99.8
123	96	257	99.93

Using Table-1, I have shown graphically the variation of percentile score as a function of marks in Figure-1. It shows that, the percentile score increases with marks and its rate of increase (with respect to marks) becomes smaller with the increase in marks. For example, for the change in marks from 61 to 106, the percentile score changes from 80 to 94, as evident from Table-1. For the change in marks from 127 to 174, the percentile score changes from 96.5 to 99. Observing this nature of change, which is also evident from Figure-1, I propose the following empirical expression representing percentile score (P) as a function of marks (m).

$$P(m) = A[1 - Exp\{-b(m + c)\}]^n \tag{2}$$

where $A, b, c, n > 0$.

To summarize the property of the above function, I would say that, $P(m)$ increases with m, approaching the value of A asymptotically. It can be shown that, for a sufficiently large value of m, the rate of change of $P(m)$ (with respect to m) decreases gradually with m. This behaviour of variation of $P(m)$, as a function of m, is very similar to what is evident from Figure-1.

Since the full marks for the question paper of JEE (Main) is 300, one must obtain $P(m) = 100$ (i.e., the highest possible percentile score) from equation (2), for $m = 300$. Based on this requirement, we get the following value for A.

$$A = \frac{100}{[1 - Exp\{-b(300 + c)\}]^n} \tag{3}$$

Using equation (3) in equation (2), one gets,

$$P(m) = 100\frac{[1 - Exp\{-b(m + c)\}]^n}{[1 - Exp\{-b(300 + c)\}]^n} \tag{4}$$

According to equation (4), $P(m) = 0$ for $m = -c$. Zero percentile score for a certain value of m means that nobody has secured marks *less than* and *equal to* that value. The parameter c therefore corresponds to the lowest marks obtainable in the examination, the lowest being $(-c + 1)$, since the value of m changes by 1. To ensure the accuracy of results obtained from this formula (eqn. 4), one needs to find a unique combination of values for the constant parameters (b, c and n), using which, the values of $P(m)$ (generated by the formula for different m values) would be as close as possible to the percentile values in Table-1 for the corresponding m values. For $m < -c$, we have negative values for $[1 - Exp\{-b(m + c)\}]$. If n is an even integer, the value of $P(m)$ increases as m decreases below $m = -c$. These values of $P(m)$ are unacceptable because $P(m)$ cannot increase as m decreases, according to its definition (eqn. 1). If n is an odd integer, one gets negative values of $P(m)$ for $m < -c$, which are unacceptable because $P(m)$ is positive by its definition (eqn. 1). For non-integral values of n, $P(m)$ remains undetermined for $m < -c$, due to the negative value of $[1 - Exp\{-b(m + c)\}]$ in that range of m. For $m > 300$, we have $P(m) > 100$, which is not acceptable because the largest percentile score is 100 (as per eqn. 1). Thus, for any analysis based on equation (4), one should consider only those values of $P(m)$ for which m is in the range expressed as, $-c + 1 \leq m \leq 300$.

The parameter b determines how fast $P(m)$ changes with m (or, how rapidly it progresses towards 100 as m increases). Let $P_1(m)$ and $P_2(m)$ be the percentile scores for $b = b_1$ and $b = b_2$ respectively, corresponding to the same marks m. According to equation (4), $P_2(m) > P_1(m)$ if $b_2 > b_1$. It has been observed that (and which is quite obvious), if the question paper of a certain session has a higher level of difficulty in comparison to that of the preceding session, one gets a higher percentile score for the same marks scored in that session. Thus, the parameter b can be regarded as a measure of the average difficulty level of the questions.

Difficulty level of a question paper is a relative concept. Its perception or estimation should vary from person to person. To quantify the level of difficulty, one may define a parameter d which denotes the fraction of questions, in a question paper, whose answers are not known to a candidate. For the competitive examinations like the JEE (Main), students generally solve previous years' question papers. The values of the parameter d, for those years, can thus be easily determined by a candidate. Based on *Marks vs. Percentile* data, the values of the parameter b can also be determined (as shown in this article) for the same years. Using these values, a correlation can be established between b and d. Based on this correlation, one can determine the value of b corresponding to the value of d for the current year.

The percentage of candidates whose marks are exactly m, is $P(m) - P(m - 1)$, based on the fact that $P(m)$ is the percentage of candidates who have secured marks $\leq m$, as per equation (1). Thus, the fraction of candidates who have secured exactly m marks can be expressed as,

$$f(m) = \frac{P(m) - P(m-1)}{100} \quad (5)$$

Using equation (4) in equation (5) we get,

$$f(m) = \frac{[1 - Exp\{-b(m+c)\}]^n - [1 - Exp\{-b(m-1+c)\}]^n}{[1 - Exp\{-b(300+c)\}]^n} \quad (6)$$

The number of examinees for each session of the JEE (Main) is close to 1 million. Due this large sample size, the function $f(m)$ can be regarded as the probability that an examinee obtains m marks in the examination. Here, m can be treated as a discrete random variable, with values in the range expressed as,$-c + 1 \leq m \leq 300$.

Using equation (6), it can be shown that, for $n \leq 1$, $f(m)$ is a monotonically decreasing function of m, having its largest value for $m = -c + 1$, the lowest obtainable marks. For $n > 1$, $f(m)$ has a peak value for a certain value of m which may be regarded as the most probable marks for the examinees. In reality, the lowest obtainable marks cannot be the most probable marks because no candidate is expected to take the examination with zero knowledge or preparation, or with an extreme urge to attempt questions whose answers are not even faintly known to him/her (causing negative marking). Therefore, $n \leq 1$ is not a permissible range of values for the parameter n. It can be shown that the most probable value of m increases as n increases.

Using the function $f(m)$, it is possible to define a *cumulative probability*, denoted by $g(m)$, which stands for the probability of scoring marks *greater than* or *equal to* m. This should be the sum of probabilities for all possible marks starting from m [15-18]. Thus, $g(m)$ can be expressed as,

$$g(m) = \sum_{x=m}^{300} f(x) \quad (7)$$

In the above equation, the upper limit of the summation index is 300 because it is the largest possible value of m.

Using equation (6) in equation (7) we get,

$$g(m) = \sum_{x=m}^{300} \frac{[1 - Exp\{-b(x+c)\}]^n - [1 - Exp\{-b(x-1+c)\}]^n}{[1 - Exp\{-b(300+c)\}]^n} \quad (8)$$

Based on the probability function $f(m)$, one can calculate the average marks (m_{av}) (i.e., the expectation value of m), using the following formula [15-18].

$$m_{av} = \sum_{m=m_{min}}^{300} m\, f(m) \quad (9)$$

In equation (9), m_{min} is the lowest possible marks that can be secured by an examinee. Here $m_{min} = -c + 1$, based on the discussion in the paragraph following equation (4).

Using equation (6) in equation (9) we get,

$$m_{av} = \sum_{m=m_{min}}^{300} m \left[\frac{[1 - Exp\{-b(m+c)\}]^n - [1 - Exp\{-b(m-1+c)\}]^n}{[1 - Exp\{-b(300+c)\}]^n}\right] \quad (10)$$

The standard deviation of marks (m_{SD}) can be calculated using the following expression [15-18].

$$m_{SD} = [(m^2)_{av} - (m_{av})^2]^{\frac{1}{2}} \quad (11)$$

The average of m^2 [denoted here by $(m^2)_{av}$] can be determined using the following formula [15-18].

$$(m^2)_{av} = \sum_{m=m_{min}}^{300} m^2\, f(m) \quad (12)$$

Using equation (12) in equation (11) we get,

$$m_{SD} = \left[\left(\sum_{m=m_{min}}^{300} m^2\, f(m)\right) - (m_{av})^2\right]^{\frac{1}{2}} \quad (13)$$

Using equations (6) and (10) in equation (13), we get,

$$m_{SD} = \left[\begin{array}{l} \left(\sum_{m=m_{min}}^{300} m^2 \frac{[1-Exp\{-b(m+c)\}]^n - [1-Exp\{-b(m-1+c)\}]^n}{[1-Exp\{-b(300+c)\}]^n} \right) \\ - \left(\sum_{m=m_{min}}^{300} m \left[\frac{[1-Exp\{-b(m+c)\}]^n - [1-Exp\{-b(m-1+c)\}]^n}{[1-Exp\{-b(300+c)\}]^n} \right] \right)^2 \end{array} \right]^{\frac{1}{2}} \quad (14)$$

The reason for the speculation that $c > 0$ for the empirical expression for $P(m)$ (eqn. 2) is that the lowest marks scored is likely to be negative. The reason for introducing *negative marking* in an examination is mainly to prevent an examinee from answering *multiple-choice-type* questions by random guessing. There are several important studies showing different aspects of negative marking [19-21]. If *negative marking* had not been introduced, the values of m would have varied in the range from 0 to 300 in the JEE (Main). The values of m_{av} and m_{SD}, for a group of examinees, are dependent not only upon their knowledge, but also upon the tendency to write answers by guessing.

3. Results and discussion

Figure 1 shows the variation of percentile score as a function of marks, based on Table-1, whose source is a datasheet prepared by Ms. Neha Agrawal based on feedbacks from 3000 examinees [11-14]. It shows that the percentile score increases with marks with a decreasing slope, approaching gradually the maximum possible percentile score (i.e., 100). The nature of this plot has led to the choice of a very simple empirical function represented by equation (2), which takes the form of equation (4) after substituting for A from equation (3).

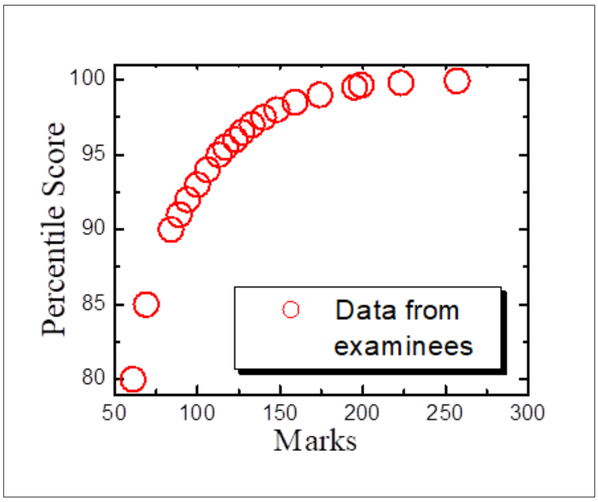

Figure 1 Plot of percentile score as a function of marks, based on the data obtained from examinees (Table-1). The choice of the empirical function $\boldsymbol{P(m)}$ (of eqn. 2) is based on its nature.

Figure 2 shows two plots of *percentile versus marks*. One of them is based on equation (4) (blue curve), and the other is based on Table-1 (red circles). The blue curve, which seems to be in very good agreement with real data (red circles), has been generated by equation (4), for $b = 0.0315$, $c = 50$ and $n = 7$. Here, n is an odd integer which causes the values of $P(m)$ to be negative for $m < -c$ (as per eqn. 4). Those values are unacceptable according to the definition of percentile score (eqn. 1) and hence they have not been shown in this figure.

Figure 3 shows the plot of probability, i.e. $f(m)$, versus marks (m), based on equation (6). It shows that the marks close to the position of the peak value of probability are around 12, with a probability of nearly 0.0124. This value is essentially an estimated fraction of examinees who have obtained marks close to 12. Assuming the number of examinees to be close to 1 million, the number of candidates with $m = 12$ is nearly $0.0124 \times 10^6 = 12400$.

Figure 4 shows the variation of cumulative probability, i.e. $g(m)$, as a function of marks (m), based on equation (8). Here $g(m)$ stands for the probability of getting marks $\geq m$. It decreases as m increases. It decreases very rapidly for marks beyond 275 (approximately), as evident from this figure. For admissions to certain institutions, it might be necessary for a candidate to get a score in the JEE (Main) beyond a certain cutoff value set by those institutions. One can calculate its probability by using the expression for $g(m)$.

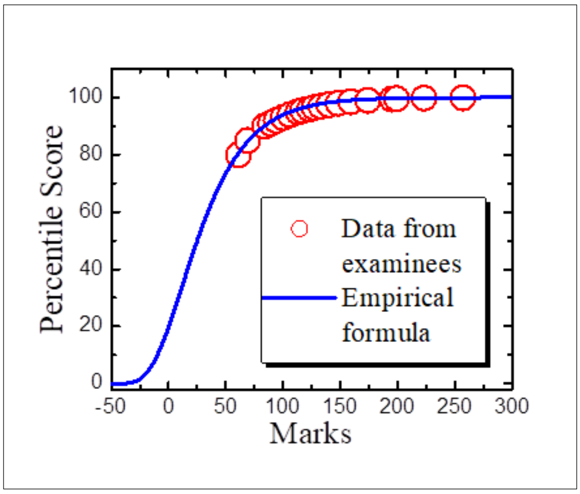

Figure 2 Plots of percentile score versus marks, with red circles based on Table-1 & the corresponding best fit (blue curve) based on equation (4).

To make the values of the variables readable from the graphs, I have used logarithmic scales along the vertical axes of Figures 3 & 4, since they have variations over 4-6 orders of magnitude.

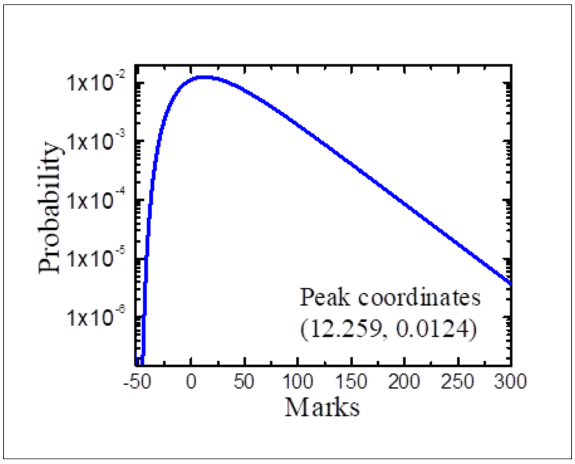

Figure 3 Plot of probability versus marks. The marks of highest probability are close to 12. The scale along the vertical axis is logarithmic.

Using equation (10), I have obtained $m_{av} = 32.78$. The standard deviation of marks has been found to be $m_{SD} = 38.90$ based on equation (14). These values, along with the most probable value of m (which is around 12), may be looked upon as indicators of the average preparation level and the dispersion in preparation standards of the candidates who took the JEE (Main) examination in January 2023.

Using the function $f(m)$, one can determine the probability of scoring marks between, say, m_1 and m_2, which are just any two values of the variable m, with $m_2 > m_1$. This probability can be expressed as, $P(m_1, m_2) = \sum_{m=m_1}^{m_2} f(m)$ [15-18]. For example, the probability of scoring marks between 150 and 200 is, $P(150, 200) = \sum_{m=150}^{200} f(m)$. Its value has come out to be, 0.0105, using equation (6).

Since $f(m)$ is regarded here as representing the probability function for the discrete random variable m, the sum of its values for all possible values of m should ideally be unity [15-18]. I have found the value of $\sum_{m=-c+1}^{300} f(m)$ to be 1.000005967, which is sufficiently close to unity. The deviation of this sum from unity would probably have been smaller if I had more data (of *marks vs. percentile*) at my disposal to be used to determine the constant parameters (b, c & n) associated with the function $f(m)$. For all calculations, discussed above, I have used the parameter values obtained by fitting equation (4) to the dataset of Table-1. These values are, $b = 0.0315$, $c = 50$ and $n = 7$.

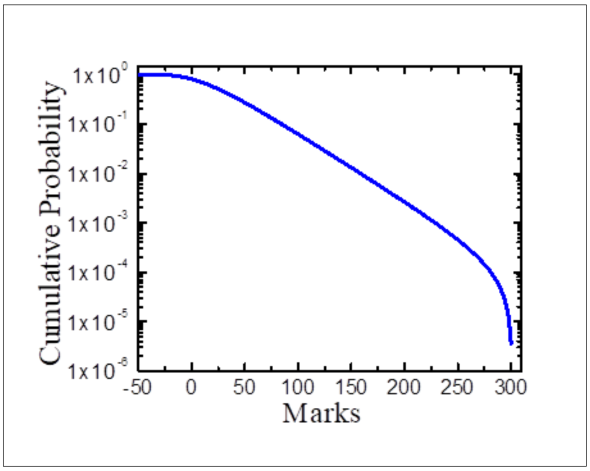

Figure 4 Plot of cumulative probability (i.e., the probability of scoring marks ≥ a certain value) versus marks. The scale along the vertical axis is logarithmic.

A shortcoming of the present study is that the dataset (*ref. no. 12*), used for this study, have percentile scores for marks ranging from 61 to 257. But the possible marks in the JEE (Main) are in the range from -75 to 300 (i.e., 376 integral values of the variable m, including zero). To impart sufficient credibility to the results of a study, based on my method, one needs to use the *Marks vs. Percentile* record for the entire range of marks obtained by the examinees in a certain session of the examination.

I have used a software named Microcal Origin (Version 6.0) for graph plotting. For Figures 2-4, the (theoretical) data for $P(m)$, $f(m)$ and $g(m)$, for the entire range of possible m values, have been generated by a software named Microsoft Excel (part of MS Office 2019).

4. Conclusion

The mathematical formulation of the present study has enabled me to determine the most probable marks, average marks and the standard deviation of marks secured by the examinees in Session-1 of JEE (Main) conducted in January, 2023. For this purpose, I have used the *Marks vs. Percentile* record compiled by Ms. Neha Agrawal [11-14]. For an exact evaluation of parameters that serve as indicators of the average preparation level of the examinees, one should use the results of all candidates of a certain session of the examination, but unfortunately, we have not found them anywhere in the internet. This article provides the reader with a simple mathematical method to formulate functions representing probability and cumulative probability of scoring marks [$f(m)$ & $g(m)$ respectively] in a certain session; and their values can be calculated with the highest accuracy if one gets access to the *Marks vs. Percentile* data of all candidates who have taken the examination in the preceding session.

After estimating one's marks, using the *final answer key* released by the NTA, a candidate can estimate the percentile score using the expression for $P(m)$ of this article (eqn. 4), instead of going through the datasheets (containing the *expected* ranges of percentile scores for different ranges of marks) available in the internet, where the mathematical basis for their compilation is not clearly stated. To get the expression for $P(m)$ ready to be used for this purpose, the constants b, c & n have to be determined first, based on the *Marks vs. Percentile* data of the preceding session of the examination. An examination, named JEE - Advanced, is conducted throughout India following the second session of the JEE (Main) which is held in April. One's performance in this examination determines one's chances of admission to various courses at the IITs (IIT: *Indian Institute of Technology*). But, to be eligible to take this examination, one has to secure, in the JEE (Main), a certain minimum marks or *cutoff* set by the NTA. Calculation of $g(m)$ (eqn. 8), based on the preceding session's data, helps one to estimate approximately the chance of securing marks beyond a certain cutoff value. For using the functions $P(m)$, $f(m)$ and $g(m)$, for predictions regarding a certain session, one needs to adjust the value of the parameter b, depending upon the difficulty level of the session relative to the preceding session. A higher difficulty level corresponds to a larger value of the parameter b, as discussed in section-2. The method of mathematical analysis discussed in this article can be applied to analyze the results of any examination of this sort, based on *Marks vs. Percentile* data, and predictions can be made with sufficient accuracy.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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[12] Link to the datasheet, mentioned in the Reference number 11, which has been used in the present study to prepare Table-1 (last accessed on June 24, 2024): https://drive.google.com/file/d/1OUKiGKDURF1UlrKcK_jafhd7YP6LmjSn/view

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