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(Research Article)

Generalized techniques for constructing the generating functions of some algebraic and rational functions

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Abstract

We construct generating functions for general forms of certain classes of algebraic and rational functions using only basic tools of Mathematics. This approach unifies, extends and generalizes several techniques of constructing generating functions.

Keywords: Power series; Algebraic function; Rational function; Sequence of numbers and differentiable function.

1. Introduction

Generating function technique in mathematics provides a way of using a single function to encode an infinite sequence of numbers by treating them as the coefficients of a formal power series. This series is called the generating function of the sequence [1]. Generating functions showcase the "power of power series", giving more depth to the word "power" in power series. However, generating function technique provides a natural and elegant way of dealing with sequence of numbers by associating a continuous function of a variable with a sequence. In this way, generating functions provide a bridge between discrete and continuous mathematics [2]. In 1730, Abraham de Moivre solved some problems of linear recurrence using generating functions. Then, James Stirling in 1730 extended this theory by using differentiation and integration to solve the same problems. In 1747, Leonhard Paul Euler extended the use of the technique to study partitions of integers. Furthermore, Pierre-Simon de Laplace in 1812 extended the technique to moment generating functions and probability theory and from there, many other researchers came up after them [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Generating functions enable the solution of sequence problems with the method used in solving problems in algebra. Furthermore, generating functions can be viewed in the perspective of the formal power series where the convergence is considered a non-issue [16]. The major idea of a generating function is to use a single function instead of an infinite sequence of numbers. Generating functions have proved to be very useful tools which facilitate the solution of various classes of counting problems as questions about the convergence are not required. Thus, this enables the solutions of difficult varieties of counting problems with little effort [17].

There are several kinds of generating functions which includes Bell Series, Lambert series, ordinary generating functions, exponential generating functions and Dirichlet series. Every sequence has one generating function or the other but the way they are handled differs except Lambert and Dirichlet series which requires its indices to start with one rather than zero. Generating functions are used to find exact formula, recurrence formulas, averages, statistical properties, prove identities and other properties of sequences [18]. Sometimes, generating functions can be regarded as generating series and can also be expressed in a closed form instead of as a series [19, 20]

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In this work, the generalized techniques for constructing the generating functions of some algebraic and rational functions are studied with the view to unify and extend these techniques and their applications. In addition, the generalized techniques for constructing the generating functions of some algebraic and rational functions were developed. Also, these generalized techniques were applied in obtaining series expansions of certain classes of functions.

2. Basic definitions

2.1. Polynomial function

A polynomial function in x is a function which contains only a finite number of positive integer powers of x, usually arranged in the order of ascending or descending powers. The degree of a Polynomial is the highest power of the variable occurring in the function; for instance

$$f(x) = 5x^4 + 7x^3 + 2x^2 + 3x + 1$$

is a $4th$ degree polynomial. Thus, the general form of a polynomial function can be seen in the form of power series like

$$f(x) = \sum_{m=0}^{\infty} f_m x^m$$

Provided that there exist $n_0 \in \mathbb{N}$ such that $f_m = 0 \; for \; all \; m > n_0$

2.2. Rational function

Let $p(x) \; and \; q(x)$ be polynomials in a variable $x. \frac{p(x)}{q(x)} \; is$ called a rational function in x provided that $q(x) \neq 0$. Thus, $\frac{p(x)}{q(x)}$ is defined only for values of x where $q(x) \neq 0$. If the degree of $p(x)$ is less than the degree of $q(x)$, then the rational function is called a proper rational function

2.3. Algebraic function

Algebraic function is a function which contains algebraic operations like addition, multiplication, subtraction and division as well as fractional or rational exponents [21].

2.4. Analytic function

A function is analytic if it is differentiable at every point in the finite plane. In other words, a function is analytic once it has a convergent local power series [4, 22, 23].

2.5. Combinatorics

It is a mathematical field that deals with the problems of arrangement and selection together with their operations in a finite set which has some constraints [3, 24, 15].

2.6. Ordinary generating function

It is a generating function which appear in the form of

$$B(x) = \sum_{n=0}^{\infty} f_n x^n = f_0 + f_1 x + f_2 x^2 + \cdots$$

Where f_n is the coefficient of the series. Ordinary generating function can also be called generating function [4, 25, 26]

2.7. Exponential generating function

Exponential generating function of the sequence $\{f_n\}_{n\geq 0}$ is any formal power series that is of the form

$$B(x) = \sum_{n\geq 0} f_n \frac{x^n}{n!}$$

Where $B(x)$ is the exponential generating function of $f_0, f_1, f_2, f_3, f_{4,} \dots$ [26].

3. Research results

3.1. Generating functions of algebraic and rational functions.

Theorem 3.1 Let $f(x) = c(a + bx)^m$. Then its generating function is given by

$$(i)\ at\ x_0 = 0,$$

$$g(x) = \begin{cases} c\sum_{r=0}^{m}\binom{m}{r} a^{m-r}b^r x^r;\ if\ b > 0, m > 0 \\ c\sum_{r=0}^{m}\binom{m}{r}(-1)^r a^{m-r}d^r x^r;\ if\ b = -d < 0, m > 0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(-1)^r a^{-(n+r)}b^r x^r;\ if\ b > 0, m = -n < 0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r} a^{-(n+r)}d^r x^r;\ if\ b = -d < 0, m = -n < 0 \end{cases}$$

$$and\ (ii)\ at\ x_0 \neq 0\ (x_0 \neq -\frac{a}{b}\ if\ m < 0)$$

$$g_0(x) = \begin{cases} c\sum_{r=0}^{m}\binom{m}{r}(a + bx_0)^{m-r}b^r(x - x_0)^r;\ if\ b > 0, m > 0 \\ c\sum_{r=0}^{m}\binom{m}{r}(-1)^r(a - dx_0)^{m-r}d^r(x - x_0)^r;\ if\ b = -d < 0, m > 0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(-1)^r(a + bx_0)^{-(n+r)}b^r(x - x_0)^r;\ if\ b > 0, m = -n < 0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(a + bx_0)^{-(n+r)}d^r(x - x_0)^r;\ if\ b = -d < 0, m = -n < 0 \end{cases}$$

Proof

Observe that $D(f) = \mathbb{R}, if\ m > 0\ and\ D(f) = \mathbb{R} \setminus \left\{-\frac{a}{b}\right\}\ and$ that $f \epsilon C^\infty$, that is f continuously differentiable i.e. f is a smooth function. Also, for any $r > 0\ integer$

$$f^{(r)}(x) = \begin{cases} c\ m_{P_r}(a + bx)^{m-r}b^r;\ if\ m > 0 \\ c\ (n - 1 + r)_{P_r}(-1)^r(a + bx)^{-(n+r)}b^r;\ if\ m = -n < 0 \end{cases}$$

So that

$$f^{(r)}(0) = \begin{cases} c\ m_{P_r}a^{m-r}b^r;\ if\ m > 0, b > 0 \\ c\ m_{P_r}(-1)^r a^{m-r}d^r;\ if\ m > 0, b = -d < 0 \\ c\ (n - 1 + r)_{P_r}(-1)^r a^{-(n+r)}b^r;\ if\ m = -n < 0, b > 0 \\ c\ (n - 1 + r)_{P_r}a^{-(n+r)}d^r;\ if\ m = -n < 0, b = -d < 0 \end{cases}$$

and

$$f^{(r)}(x_0) = \begin{cases} c\, m_{P_r}(a+bx_0)^{m-r}b^r;\ if\ m>0, b>0 \\ c\, m_{P_r}(-1)^r(a+bx_0)^{m-r}d^r;\ if\ m>0, b=-d<0 \\ c\,(n-1+r)_{P_r}(-1)^r(a+bx_0)^{-(n+r)}b^r;\ if\ m=-n<0, b>0 \\ c\,(n-1+r)_{P_r}(a+bx_0)^{-(n+r)}d^r;\ if\ m=-n<0, b=-d<0 \end{cases}$$

Hence, the generating function has the stated forms

$$(i)\ at\ x_0 = 0$$

$$g(x) = \begin{cases} c\sum_{r=0}^{m}\binom{m}{r}a^{m-r}b^r x^r;\ if\ b>0, m>0 \\ c\sum_{r=0}^{m}\binom{m}{r}(-1)^r a^{m-r}d^r x^r;\ if\ b=-d<0, m>0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(-1)^r a^{-(n+r)}b^r x^r;\ if\ b>0, m=-n<0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}a^{-(n+r)}d^r x^r;\ if\ b=-d<0, m=-n<0 \end{cases}$$

$$and\ (ii)\ at\ x_0 \neq 0\ (x_0 \neq -\frac{a}{b}\ if\ m<0)$$

$$g_0(x) = \begin{cases} c\sum_{r=0}^{m}\binom{m}{r}(a+bx_0)^{m-r}b^r(x-x_0)^r;\ if\ b>0, m>0 \\ c\sum_{r=0}^{m}\binom{m}{r}(-1)^r(a-dx_0)^{m-r}d^r(x-x_0)^r;\ if\ b=-d<0, m>0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(-1)^r(a+bx_0)^{-(n+r)}b^r(x-x_0)^r;\ if\ b>0, m=-n<0 \\ c\sum_{r\geq 0}\binom{n-1+r}{r}(a+bx_0)^{-(n+r)}d^r(x-x_0)^r;\ if\ b=-d<0, m=-n<0 \end{cases}$$

This completes the proof.

COROLLARY 3.1 Let the function

$$f(x) = \frac{ax}{(1-bx)^m}\ \forall\ b>0\ and\ m>0.$$

Then its generating function is given by

$$g(x) = \begin{cases} a\sum_{r\geq 0}\binom{m+r-2}{r-1}b^{r-1}x^r\ if\ x_0 = 0 \\ a\sum_{r\geq 0}b^{r-1}(1-bx_0)^{-(m+r-1)}\left[\binom{m+r-2}{r-1} + b\binom{m+r-1}{r}x_0(1-bx_0)^{-1}\right](x-x_0)^r\ if\ x_0 \neq 0 \end{cases}$$

Proof

We observe that f is undefined for $x = \frac{1}{b}$ $so\ D(f) = \mathbb{R}\setminus\left\{\frac{1}{b}\right\}$

$$f(x) = \frac{ax}{(1-bx)^m} = ax(1-bx)^{-m}$$

Now, for $r \in \mathbb{N}$, we have

$$f^r(x) = rab^{r-1}(m+r-2)_{P_{(r-1)}}(1-bx)^{-(m+r-1)} + ab^r(m+r-1)_{P_r}x(1-bx)^{-(m+r)}$$

$$f^r(0) = rab^{r-1}(m+r-2)_{P_{(r-1)}}$$

$$f^r(x_0) = rab^{r-1}(m+r-2)_{P_{(r-1)}}(1-bx_0)^{-(m+r-1)} + ab^r(m+r-1)_{P_r}x_0(1-bx_0)^{-(m+r)}$$

$$= ab^{r-1}(m+r-2)_{P_{(r-1)}}(1-bx_0)^{-(m+r-1)}[r + b(m+r-1)x_0(1-bx_0)^{-1}]$$

Then, the generating function becomes

$$g(x) = \begin{cases} a\sum_{r\geq 0}\binom{m+r-2}{r-1} b^{r-1}\, x^r \; if\; x_0 = 0 \\ a\sum_{r\geq 0}\frac{(m+r-2)_{P_{(r-1)}}}{r!} b^{r-1}(1-bx_0)^{-(m+r-1)}[r + b(m+r-1)x_0(1-bx_0)^{-1}](x-x_0)^r \; if\; x_0 \neq 0 \end{cases}$$

$$g(x) = \begin{cases} a\sum_{r\geq 0}\binom{m+r-2}{r-1} b^{r-1}\, x^r \; if\; x_0 = 0 \\ a\sum_{r\geq 0} b^{r-1}(1-bx_0)^{-(m+r-1)}\left[\binom{m+r-2}{r-1} + b\binom{m+r-1}{r} x_0(1-bx_0)^{-1}\right](x-x_0)^r \; if\; x_0 \neq 0 \end{cases}$$

This completes the proof.

Similarly, (i) for the function

$$f(x) = \frac{a}{(1-bx)^m}\; for\; all\; b > 0\; and\; m > 0$$

its generating function is

$$g(x) = \begin{cases} a\sum_{r\geq 0}\binom{m+r-1}{r} b^r x^r \; if\; x_0 = 0 \\ a\sum_{r\geq 0}\binom{m+r-1}{r} b^r(1-bx_0)^{-(m+r)}(x-x_0)^r \; if\; x_0 \neq 0 \end{cases}$$

(ii) for the function

$$f(x) = \frac{a}{1-rx}\; \forall\, a, r \in \mathbb{R}\setminus\{0\}$$

its generating function is

$$g(x) = \begin{cases} a\sum_{k\geq 0} r^k x^k \; if\; x_0 = 0 \\ a\sum_{k\geq 0} r^k(1-rx_0)^{-(k+1)}(x-x_0)^k \; if\; x_0 \neq 0 \end{cases}$$

THEOREM 3.2 Let m be a positive integer, $0 < l < m\; integer, let\; \lambda_i\; (i = 0, \dots, l), a, b \epsilon \mathbb{R}$, let

$$q_l(x) = \sum_{i=0}^{l} \lambda_i x^i\; and\; let\; f(x) = \frac{q_l(x)}{(a+bx)^m}$$

Then, the generating function for f is given by

$$g(x) = \sum_{r\geq 0}\left\{\left[\sum_{j=0}^{l} A_{m-l+j} b^{l-j}\binom{m-1-l+j+r}{r}\right](-1)^r b^{-(m+r)} a^r x^r\right\}$$

Proof

$$\frac{q_l(x)}{(a+bx)^m} \equiv \sum_{i=1}^{m}\frac{A_i}{(a+bx)^i} = \frac{A_1}{(a+bx)} + \frac{A_2}{(a+bx)^2} + \cdots + \frac{A_m}{(a+bx)^m}$$

This produces

$$A_k = 0\ for\ k = 1, 2, \dots, m-l-1$$

Hence,

$$\frac{q_l(x)}{(a+bx)^m} = \sum_{j=0}^{l}\frac{A_{m-l+j}}{(a+bx)^{m-l+j}}$$

The generating function for

$$f_{m-l+j}(x) = \frac{A_{m-l+j}}{(a+bx)^{m-l+j}}\ is\ given\ by$$

$$g_j(x) = A_{m-l+j}\sum_{r\geq 0}\binom{m-1-l+j+r}{r}(-1)^r b^{-(m-l+j+r)} a^r x^r$$

Hence, the generating function for f is given by

$$g(x) = \sum_{j=0}^{l} g_j(x) = \sum_{j=0}^{l}\left\{A_{m-l+j}\sum_{r\geq 0}\binom{m-1-l+j+r}{r}(-1)^r b^{-(m-l+j+r)} a^r x^r\right\}$$

$$g(x) = \sum_{r\geq 0}\left[\sum_{j=0}^{l} A_{m-l+j} b^{l-j}\binom{m-l-1+j+r}{r}\right](-1)^r b^{-(m+r)}\, a^r x^r$$

$$g(x) = \sum_{r\geq 0}\left\{A_{m-l} b^l \binom{m-l-1+r}{r} + A_{m-l+1} b^{l-1}\binom{m-l+r}{r} + \cdots + A_{m-1} b\binom{m-2+r}{r} + A_m\binom{m-1+r}{r}\right\}(-1)^r b^{-(m+r)} a^r x^r$$

This completes the proof.

3.2. Applications of the generalized techniques in obtaining series expansion of some functions

EXAMPLE 3.1 Find the series expansion of the function

$$f(x) = \frac{3}{(1-5x)^6}\ at\ (i)\ x_0 = 0\ (ii)\ x_0 = 2$$

Solution

The generalized technique for obtaining the generating function of

$$f(x)=\frac{a}{(1-bx)^m}\ for\ all\ b>0\ and\ m>0$$

is

$$g(x)=\begin{cases} a\sum_{r\geq 0}\binom{m+r-1}{r}b^r x^r\ if\ x_0=0 \\ a\sum_{r\geq 0}\binom{m+r-1}{r}b^r(1-bx_0)^{-(m+r)}(x-x_0)^r\ if\ x_0\neq 0 \end{cases}$$

Here, we observe that $a=3, b=5\ and\ m=6$. Thus, the series is given by

$$g(x)=\begin{cases} 3\sum_{r\geq 0}\binom{6+r-1}{r}5^r x^r\ if\ x_0=0 \\ 3\sum_{r\geq 0}\binom{6+r-1}{r}5^r(1-5(2))^{-(6+r)}(x-2))^r\ if\ x_0=2 \end{cases}$$

$$g(x)=\begin{cases} 3\sum_{r\geq 0}\binom{5+r}{r}5^r x^r\ if\ x_0=0 \\ 3\sum_{r\geq 0}\binom{5+r}{r}5^r(-9)^{-(6+r)}(x-2))^r\ if\ x_0=2 \end{cases}$$

$$g(x)=\begin{cases} 3\left[\binom{5}{0}5^0x^0+\binom{6}{1}5^1x^1+\binom{7}{2}5^2x^2+\binom{8}{3}5^3x^3+\cdots\right]\ if\ x_0=0 \\ 3\left[\binom{5}{0}5^0(-9)^{-(6)}(x-2)^0+\binom{6}{1}5^1(-9)^{-(7)}(x-2)^1+\binom{7}{2}5^2(-9)^{-(8)}(x-2)^2+\cdots\right]\ if\ x_0=2 \end{cases}$$

$$g(x)=\begin{cases} 3+90x+1575x^2+21000x^3+\cdots\ if\ x_0=0 \\ \frac{1}{177147}-\frac{10}{531441}(x-2)+\frac{175}{4782969}(x-2)^2+\cdots\ if\ x_0=2 \end{cases}$$

Thus, the series is

$$g(x)=\begin{cases} 3+90x+1575x^2+21000x^3+\cdots\ if\ x_0=0 \\ \frac{1}{177147}-\frac{10}{531441}(x-2)+\frac{175}{4782969}(x-2)^2+\cdots\ if\ x_0=2 \end{cases}$$

EXAMPLE 3.2 Find the series expansion of the function

$$f(x)=\frac{3}{1-2x}\ at\ (i)\ x_0=0\ (ii)\ x_0=4$$

Solution

The generalized technique for obtaining the generating function of

$$f(x)=\frac{a}{1-rx}\ \forall\ a,r\in\mathbb{R}\setminus\{0\}$$

is given by

$$g(x)=\begin{cases} a\sum_{k\geq 0}r^k x^k\ if\ x_0=0 \\ a\sum_{k\geq 0}r^k(1-rx_0)^{-(k+1)}(x-x_0)^k\ if\ x_0\neq 0 \end{cases}$$

Here, we observe that $a = 3$ and $r = 2$. Thus, the series is given by

$$g(x) = \begin{cases} 3\sum_{k\geq 0} 2^k x^k \; if \; x_0 = 0 \\ 3\sum_{k\geq 0} 2^k (1-2(4))^{-(k+1)}(x-4)^k \; if \; x_0 = 4 \end{cases}$$

$$g(x) = \begin{cases} 3\sum_{k\geq 0} 2^k x^k \; if \; x_0 = 0 \\ 3\sum_{k\geq 0} 2^k (-7)^{-(k+1)}(x-4)^k \; if \; x_0 = 4 \end{cases}$$

$$g(x) = \begin{cases} 3[2^0x^0 + 2^1x^1 + 2^2x^2 + 2^3x^3 + 2^4x^4 + \cdots] \; if \; x_0 = 0 \\ 3[2^0(-7)^{-1}(x-4)^0 + 2^1(-7)^{-2}(x-4)^1 + 2^2(-7)^{-3}(x-4)^2 + \cdots] \; if \; x_0 = 4 \end{cases}$$

$$g(x) = \begin{cases} 3 + 6x + 12x^2 + 24x^3 + 48x^4 + \cdots \; if \; x_0 = 0 \\ -\frac{3}{7} + \frac{6}{49}(x-4) - \frac{12}{343}(x-4)^2 + \cdots \; if \; x_0 = 4 \end{cases}$$

Thus, the series is

$$g(x) = \begin{cases} 3 + 6x + 12x^2 + 24x^3 + 48x^4 + \cdots \; if \; x_0 = 0 \\ -\frac{3}{7} + \frac{6}{49}(x-4) - \frac{12}{343}(x-4)^2 + \cdots \; if \; x_0 = 4 \end{cases}$$

EXAMPLE 3.3 Find the series expansion of the function

$$f(x) = \frac{5x}{(1-3x)^7}$$

Solution

The generalized technique for obtaining the generating function of

$$f(x) = \frac{ax}{(1-bx)^m} \; for \; all \; b > 0 \; and \; m > 0$$

is

$$g(x) = a\sum_{r\geq 0}\binom{m+r-2}{r-1} b^{r-1} x^r \; if \; x_0 = 0$$

$Here, a = 5, b = 3$ and $m = 7.$ Thus, the series is given by

$$g(x) = 5\sum_{r\geq 0}\binom{7+r-2}{r-1}3^{r-1}x^r = 5\sum_{r\geq 0}\binom{5+r}{r-1}3^{r-1}x^r$$

$$g(x) = 5\left[\binom{5}{-1}3^{-1}x^0 + \binom{6}{0}3^0x^1 + \binom{7}{1}3^1x^2 + \binom{8}{2}3^2x^3 + \cdots\right]$$

$$g(x) = 5[0 + x + 7(3)x^2 + 28(9)x^3 + \cdots]$$

$$g(x) = 5[x + 21x^2 + 252x^3 + \cdots]$$

$$g(x) = 5x + 105x^2 + 1260x^3 + \cdots$$

Thus, the series is $g(x) = 5x + 105x^2 + 1260x^3 + \cdots$

EXAMPLE 3.4 Find the series expansion of the function

$$f(x) = 4(1 - 5x)^{-7}$$

Solution

The generalized technique for obtaining the generating function of

$$f(x) = c(a + bx)^m \; at \; x_0 = 0$$

is

$$g(x) = c\sum_{r\geq 0}\binom{n-1+r}{r}(-1)^r a^{-(n+r)} b^r x^r; \; if \; b > 0, m = -n < 0$$

$Here, c = 4, a = 1, b = -5 \; and \; n = 7 \; (i.e - n = -7).$ Thus, the series is given by

$$g(x) = 4\sum_{r\geq 0}\binom{7-1+r}{r}(-1)^r 1^{-(7+r)}(-5)^r x^r = 4\sum_{r\geq 0}\binom{6+r}{r}(-1)^r 1^{-(7+r)}(-5)^r x^r$$

$$= 4\left[\binom{6}{0}(-5)^0 x^0 + \binom{7}{1}(-1)(-5)^1 x^1 + \binom{8}{2}(-1)^2(-5)^2 x^2 + \binom{9}{3}(-1)^3(-5)^3 x^3 + \cdots\right]$$

$$g(x) = 4[1 + 7(-1)(-5)x + 28(1)25x^2 + 84(-1)(-125)x^3 + \cdots]$$

$$g(x) = 4[1 + 35x + 700x^2 + 10500x^3 + \cdots]$$

$$g(x) = 4 + 140x + 2800x^2 + 42000x^3 + \cdots$$

Thus, the series is

$$g(x) = 4 + 140x + 2800x^2 + 42000x^3 + \cdots$$

4. Conclusion

Generating functions provide a bridge between discrete and continuous mathematics by associating a continuous function of a variable with a sequence. Generating functions have proved to be very useful tools which facilitate the solution of various classes of counting problems. Its roles can be appreciated because it treats any given power series as an algebraic object where the issue of convergence is de-emphasized. Thus, this enables the solutions of difficult varieties of counting problems with little effort. In this study, the generalized techniques for constructing the generating functions of some algebraic and rational functions were developed. Furthermore, these generalized techniques were applied in obtaining series expansion of certain classes of functions.

Recommendation

Additional research on generating function should be done so that it can be used to manipulate certain problems in Laurent and Fourier series.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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