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Examining multiplication and division on fuzzy numbers using composite tables

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Abstract

A fuzzy number is a concept that represents a real number abstractly. It doesn't assign a single value, but instead a range of uncertain values, each with its own weight between 0 and 1. It is essential to comprehend the arithmetic operations of fuzzy numbers in the real number of fuzzy mathematics. One of the fundamental concepts that should not be ignored is the explanation of the interval operation. In this paper, a composition table is constructed to demonstrate the multiplication and division of fuzzy numbers. Fuzzy arithmetic operations are commonly utilized to solve mathematical equations that involve fuzzy numbers. In this paper, we explore the relationship between multiplication and division operations and fuzzy numbers. We also demonstrate that the shape of their membership functions heavily influences the outcome of our calculations when using fuzzy numbers.

Keywords: Fuzzy number; Membership function; α cut approach; Fuzzy Arithmetic

1. Introduction

Professor L.A. Zadeh [11] proposed the concept of fuzzy set theory as a way to quantify the uncertainty of real-world issues that cannot be resolved using classical sets (crisp sets). Any fuzzy number was defined as a fuzzy subset of the real line by Dubois and Prade in 1978 [4]. An ordinary number with a somewhat ambiguous, precise value is known as a fuzzy number. Fuzzy numbers are utilized in experimental research, computer programming, engineering, and statistics [5]. Basic concepts in fuzzy mathematics include the arithmetic operators of fuzzy numbers. From the operation of a crisp interval, fuzzy number operations may be generalized. It describes how intervals operate. The extension principle [10] or interval arithmetic [8] are the primary foundations for arithmetic operations on fuzzy numbers, which have also been developed.

We developed multiplication and division of fuzzy numbers with a composition table and a graph. Using the cut approach and the extension principle, we constructed multiplication and division operations that are connected to fuzzy numbers [6]. The outcome of our computations while working with fuzzy numbers is significantly influenced by the shape of their membership functions [1]. The α cut approach makes it feasible to create a graphical representation of fuzzy number multiplication and division that allows for a vast range of possible shapes and is incredibly easy to use, with the benefit of producing a considerably larger family of fuzzy numbers.

The paper is structured as follows: in section 2, we discuss some fundamental terms related to fuzzy numbers; in section 3, we discuss fuzzy arithmetic multiplication using a composition table with a MATLAB-based graph representation; and in section 4, we discuss fuzzy arithmetic division using a composition table with a MATLAB-based graph representation.

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2. Some basic terminologies on Fuzzy numbers

2.1. Interval arithmetic

The four arithmetic operations that can be carried out on closed intervals are addition (+), subtraction (-), multiplication (•), and division (/). Let * stand for any of these four operations.

Then

$$[s,t] * [x,y] = \{f * g \mid s \le f \le t, x \le g \le y\} \text{ except when } 0 = [x,y].$$

Most arithmetic operations on closed intervals have this possess, with the exception of $\frac{[s,t]}{[x,y]}$ being not defined when $0 = [x,y]$

2.2. Closed-interval arithmetic operations

$$[x,y] + [t,r] = [x+t, y+r]$$

$$[x,y] - [t,r] = [x-r, y-t]$$

$$[x,y] \cdot [t,r] = [\min(xt, xr, yt, yr), \max(xt, xr, yt, yr)]$$

$$\frac{[x,y]}{[t,r]} = [\min\left(\frac{x}{t}, \frac{x}{r}, \frac{y}{t}, \frac{y}{r}\right), \max(\frac{x}{t}, \frac{x}{r}, \frac{y}{t}, \frac{y}{r})$$

2.3. Operations of fuzzy sets

- **Union:** $A \cup B \Leftrightarrow \mu_A \vee \mu_B$
- **Intersection**: $A \cap B \Leftrightarrow \mu_A \wedge \mu_B$
- **Complement:** $\bar{A} \Leftrightarrow \mu_{\bar{A}} = 1 - \mu_A$
- **Algebraic Product:** μA∗μB. $A * B \Leftrightarrow \mu_{A*B} = \mu_A * \mu_B$
- **Algebraic Sum**: $A + B \Leftrightarrow \mu_{A+B} = \mu_A + \mu_A - \mu_A * \mu_B$

2.4. Fuzzy Number [3]

A Fuzzy number is a number without a precise value. In crisp set theory, we use fixed values that are exact, but in Fuzzy Set theory, fuzzy numbers have imprecise values. These imprecise values are linked to specific weights called the membership function. Fuzzy numbers are an extension of real numbers. For a number to be considered a fuzzy set B on R must satisfy at least the following conditions.

- B must be a normal fuzzy set.
- B^α must be a closed interval for every $\alpha\epsilon$ (0,1] where $B^\alpha = \{x \in \mathbb{R} | B(x) \ge \alpha\}$ is called the alpha - cut of B.
- The support of B, B^{0+} must be bounded where $B^{0+} = \{x \in \mathbb{R} | B(x) \ge \alpha\}$ } is called the support of B.

2.5. Normal fuzzy set [9]

Assume there is a fuzzy set B in X. The height of B, denoted as h(B), is defined as follows:

$$h(B) = \sup_{x \epsilon X} \mu_B(x)$$

B is a normal fuzzy set if h(B) = 1, otherwise it is subnormal.

If $0 < h(B) < 1$ then the subnormal fuzzy set B can be normalized by defining its membership function as $\frac{\mu_B(x)}{h(B)}. x \in X$.

2.6. Membership Function

The fuzzy membership function is a graphical method for representing the degree to which a value belongs to a given fuzzy set B on the universe of discourse X. It is defined as

$$\mu_B: X \to [0,1]$$

where each element of X is assigned a value between 0 and 1. X represents the universal set, while B is the fuzzy set derived from X.

2.7. Extension Theory and Fuzzy Arithmetic:

The extension theory will now be used to execute algebraic operations on fuzzy numbers. A normal, convex set on the real line is used to represent the fuzzy number $T_{\tilde{x}}$. Consider two fuzzy numbers $T_{\tilde{x}}$ and $T_{\tilde{y}}$, defined on the real line in the universes X and Y respectively, and the symbol $*$ denotes a general arithmetic operation that is,$* \in \{+,-,\cdot,/\}$

To perform an arithmetic operation between two numbers in universe Z denoted by $T_{\tilde{x}} * T_{\tilde{y}}$, which can be accomplished using the extension principle by

$$\mu_{T_{\tilde{z}}}(z) = \sup_{z=x*y} \{\mu_{T_{\tilde{x}}}(x) \wedge \mu_{T_{\tilde{y}}}(y)\}$$

3. Multiplication of fuzzy arithmetic using a table of composition:

$$Let\ z = F(x,y) = x \cdot y. Then\ \tilde{x} \cdot \tilde{y}\ with$$

$$T_{\tilde{z}} = \{z \in Z \mid z = x \cdot y, x \in T_{\tilde{x}}, y \in T_{\tilde{y}}$$

$$\text{And } \mu_{T_{\tilde{z}}}(z) = \sup_{z=x*y} \{\mu_{T_{\tilde{x}}}(x) \wedge \mu_{T_{\tilde{y}}}(y)\}$$

$$\text{By } \alpha-\text{cut notation}, (T_{\tilde{z}})^{\alpha} = F\left((T_{\tilde{x}})^{\alpha}, (T_{\tilde{y}})^{\alpha}\right) = (T_{\tilde{x}})^{\alpha} \cdot (T_{\tilde{y}})^{\alpha}.$$

Example: Let $\tilde{x}$ and $\tilde{y}$ be such that $T_{\tilde{x}} = [2,5], T_{\tilde{y}} = [3,6]$ with the membership functions

$$\mu_{T_{\tilde{x}}}(x) = \begin{cases} x-2, 2 \le x \le 3 \\ -\frac{x}{2}+\frac{5}{2}, 3 \le x \le 5 \end{cases} \text{ and } \mu_{T_{\tilde{y}}}(y) = \begin{cases} \frac{y}{2}-\frac{3}{2}, 3 \le y \le 5 \\ -y+6, 5 \le y \le 6 \end{cases}$$

We have $\mu_{T_{\tilde{z}}}(z) = \sup_{z=x\cdot y} \{\mu_{T_{\tilde{x}}}(x) \wedge \mu_{T_{\tilde{y}}}(y)\}$.

We select an integer from the range [2, 5] and [3, 6] and create the following composition table

·	2	3	4	5
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25
6	12	18	24	30

Using the above table, let's consider when z=12. It is feasible to multiply such that z = 12 under the following circumstances:

$$(2 \cdot 6, 3 \cdot 4, 4 \cdot 3, \cdots\cdots\cdots)$$

$$\text{So } \mu_{T_{\tilde{z}}}(12) = \sup_{x\cdot y=12} \{\mu_{T_{\tilde{x}}}(2) \wedge \mu_{T_{\tilde{y}}}(6), \mu_{T_{\tilde{x}}}(3) \wedge \mu_{T_{\tilde{y}}}(4), \mu_{T_{\tilde{x}}}(4) \wedge \mu_{T_{\tilde{y}}}(3), \cdots\cdots\cdots\}$$

$$= \underset{x\cdot y=12}{\text{Sup}} \{0 \wedge 0, 1 \wedge 3.5, 0.5 \wedge 3, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\}.$$

$$= \underset{x\cdot y=12}{\text{Sup}} \{0, 1, 0.5, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\} = 1.$$

Again, consider that when z=20. The following situations allow for multiplication to make z=20:

(4 · 5, 5 · 4, …….).

$$\text{Now } \mu_{T_{\tilde{z}}}(20) = \underset{x\cdot y=20}{\text{Sup}} \{\mu_{T_{\tilde{x}}}(4) \wedge \mu_{T_{\tilde{y}}}(5), \mu_{T_{\tilde{x}}}(5) \wedge \mu_{T_{\tilde{y}}}(4), \cdots\cdots\cdots\}$$

$$= \underset{x\cdot y=20}{\text{Sup}} \{0.5 \wedge 4, 0 \wedge 3.5, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\}.$$

$$= \underset{x\cdot y=20}{\text{Sup}} \{0.50, 0, \cdots\cdots\cdots\cdots\} = 0.50.$$

If you get the membership function for all $z \in T_{\tilde{x}} \cdot T_{\tilde{y}}$ from this kind of method. we may conveniently definite it is a fuzzy number by approximating where $T_{\tilde{z}} = [6,30]$.

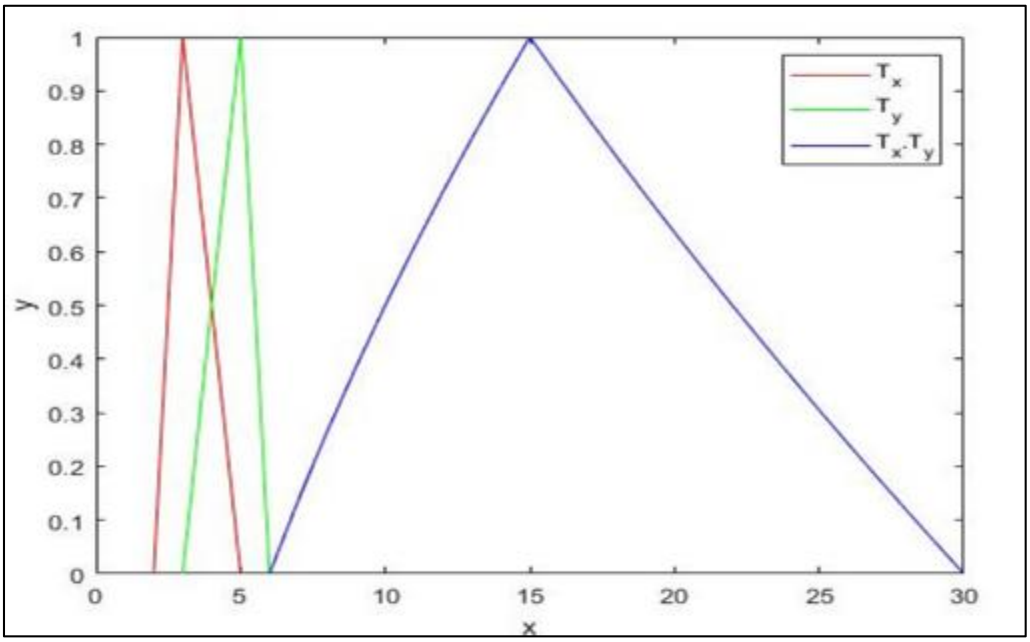

Figure 1 Multiplication of Two fuzzy numbers

By α − cut notation, for any α values, Letting,

$$\text{Gives } x_1 = \alpha + 2 \text{ and } x_2 = -2\alpha + 5$$

So that $(T_{\tilde{x}})^{\alpha} = [\alpha + 2, -2\alpha + 5]$ *Similarly* $(T_{\tilde{y}})^{\alpha} = [2\alpha + 3, -\alpha + 6]$

It then follows that $T_{\tilde{z}} = [6,30]$ and $\mu_{T_{\tilde{z}}}(z) = \underset{z=x\cdot y}{\text{Sup}} \{\mu_{T_{\tilde{x}}}(x) \cdot \mu_{T_{\tilde{y}}}(y)\}$

$$=[\alpha + 2, -2\alpha + 5] \cdot [2\alpha + 3, -\alpha + 6].$$

$$=[2\alpha^2 + 7\alpha + 6, -\alpha^2 + 4\alpha + 12, -4\alpha^2 + 4\alpha + 15, 2\alpha^2 - 17\alpha + 30]$$

$$= [p(\alpha), \bar{p}(\alpha)]$$

Where $p(\alpha) = \min\{2\alpha^2 + 7\alpha + 6, -\alpha^2 + 4\alpha + 12, -4\alpha^2 + 4\alpha + 15, 2\alpha^2 - 17\alpha + 30\}$,

$$\bar{p}(\alpha) = \max\{2\alpha^2 + 7\alpha + 6, -\alpha^2 + 4\alpha + 12, -4\alpha^2 + 4\alpha + 15, 2\alpha^2 - 17\alpha + 30\}$$

Hence $p(\alpha) = 2\alpha^2 + 7\alpha + 6$ and $\bar{p}(\alpha) = 2\alpha^2 - 17\alpha + 30$

So that $(T_{\tilde{z}})^{\alpha} = [p(\alpha), \bar{p}(\alpha)] = [2\alpha^2 + 7\alpha + 6, 2\alpha^2 - 17\alpha + 30]$.

Let moreover, $z_1 = 2\alpha^2 + 7\alpha + 6$ and $z_2 = 2\alpha^2 - 17\alpha + 30$.

We solve them for α, subject to $0 \le \alpha \le 1$ and obtain $\alpha = \frac{-7+\sqrt{1+8z_1}}{4}$ or $\alpha = \frac{17-\sqrt{49+8z_2}}{4}$

Consequently, we have the membership function

$$\mu_{T_{\tilde{z}}}(z) = \begin{cases} \frac{-7+\sqrt{1+8z_1}}{4}, & 6 \le z_1 \le 15 \\ \frac{17-\sqrt{49+8z_2}}{4}, & 15 \le z_2 \le 30 \end{cases}.$$

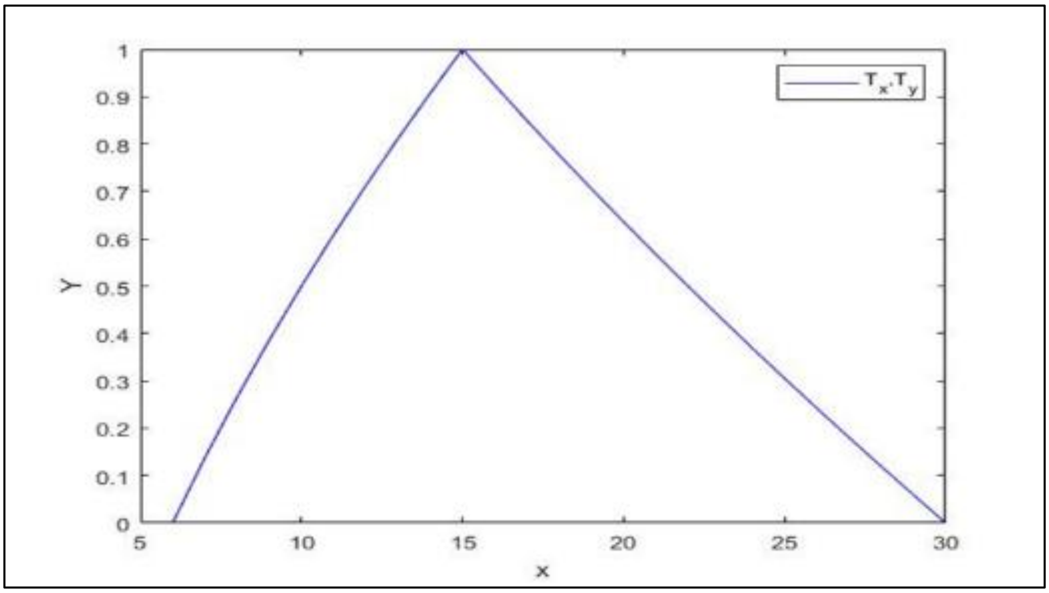

Figure 2 The resulting membership function

4. Division of fuzzy arithmetic using a table of composition:

$$Let\ z = F(x, y) = \frac{x}{y}. Then \frac{\tilde{x}}{\tilde{y}}\ with$$

$$T_{\tilde{z}} = \{z \in Z \mid z = \frac{x}{y}, x \in T_{\tilde{x}}, y \in T_{\tilde{y}}$$

$$\text{And } \mu_{T_{\tilde{z}}}(z) = \sup_{z=x*y} \{\mu_{T_{\tilde{x}}}(x) \wedge \mu_{T_{\tilde{y}}}(y)\}$$

$$\text{By } \alpha-\text{cut notation}, (T_{\tilde{z}})^{\alpha} = F\left((T_{\tilde{x}})^{\alpha}, (T_{\tilde{y}})^{\alpha}\right) = \frac{(T_{\tilde{x}})^{\alpha}}{(T_{\tilde{y}})^{\alpha}}.$$

Example: Let $\tilde{x}$ and $\tilde{y}$ be such that $T_{\tilde{x}} = [18, 33], T_{\tilde{y}} = [5, 8]$ with the membership functions

$$\mu_{T_{\tilde{x}}}(x) = \begin{cases} \frac{x}{4} - \frac{18}{4}, 18 \le x \le 22 \\ -\frac{x}{11} + 3, 22 \le x \le 33 \end{cases} \text{ and } \mu_{T_{\tilde{y}}}(y) = \begin{cases} y - 5, 5 \le x \le 6 \\ -\frac{y}{2} + 4, 6 \le x \le 8 \end{cases}$$

We have $\mu_{T_{\tilde{z}}}(z) = \underset{z=\frac{x}{y}}{\text{Sup}}\{\mu_{T_{\tilde{x}}}(x) \wedge \mu_{T_{\tilde{y}}}(y)\}$.

We pick some integers from the ranges [18, 33] and [5, 8] and create the following composition table:

/	18	21	24	27	30	33
5	9/4	21/8	3	27/8	15/4	33/8
6	18/7	3	24/7	27/7	30/7	33/7
7	3	21/6	4	9/2	5	33/6
8	18/5	21/5	24/5	27/5	6	33/5

Let's consider the case where z = 3 based on the table above. It is possible to divide to get z = 3 in the following circumstances:

$$(\frac{18}{6}, \frac{21}{7}, \cdots\cdots\cdots)$$

$$\text{So } \mu_{T_{\tilde{z}}}(20) = \underset{x/y=3}{\text{Sup}}\{\mu_{T_{\tilde{x}}}(18) \wedge \mu_{T_{\tilde{y}}}(6), \mu_{T_{\tilde{x}}}(21) \wedge \mu_{T_{\tilde{y}}}(7), \cdots\cdots\cdots\}$$

$$= \underset{x/y=3}{\text{Sup}}\{0 \wedge 1, 0.75 \wedge 0.5, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\}.$$

$$= \underset{x/y=3}{\text{Sup}}\{0, 0.5, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\} = 0.5.$$

If using this method, we determine the membership function for every $z\epsilon\frac{T_{\tilde{x}}}{T_{\tilde{y}}}$, we may conveniently definite it as a fuzzy number by approximating where $T_{\tilde{z}} = \left[\frac{9}{4}, \frac{33}{5}\right]$

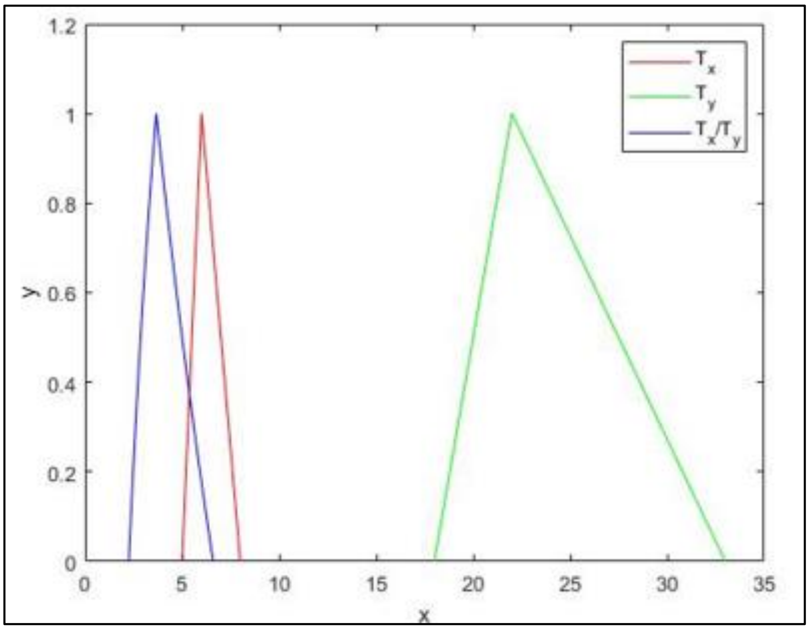

Figure 3 Division of Two fuzzy numbers

By $\alpha - cut$ notation, Let $\alpha = \frac{x}{4} - \frac{18}{4}$ and $\alpha = \frac{-x}{11} + 3$,

We obtain $x_1 = 4\alpha + 18$ and $x_2 = -11\alpha + 33$.

So that $(T_{\tilde{x}})^{\alpha} = [4\alpha + 18, -11\alpha + 33]$

Similarly, $(T_{\tilde{y}})^{\alpha} = [\alpha + 5, -2\alpha + 8]$.

We have $(T_{\tilde{z}})^{\alpha} = \frac{(T_{\tilde{x}})^{\alpha}}{(T_{\tilde{y}})^{\alpha}} = \frac{[4\alpha+18, -11\alpha+33]}{[\alpha+5, -2\alpha+8]} = [\frac{4\alpha+18}{-2\alpha+8}, \frac{-11\alpha+33}{\alpha+5}]$.

Next letting $z_1 = \frac{4\alpha+18}{-2\alpha+8}$ and $z_2 = \frac{-11\alpha+33}{\alpha+5}$

Which implies that $\alpha = \frac{8z_1-18}{2z_1+4}$ and $\alpha = \frac{-5z_2+33}{z_2+11}$.

Consequently, we have the membership function $\mu_{T_{\tilde{z}}}(z) = \begin{cases} \frac{8z_1-18}{2z_1+4}, \frac{9}{4} \leq z_1 \leq \frac{11}{3} \\ \frac{-5z_2+33}{z_2+11}, \frac{11}{3} \leq z_2 \leq \frac{33}{5} \end{cases}$.

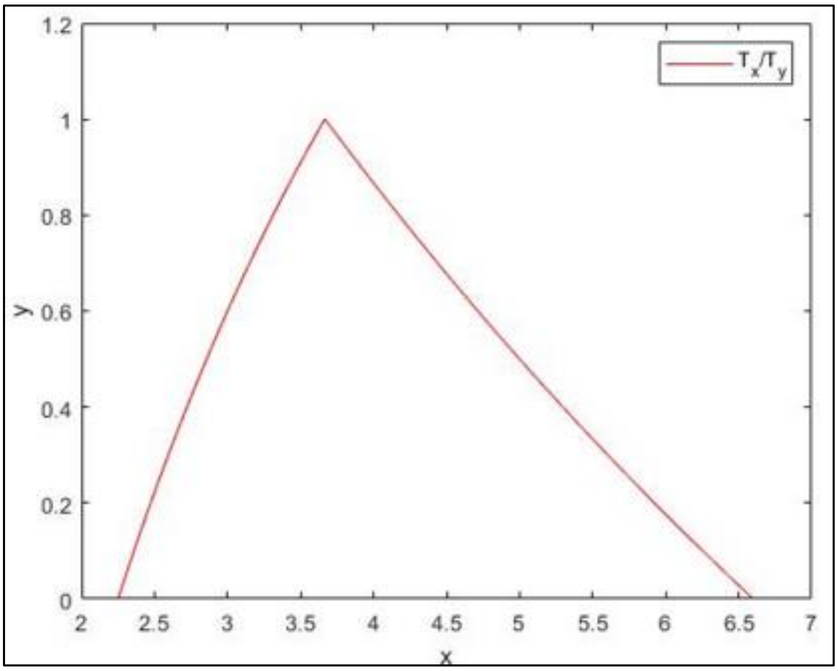

Figure 4 The resulting membership function

5. Conclusion

In this paper, we investigate fuzzy numbers with arithmetic, a particular kind of fuzzy set. By creating a composition table and displaying a graphical representation, we have carefully studied several operations, such as the multiplication and division of fuzzy integers.

Fuzzy numbers are a type of subset within the real number set that have additional conditions. There are arithmetic operations that have been developed for fuzzy numbers using Extension Theory and Fuzzy Arithmetic. If working with fuzzy numbers, the shape of their membership functions greatly affects the outcome of our calculations. The operations of multiplication and division are also applicable to fuzzy numbers.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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