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Application of modified first order shear deformation theory to vibration analysis of SSSS and CCSS thick anisotropic rectangular plates

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Abstract

This research presents a vibration analysis of a thick anisotropic rectangular plate using modified first shear deformation theory. Modified first shear deformation theory, which is not built upon the classical plate theory, was used to develop the kinematic equations and constitutive relations of a deformed section of thick anisotropic rectangular plate from which the generalized stress equations were determined. Using the assumptions of the theory, the strain energy and external work equations were formulated, and by employing the principles of total minimum potential energy, the total potential energy functional of a thick anisotropic plate was developed. Minimization of the total potential energy functional with respect to the deflection function (w) and with respect to the shear rotations (ϕ_x)and (ϕ_y), respectively, resulted in the governing differential equation and the two compatibility equations of the plate. The displacement functions that satisfy the governing and compatibility equations were obtained by solving the governing and compatibility equations. From the general displacement function, the peculiar deflection equations (shape functions) were obtained for the boundary conditions considered, which are simply supported on its four edges (SSSS), clamped on two adjacent edges, and simply supported on the other two (CCSS). Using the displacement equation and the equation for rotation in the x-direction (ϕ_x) and equation for rotation in y-direction (ϕ_y) the direct governing and two direct compatibility equations were obtained, from which coefficients that enable the formula for calculating fundamental natural frequencies to be obtained. For the boundary condition analyzed in this work, the stiffness coefficients (kR, kQ, kRQ, kq, kRRQ , kRQQ , kNR, kNQ, kNRQ and kλ) were computed and used in determining the fundamental natural frequency parameter values at various span to depth ratios (5,10,20, 25 and 100), aspect ratios (1 to 2 at the increment of 0.1) and angle of fibre orientation (0^o, 15^o, 45^o). The solutions of this study were compared with those from previous researchers. The fundamental natural frequency parameter values obtained in this study were compared with the work of Reddy (1984) for 0^o angle of fibre orientation at span to depth ratios of 5,10,20,25,50 and 100 at an aspect ratio of 1. The percentage difference values were 6.073%, 3.197%, 1.132%, 0.788%, 0.255%, and 0.112%, respectively. These differences revealed the closeness of the results of this present study to the results of Reddy (1984). This shows that the present theory provides good and acceptable solutions to the vibration problems of thick anisotropic rectangular plates.

Keywords: Thick Plate; Anisotropic; Fundamental natural frequency; Governing Equations

1. Introduction

For vibration analysis of thick plates, shear deformation effects are very important; these effects are what the classical plate theory (CPT) did not take into account. The classical plate theory ignored the effect of through-thickness shear deformation and thus overrated the stiffness of the plate, which is of very considerable significance for thick plates. In order to define the correct behavior of thick plates, including shear deformation effects and the associated cross-

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sectional warping are required and many theories have evolved to address this dearth. Refined traditional theories of plate analysis were advanced to overwhelm the shortcomings of classical plate theories (otherwise called Kirchhoff's plate theory). The first refined traditional theories are Mindlin theory and Reissner theory (Sadrnejad et al., 2009). These theories are called first-order shear deformation theories. They are called first-order theories because the straight line, which was normal to the middle surface before bending, remained straight but was no longer normal to the middle surface after bending. Hence, the profile equation of that line as used in the first-order theories is z coordinate. A major constraint of these first-order theories is the assumption of constant shear stress across the thickness of the plate. Correction factors are usually employed when these theories are used. To overcome the limitations of first-order theories, second, third and higher-order theories evolved (Sayyad and Ghugal, 2012; Sayyad, 2011). Popular among these theories is third-order theories (third-order shear deformation, exponential shear deformation, hyperbolic shear deformation, trigonometric shear deformation, etc.). Since the development of this theory, many researchers have used it in their respective studies on thick plates. These include the works of Qian et al. (2003), Batra and Vidoli (2002), Abdul-Razzak and Haido (2002), Kank and Swaminathan (2001), and Kocak and Hassis (2002). These higher-order theories have a shear deformation profile line (usually called f(z)) that is not linear (not equal to the z coordinate). The underlying reason for the nonlinear shear deformation line is that shear stress was assumed to relate to the first derivative of the profile line equation. That is, shear stress is the product of the first derivative of F(z) and nominal stress (CPT shear stress and the first derivative of the profile line equation). The implication of this assumption is that the shear deformation profile line equation, F(z) is related to the shear stress profile equation, G(z). That is, G(z) is the first derivative of F(z) (Shimpi and Patel, 2006; Chikalthankar et al., 2013; Ibearugbulem et al., 2016a). A better assumption ought to have been that, whereas the vertical shear is a product of nominal shear stress and G(z), no relationship exists between G(z) and F(z), meaning that G(z) cannot be obtained by the first derivative of F(z). Another, better assumption ought to be that the rotation of the middle surface is not divided into a classical part and a shear deformation part. The same assumption goes for in-plane displacements, u and v. They are whole each and are not divided into classical and shear deformation parts (Ibearugbulem et al., 2016b). Ibearugbulem et al. (2016b) assumed that F(z) is equal to z (that the profile is a straight line after deformation) and that there is no relationship between F(z) and G(z). With these assumptions, Kirchhoff's assumption of zero vertical shear strain or stress on the classical theory was avoided completely. In their opinion, the vertical shear strains and stresses can be very small to be ignored but can never be equal to zero (no matter how small). This erroneous assumption that, for classical theory, the vertical shear strain and stress are absolutely equal to zero was introduced in the third and higher-order theories. Ordinarily, in-plane displacements and stresses are not functions of span-to-depth ratios. It is only the out-of-plane displacement and stresses that are functions of the span-to-depth ratio. But this is not the case from the results obtained using third- and higher-order theories (Pagano, 1970; Pagano and Hatfield, 1972; Reddy, 1984; Idibi et al., 1997; Aydogdu, 2009; Daouadji et al., 2012). Ibearugbulem (2016b), in their work titled "Full shear deformation for analysis of thick plates," analyzed only isotropic plates. Another shortcoming of their work is the assumption of the displacement function. The present work tried to extend their work (Ibearugbulem, 2016b) to the case of vibration analysis of thick anisotropic plates. Theoretical formulation of natural frequency

2. Theoretical formulation of natural frequency

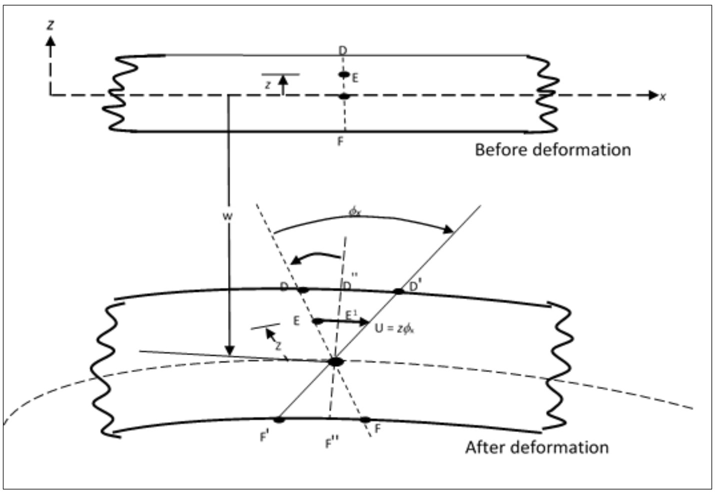

Figure 1 Deformation of a section of a thick anisotropic plate

The displacement field consists of one out-of-plane displacement(w) and two in-plane displacements (u and v). From Figure 1, a fibre of the plate DF oriented in the z direction takes the positions D′′F′′ and D′F′ as a result of bending and shear deformations in the x-z plane. Let the rotation in the x-z plane of a line initially normal to the middle surface before deformation be ϕ_x The displacement of a point E having a distance z from the mid-plane in the line of action of the x axis is $+z\phi_x$. Likewise, the displacement of point E along the y axis is $+z\phi_y$. Where ϕ_y is the rotation in the y-z plane of a line that was previously normal to the middle plane before the deformation of the plate.

The kinematic equations of the present theory are as follows:

The in-plane displacements of any point (like E) from Figure 1 is given by Equations (1) and (2) respectively.

$$u = z\phi_x \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$v = z\phi_y \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equations (3) to (7) are the equations of the five engineering strain components

$$\varepsilon_x = \frac{\partial u}{\partial x} = z\frac{\partial \phi_x}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = z\frac{\partial \phi_y}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\gamma_{xy} = 2\varepsilon_{xy} = 2\varepsilon_{yx} = 2z\frac{\partial \phi_x}{\partial y} = 2\,z\frac{\partial \phi_y}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\gamma_{xz} = \varepsilon_{xz} + \varepsilon_{zx} = \phi_x + \frac{\partial w}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\gamma_{yz} = \varepsilon_{yz} + \varepsilon_{zy} = \phi_y + \frac{\partial w}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The equations of the vertical rotations (ϕ_x *and* ϕ_y) are expressed in Equations (8) and (9).

$$\phi_x = \gamma_{xz} - \frac{\partial w}{\partial x} = c_x\frac{\partial w}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\phi_y = \gamma_{yz} - \frac{\partial w}{\partial y} = c_y\frac{\partial w}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The constitutive relationships are formulated as follows:

Applying Hooke's law, the anisotropic material engineering strains are expressed in terms of stress, Young's modulus, Poisson's ratios, and stress and are as given in Equations (10) to (14).

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_{11}} - \mu_{21}\frac{\sigma_{22}}{E_{22}} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\varepsilon_{22} = -\mu_{12}\frac{\sigma_{11}}{E_{11}} + \frac{\sigma_{22}}{E_{22}} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\gamma_{12} = \frac{1}{G_{12}}.\tau_{12} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\gamma_{13} = \frac{1}{G_{13}}.\tau_{13} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\gamma_{23} = \frac{1}{G_{23}}.\tau_{23} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Note: E_{11} and E_{22} represents Young's moduli of elasticity of the anisotropic plate. μ_{12} and μ_{21} stands for Poisson's ratios of the anisotropic plate. G_{12}, G_{13} and G_{23} are the shear moduli of elasticity of the anisotropic plate. σ_{11} and σ_{22} are normal stresses.

By simultaneously solving Equations (10) and (11) and rearranging Equations (12), (13) and (14) respectively gave Equation (15a)

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{bmatrix} = E_{00}\begin{bmatrix} e_{11} & e_{12} & 0 & 0 & 0\\ e_{12} & e_{22} & 0 & 0 & 0\\ 0 & 0 & e_{33} & 0 & 0\\ 0 & 0 & 0 & e_{44} & 0\\ 0 & 0 & 0 & 0 & e_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

$Where$:

$$E_{00} = E_0/(1-\mu_{12}\mu_{21}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

$$e_{11} = \frac{E_{11}}{E_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15c)$$

$$e_{12} = \frac{\mu_{21}.E_{11}}{E_0} = \frac{\mu_{12}.E_{22}}{E_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15d)$$

$$e_{22} = \frac{E_{22}}{E_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15e)$$

$$e_{33} = \frac{G_{12}}{E_0}.(1-\mu_{12}\mu_{21}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15f)$$

$$e_{44} = \frac{G_{13}}{E_0}.(1-\mu_{12}\mu_{21}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15i)$$

$$e_{55} = \frac{G_{23}}{E_0}.(1-\mu_{12}\mu_{21}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15j)$$

Note: E_0 $can\ be$ E_{11} or E_{22}

By arranging Equations (3), (4), (5), (6) and (7) in matrix form gives Equation (16).

$$\varepsilon = \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} = \begin{bmatrix} z\frac{\partial\phi_x}{\partial x}\\ z\frac{\partial\phi_y}{\partial y}\\ z\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\\ \left(\phi_x+\frac{\partial w}{\partial x}\right)\\ \left(\phi_y+\frac{\partial w}{\partial y}\right)\end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Transforming Equation ($15a$) from local coordinate (1-2 coordinate) system to global coordinate (x-y coordinate) system using the transformation matrix [T] yields Equation (17).

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix} = E_{00}\left\{[T]^{-1}\begin{bmatrix}e_{11} & e_{12} & 0 & 0 & 0\\ e_{12} & e_{22} & 0 & 0 & 0\\ 0 & 0 & e_{33} & 0 & 0\\ 0 & 0 & 0 & e_{44} & 0\\ 0 & 0 & 0 & 0 & e_{55}\end{bmatrix}[T]^{-T}\right\}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \dots\dots\dots\dots\dots\dots\dots (17)$$

Where: the transformation matrix [T] is defined as:

$$[T] = \begin{bmatrix}m^2 & n^2 & 2mn & 0 & 0\\ n^2 & m^2 & -2mn & 0 & 0\\ -mn & mn & (m^2-n^2) & 0 & 0\\ 0 & 0 & 0 & m & n\\ 0 & 0 & 0 & -n & m\end{bmatrix} \dots\dots\dots\dots\dots\dots\dots (18)$$

Where: "m" and "n" are respectively Cos θ and Sin θ, and θ is the angle of orientation of the fibers.

When Equation (16) and (18) are substituted into Equation (17), Equation ($19a$) is obtained.

$$\sigma = \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix} = E_{00}\begin{bmatrix}a_{11} & a_{12} & a_{13} & 0 & 0\\ a_{12} & a_{22} & a_{23} & 0 & 0\\ a_{13} & a_{23} & a_{33} & 0 & 0\\ 0 & 0 & 0 & a_{44} & a_{45}\\ 0 & 0 & 0 & a_{45} & a_{55}\end{bmatrix}\begin{bmatrix} z\frac{\partial\phi_x}{\partial x}\\ z\frac{\partial\phi_y}{\partial y}\\ z\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\\ \left(\phi_x+\frac{\partial w}{\partial x}\right)\\ \left(\phi_y+\frac{\partial w}{\partial y}\right)\end{bmatrix} \dots\dots\dots\dots\dots\dots\dots (19a)$$

Where:

$$a_{11} = m^4e_{11} + 2m^2n^2(e_{12}+2e_{33}) + n^4e_{22} \dots\dots\dots\dots\dots\dots\dots (19b)$$

$$a_{12} = e_{12}(n^4+m^4) + m^2n^2(e_{11}+e_{22}-4e_{33}) \dots\dots\dots\dots\dots\dots\dots (19c)$$

$$a_{13} = m^3n(e_{11}-e_{12}-2e_{33}) + mn^3(e_{12}-e_{22}+2e_{33}) \dots\dots\dots\dots\dots\dots\dots (19d)$$

$$a_{22} = n^4e_{11} + 2m^2n^2(e_{12}+2e_{33}) + m^4e_{22} \dots\dots\dots\dots\dots\dots\dots (19e)$$

$$a_{23} = mn^3e_{11} - m^3ne_{22} + (m^3n - mn^3)(e_{12}+2e_{33}) \dots\dots\dots\dots\dots\dots\dots (19f)$$

$$a_{33} = m^2n^2(e_{11}-2e_{12}+e_{22}-2e_{33}) + e_{33}(m^4+n^4) \dots\dots\dots\dots\dots\dots\dots (19g)$$

$$a_{44} = m^2e_{44} - 2mn\,e_{45} + n^2e_{55} \dots\dots\dots\dots\dots\dots\dots (19h)$$

$$a_{45} = mn\,(e_{44}-e_{55}) + (m^2-n^2)e_{45} \dots\dots\dots\dots\dots\dots\dots (19i)$$

$$a_{55} = n^2e_{44} + 2mn\,e_{45} + m^2e_{55} \dots\dots\dots\dots\dots\dots\dots (19j)$$

The total potential energy functional is defined as Equation (20).

$$\Pi = U + V \dots\dots\dots\dots\dots\dots\dots (20)$$

Where: V is the work done on the thick plate and U is the internal energy of the thick rectangular anisotropic plate. strain energy of the plate is given as Equation (21)

$$\boldsymbol{U = \frac{1}{2}\iiint \varepsilon^T \sigma \; dx\,.\mathrm{dy}\,.dz} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (\mathbf{21})$$

By filling in Equations (16) and (19a) into Equation (21) and rearranging the resulting equation, Equation (22a) is obtained. Equation (22a) is the strain energy equation of anisotropic rectangular plate based on modified first order shear deformation theory.

$$U = \frac{D_{00}}{2}\iint \{\left(\frac{\partial \phi_x}{\partial x}\right)^2 . a_{11} + 2a_{xy}\frac{\partial \phi_x}{\partial x}.\frac{\partial \phi_y}{\partial y} + \left(\frac{\partial \phi_y}{\partial y}\right)^2 a_{22}$$

$$+2\left[\frac{\partial \phi_x}{\partial y}\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial x}\frac{\partial \phi_x}{\partial x}\right] a_{13} + 2\left[\frac{\partial \phi_x}{\partial y}\frac{\partial \phi_y}{\partial y} + \frac{\partial \phi_y}{\partial x}\frac{\partial \phi_y}{\partial y}\right] a_{23}$$

$$+\frac{12}{t^2}\left({\phi_x}^2 + 2\frac{\partial w}{\partial x}\phi_x + \left(\frac{\partial w}{\partial x}\right)^2\right) a_{44} + \frac{24}{t^2}\left(\frac{\partial w}{\partial x}.\phi_y + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} + \phi_y\phi_x + \frac{\partial w}{\partial y}\phi_x\right) a_{45}$$
$$+\frac{12}{t^2}\left(2\frac{\partial w}{\partial y}\phi_y + \left(\frac{\partial w}{\partial y}\right)^2 + {\phi_y}^2\right) a_{55}\} \, dxdy \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (22a)$$

Where:

$$D_{00} = \frac{E_{00}t^3}{12} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

$$a_{xy} = (a_{12} + 2a_{33}) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (22c)$$

The external work induced on a rectangular plate undergoing free vibration as given by Ibearugbulem et al. (2014) will be adopted in this study and is herein written as Equation (23).

$$V_\lambda = \frac{M.\lambda^2}{2}.\int_0^a\int_0^b w^2 \, dxdy \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Where 'M' is the inertia mass of the plate while 'λ' is the fundamental natural frequency of the plate.

This inertia work when written in non-dimensional coordinate terms yields Equation (24).

$$V_\lambda = \frac{M\,.\lambda^2 ab}{2}.\int_0^a\int_0^b w^2 \, dRdQ \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (24)$$

By filling in Equations $(22a)$ and (24) into Equation (20), and rearranging the resulting equation in non-dimensional coordinate form, Equation (25a) is obtained.

$$\Pi = \frac{\beta D_{00}}{2}\iint \{\left(\frac{\partial \phi_R}{\partial R}\right)^2 . a_{11} + 2\frac{a_{xy}}{\beta}\frac{\partial \phi_R}{\partial R}.\frac{\partial \phi_Q}{\partial Q} + \frac{1}{\beta^2}\left(\frac{\partial \phi_Q}{\partial Q}\right)^2 a_{22} + 2\left[\frac{1}{\beta}\frac{\partial \phi_R}{\partial Q}\frac{\partial \phi_R}{\partial R} + \frac{\partial \phi_Q}{\partial R}\frac{\partial \phi_R}{\partial R}\right] a_{13}$$
$$+2\left[\frac{1}{\beta^2}\frac{\partial \phi_R}{\partial Q}\frac{\partial \phi_Q}{\partial Q} + \frac{1}{\beta}\frac{\partial \phi_Q}{\partial R}\frac{\partial \phi_Q}{\partial Q}\right] a_{23} + \frac{12}{t^2}\left(a^2{\phi_R}^2 + 2a\frac{\partial w}{\partial R}\phi_R + \left(\frac{\partial w}{\partial R}\right)^2\right) a_{44}$$
$$+\frac{24}{t^2}\left(a\frac{\partial w}{\partial R}.\phi_Q + \frac{1}{\beta}\frac{\partial w}{\partial R}.\frac{\partial w}{\partial Q} + a^2\phi_Q\phi_R + \frac{a}{\beta}\frac{\partial w}{\partial Q}\phi_R\right) a_{45} + \frac{12}{t^2}\left(\frac{2a}{\beta}\frac{\partial w}{\partial Q}\phi_Q + \frac{1}{\beta^2}\left(\frac{\partial w}{\partial Q}\right)^2 + a^2{\phi_Q}^2\right) a_{55}$$
$$-\frac{M\,.\lambda^2 w^2 a^2}{D_{00}}\} \, dRdQ \; (25a)$$

Where:

$$\beta = \frac{b}{a};\; \partial x = a\partial R \; and \; \partial y = b\partial Q \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

The governing and two compatibility equations are obtained follows:

By differentiating Equation ($25a$) with respect to the deflection (w) and equating the resulting equation to zero, the governing equation is obtained, which is given in Simplified form as Equation (26).

$$\frac{12}{t^2}\left(a\frac{\partial\phi_R}{\partial R}+\frac{\partial^2 w}{\partial R^2}\right)a_{44}+\frac{12}{t^2}\left(a\frac{\partial\phi_Q}{\partial R}+\frac{a}{\beta}\frac{\partial\phi_R}{\partial Q}+\frac{2}{\beta}\frac{\partial^2 w}{\partial R\partial Q}\right)a_{45}+\frac{12}{t^2}\left(\frac{a}{\beta}\frac{\partial\phi_Q}{\partial Q}+\frac{1}{\beta^2}\frac{\partial^2 w}{\partial Q^2}\right)a_{55}-\frac{M\,.\lambda^2 w^2 a^2}{D_{00}}=0 \quad \ldots.(26)$$

Differentiating Equation ($25a$) with respect to ϕ_R and equating the resulting equation to zero, the compatibility equation in *x-z* plane is obtained as Equation (27).

$$\frac{\partial^2\phi_R}{\partial R^2}.a_{11}+\frac{a_{xy}}{\beta}\frac{\partial^2\phi_Q}{\partial R\partial Q}+\left[\frac{2}{\beta}\frac{\partial^2\phi_R}{\partial R\partial Q}+\frac{\partial^2\phi_Q}{\partial R^2}\right]a_{13}+\left[\frac{1}{\beta^2}\frac{\partial^2\phi_Q}{\partial Q^2}\right]a_{23}+\frac{12}{t^2}\left(a^2\phi_R+a\frac{\partial w}{\partial R}\right)a_{44}+\frac{12}{t^2}\left(a^2\phi_Q+\frac{a}{\beta}\frac{\partial w}{\partial Q}\right)a_{45}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\;(27)$$

Differentiating Equation ($25a$) with respect to ϕ_Q gives compatibility equation in *y-z* plane as Equation (28).

$$\frac{a_{xy}}{\beta}\frac{\partial^2\phi_R}{\partial R\partial Q}+\frac{1}{\beta^2}\frac{\partial^2\phi_Q}{\partial Q^2}a_{22}+\frac{\partial^2\phi_R}{\partial R^2}a_{13}+\left[\frac{1}{\beta^2}\frac{\partial^2\phi_R}{\partial Q^2}+\frac{2}{\beta}\frac{\partial^2\phi_Q}{\partial R\partial Q}\right]a_{23}+\frac{12}{t^2}\left(a^2\phi_R+a\frac{\partial w}{\partial R}\right)a_{45}+\frac{12}{t^2}\left(a^2\phi_Q+\frac{a}{\beta}\frac{\partial w}{\partial Q}\right)a_{55}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\;(28)$$

The general displacement equations are determined as follows:

By working out equations (26), (27) and (28) and reducing the resulting equation, Equation (29a) is obtained.

$$w\;=\;(a_{00}+a_{01}R+a_{02}R^2+a_{03}R^3+a_{04}R^4)(b_{00}+b_{01}Q+b_{02}Q^2+b_{03}Q^3+b_{04}Q^4)\ldots\ldots\ldots\ldots\ldots\ldots\;(29a)$$

Equation ($29a$) can be represented as Equation ($29b$)

$$w=\;A_1h\ldots\ldots\ldots\ldots\ldots\ldots\;(29b)$$

Where:

$$h=\;[1\quad R\quad R^2\quad R^4\quad R^4].[1\quad Q\quad Q^2\quad Q^4\quad Q^4]\;\ldots\ldots\ldots\ldots\ldots\ldots(29c)$$

$A_1$is the coefficient of deflection; h is the shape function

By substituting Equation ($29b$) into the non-dimensional form of Equations (8) and (9) respectively gives equations (30) and (31).

$$\phi_R=\frac{c_x}{a}.A_1.\frac{\partial h}{\partial R}\;=\;\frac{A_2}{a}.\frac{\partial h}{\partial R}\;\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

$$\phi_Q=\frac{A_3}{a\beta}.\frac{\partial h}{\partial Q}\;\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

Where: $A_2=A_1.\,c_x$; $A_3=A_1.\,c_y$ (A_2 *and* A_3 are the coefficients of vertical rotations ϕ_x *and* ϕ_y respectively)

In split deflection form, Equations ($29a$) can be written as Equation (32)

$$w\;=\;w_R\times w_Q\;\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

w_R and w_Q are represented respectively as Equations ($33a$) and ($33b$).

$$w_R\;=\;(a_{00}+a_{01}R+a_{02}R^2+a_{03}R^3+a_{04}R^4)\;\ldots\ldots\ldots\ldots\ldots\ldots\;(33a)$$

$$w_Q = (b_{00} + b_{01}Q + b_{02}Q^2 + b_{03}Q^3 + b_{04}Q^4) \dots\dots\dots\dots\dots\dots (33b)$$

Equations $(33a)$ and $(33b)$ are the deflection equations of a strip of the rectangular plate along the x *and* y axes respectively.

In generalized form, Equations $(33a)$ and $(33b)$ are given as Equation (34).

$$w_\propto = (\Delta_{00} + \Delta_{01}\propto + \Delta_{02}\propto^2 + \Delta_{03}\propto^3 + \Delta_{04}\propto^4) \dots\dots\dots\dots\dots\dots (34)$$

Equation (34) is the generalized split deflection equation and $\propto$ can be R or Q and Δ can be a or b as the case may be.

By substituting the boundary conditions of a particular strip into the generalized deflection equation of the plate, the deflection equations of the plate along simply supported (S-S) strip and clamped at one end and simply supported at the other end (C-S) strip are given in Equations (35) and (36) respectively.

$$\mathrm{S-S} = \Delta_{04}(\propto - 2\propto^3 + \propto^4) \dots\dots\dots\dots\dots\dots (35)$$

$$\mathrm{C-S} = \Delta_{04}(1.5\propto^2 - 2.5\propto^3 + \propto^4) \dots\dots\dots\dots\dots\dots (36)$$

By combining the peculiar deflections along various strips, the peculiar deflection equations for plate of various support conditions are obtained as given in Equations (37) and (38).

$$\mathrm{SSSS} = \boldsymbol{A_1(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)} \dots\dots\dots\dots\dots\dots (37)$$

$$\mathrm{CCSS} = \boldsymbol{A_1(1.5R^2 - 2.5R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)} \dots\dots\dots\dots\dots\dots (38)$$

The formula for calculating fundamental natural frequency is determined as follows:

By filling in Equations $(29a)$, (30) and (31) into Equation $(25a)$ and minimizing the resulting equation with respect A_1 (coefficient of deflection) and A_2 (coefficient of *x-z* shear rotation along *x*-direction) and A_3(coefficient of *y-z* shear rotation along *y*-direction) respectively, Equations (39), (40) and (41) are obtained.

$$\frac{d\Pi}{dA_1} = \frac{\beta D_{00}}{2}\{24a_{44}\left(\frac{a}{t}\right)^2(A_1 + A_2)k_{N_R} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(2A_1 + A_2 + A_3)k_{N_{RQ}} + \frac{24a_{55}}{\beta^2}\left(\frac{a}{t}\right)^2(A_1 + A_3)k_{N_Q} - A_1\frac{M\,.\lambda^2a^4}{D_{00}}k_\lambda\} = 0 \dots\dots\dots\dots\dots\dots (39)$$

$$\frac{d\Pi}{dA_2} = \frac{\beta D_{00}}{2}\{2A_2a_{11}k_R + 2A_3\frac{a_{xy}}{\beta^2}k_{RQ} + \frac{2a_{13}}{\beta}[2A_2 + A_3]k_{RRQ} + \frac{2a_{23}}{\beta^3}[A_3]k_{RQQ} + 12a_{44}\left(\frac{a}{t}\right)^2(2A_1 + 2A_2)k_{N_R} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(A_1 + A_3)k_{N_{RQ}}\} = 0 \dots\dots\dots\dots\dots\dots (40)$$

$$\frac{d\Pi}{dA_3} = \frac{\beta D_{00}}{2}\{2A_2\frac{a_{xy}}{\beta^2}k_{RQ} + 2A_3\frac{a_{22}}{\beta^4}k_Q + \frac{2a_{13}}{\beta}[A_2]k_{RRQ} + \frac{2a_{23}}{\beta^3}[A_2 + 2A_3]k_{RQQ} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(A_1 + A_2)k_{N_{RQ}} + \frac{12a_{55}}{\beta^2}\left(\frac{a}{t}\right)^2(2A_1 + 2A_3)k_{N_Q}\} = 0 \dots\dots\dots\dots\dots\dots (41)$$

Note:

$$k_R = \int_0^1\int_0^1\left(\frac{\partial^2h}{\partial R^2}\right)^2 dRdQ;\ k_{RQ} = \int_0^1\int_0^1\left(\frac{\partial^2h}{\partial R\partial Q}\right)^2 dRdQ;\ k_Q = \int_0^1\int_0^1\left(\frac{\partial^2h}{\partial Q^2}\right)^2 dRdQ;$$

$$k_{RRQ} = \int_0^1\int_0^1\frac{\partial^2h}{\partial R\partial Q}\frac{\partial^2h}{\partial R^2}dRdQ;\ k_{RQQ} = \int_0^1\int_0^1\frac{\partial^2h}{\partial R\partial Q}\frac{\partial^2h}{\partial Q^2}dRdQ;\ k_{N_R} = \int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2 dRdQ\ ;$$

$$k_{N_Q} = \int_0^1\int_0^1\left(\frac{\partial h}{\partial Q}\right)^2 dRdQ;\ k_{N_{RQ}} = \int_0^1\int_0^1\frac{\partial h}{\partial R}.\frac{\partial h}{\partial Q}dRdQ;\ k_\lambda = \int_0^1\int_0^1 \mathrm{h}^2\, dRdQ$$

By solving simultaneously, the simplified form of Equations (40) and (41) gave Equations (42) and (43).

$$A_2 = T_2 A_1 = A_1 \frac{(d_{12}.d_{23} - d_{13}.d_{22})}{\left(d_{12}{}^2 - d_{11} d_{22}\right)} \quad \dots\dots\dots\dots\dots\dots (42)$$

$$A_3 = T_3 A_1 = A_1 \frac{(d_{12}.d_{13} - d_{11} d_{23})}{\left(d_{12}{}^2 - d_{11} d_{22}\right)} \quad \dots\dots\dots\dots\dots\dots (43)$$

Substituting Equations (42) and (43) into Equation (39) and making square of fundamental natural frequency (λ^2) the subject gives Equation (44).

$$\lambda^2 = \frac{k_T}{k_\lambda} \cdot \frac{D_{00}}{Ma^4} \quad \dots\dots\dots\dots\dots\dots (44)$$

simplifying Equation (44) gives Equation (45).

$$\lambda = \frac{1}{a^2} \sqrt{\frac{k_T}{k_\lambda} \cdot \frac{D_{00}}{M}} \quad \dots\dots\dots\dots\dots\dots (45)$$

Equation (45) can be expressed in the form of fundamental natural frequency parameter as Equation (46)

$$\lambda a^2 \sqrt{\frac{M}{D_{00}}} = \sqrt{\frac{k_T}{k_\lambda}} \quad \dots\dots\dots\dots\dots\dots (46)$$

Where:

$$d_{11} = a_{11} k_R + 2 \frac{a_{13}}{\beta} k_{RRQ} + 12 a_{44} \left(\frac{a}{t}\right)^2 k_{N_R}; \; d_{12} = \frac{a_{xy}}{\beta^2} k_{RQ} + \frac{a_{13}}{\beta} k_{RRQ} + \frac{a_{23}}{\beta^3} k_{RQQ} + \frac{12 a_{45}}{\beta} \left(\frac{a}{t}\right)^2 k_{N_{RQ}};$$

$$d_{13} = -12 \left(\frac{a}{t}\right)^2 \left[a_{44} k_{N_R} + \frac{a_{45}}{\beta} k_{N_{RQ}}\right]; \; d_{22} = \frac{a_{22}}{\beta^4} k_Q + 2 \frac{a_{23}}{\beta^3} k_{RQQ} + \frac{12 a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 k_{N_Q};$$

$$d_{23} = -12 \left(\frac{a}{t}\right)^2 \left[\frac{a_{45}}{\beta} k_{N_{RQ}} + \frac{a_{55}}{\beta^2} k_{N_Q}\right];$$

$$k_T = 12 a_{44} \left(\frac{a}{t}\right)^2 (1 + T_2) k_{N_R} + \frac{12 a_{45}}{\beta} \left(\frac{a}{t}\right)^2 (2 + T_2 + T_3) k_{N_{RQ}} + \frac{12 a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 (1 + T_3) k_{N_Q}$$

3. Numerical problems

Using the above described theory, rectangular and square anisotropic thick plates that are simply supported (SSSS) and clamped on two adjacent edges and simply supported on the other two edges (CCSS) are analysed for fundamental natural frequency (λ), at various span to depth ratios ($^a/_t$ = 5, 10, 20, 25, 50 and 100), aspect ratios (β = 1 to 2 at increaments of 0.1) and fibre orientation angles (θ = 0^0, 15^0 , 45^0) with the following material properties: $\frac{E_2}{E_1} = 0.52500$, $\frac{G_{12}}{E_1} = 0.26293$, $\frac{G_{13}}{E_1} = 0.15991$, $\frac{G_{23}}{E_1} = 0.26681$, $\mu_{12} = 0.44049$ and $\mu_{21} = 0.23124$.

4. Results

The numerical results for the vibration analysis of SSSS and CCSS thick anisotropic rectangular plates with the material properties given in the section above and for span-depth ratios (a/t = 5, 10, 20, 25, 50, and 100), aspect ratios (β = b/a = 1 to 2), and grain fiber orientation angles (θ = 0^0, 15^0 , 45^0) obtained from Equation (46) are presented on Tables 1 to 6. The present solution is compared with solutions obtained from previous works by Srinivas and Roa (1970), Shimpi and Patel (2006), and Reddy (1984). The fundamental natural frequency parameter values ($\bar{\lambda}$) were calculated for SSSS plate having the same material properties as given in the section above using the following formula: $\bar{\lambda}$ =

$\lambda t\sqrt{(\frac{\rho(1-\mu_{12}\mu_{21})}{E_o}}$). If β = 0.5 for previous work, β for this study is $\frac{1}{\beta} = \frac{1}{0.5}$ = 2.0. If β = 1.0 for previous work, β for this study is $\frac{1}{\beta} = \frac{1}{1}$ = 1.0 and if β = 2.0 for previous work; for this study β is $\frac{1}{\beta} = \frac{1}{2}$ = 0.5. Also, If $\frac{t}{a}$ = 0.05 for previous work; for this present study $\frac{a}{t} = \frac{1}{0.05}$ = 20.

The present study solution was further validated by comparing its solutions with those obtained from Reddy (2004) for SSSS thick orthotropic rectangular plate with the following material properties: $E_1 = 25E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \mu_{12} = 0.25$. The fundamental natural frequency parameter values were calculated using the following formula: $\bar{\lambda} = \lambda\frac{a^2}{t}\sqrt{\frac{\rho}{E}}$. The compared results are presented in Tables 7 and 8.

Table 1 Fundamental natural frequency parameter values for SSSS plate at θ = $\mathbf{0^o}$

β	$\lambda a^2\sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	14.8434	16.3116	16.7524	16.8077	16.8824	16.9012
1.1	13.8287	15.1476	15.5410	15.5903	15.6568	15.6735
1.2	13.0520	14.2616	14.6206	14.6655	14.7260	14.7413
1.3	12.4447	13.5718	13.9049	13.9466	14.0027	14.0169
1.4	11.9612	13.0244	13.3377	13.3768	13.4296	13.4429
1.5	11.5701	12.5828	12.8805	12.9177	12.9677	12.9804
1.6	11.2493	12.2215	12.5067	12.5423	12.5902	12.6023
1.7	10.9831	11.9221	12.1971	12.2314	12.2776	12.2892
1.8	10.7597	11.6713	11.9379	11.9711	12.0159	12.0271
1.9	10.5704	11.4591	11.7187	11.7510	11.7945	11.8055
2	10.4087	11.2780	11.5316	11.5632	11.6057	11.6164

Table 2 Fundamental natural frequency parameter values for SSSS plate at θ = $\mathbf{15^o}$

β	$\lambda a^2 \sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	14.9711	16.4112	16.8412	16.8951	16.9677	16.9860
1.1	13.9253	15.2089	15.5893	15.6369	15.7011	15.7172
1.2	13.1229	14.2921	14.6367	14.6797	14.7377	14.7523
1.3	12.4941	13.5771	13.8949	13.9346	13.9880	14.0015
1.4	11.9925	13.0090	13.3062	13.3433	13.3932	13.4057
1.5	11.5861	12.5501	12.8311	12.8662	12.9133	12.9251
1.6	11.2523	12.1741	12.4422	12.4756	12.5206	12.5319
1.7	10.9749	11.8623	12.1199	12.1520	12.1951	12.2060
1.8	10.7418	11.6008	11.8497	11.8807	11.9224	11.9329
1.9	10.5442	11.3793	11.6211	11.6511	11.6916	11.7017
2	10.3752	11.1902	11.4258	11.4551	11.4945	11.5044

Table 3 Fundamental natural frequency parameter values for SSSS plate at θ = $\mathbf{45^o}$

β	$\lambda a^2 \sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	15.3497	16.7053	17.1042	17.1540	17.2211	17.2380
1.1	14.1438	15.2940	15.6282	15.6698	15.7258	15.7399
1.2	13.2092	14.2116	14.5001	14.5359	14.5840	14.5961
1.3	12.4706	13.3633	13.6181	13.6497	13.6921	13.7028
1.4	11.8769	12.6859	12.9153	12.9437	12.9819	12.9914
1.5	11.3928	12.1364	12.3462	12.3722	12.4070	12.4157
1.6	10.9929	11.6846	11.8789	11.9028	11.9350	11.9431
1.7	10.6589	11.3085	11.4903	11.5127	11.5428	11.5504
1.8	10.3770	10.9921	11.1638	11.1849	11.2133	11.2204
1.9	10.1370	10.7234	10.8867	10.9068	10.9337	10.9405
2	9.9310	10.4934	10.6495	10.6688	10.6945	10.7010

Table 4 Fundamental natural frequency parameter values for CCSS plate at θ = $\mathbf{0^o}$

β	$\lambda a^2\sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	19.0773	22.0123	23.0050	23.1340	23.3097	23.3543
1.1	17.8222	20.5396	21.4548	21.5735	21.7352	21.7762
1.2	16.8887	19.4541	20.3154	20.4271	20.5790	20.6175
1.3	16.1799	18.6349	19.4571	19.5636	19.7084	19.7452
1.4	15.6314	18.0036	18.7963	18.8989	19.0385	19.0739
1.5	15.1998	17.5078	18.2779	18.3775	18.5130	18.5473
1.6	14.8549	17.1121	17.8642	17.9614	18.0937	18.1272
1.7	14.5753	16.7916	17.5291	17.6245	17.7541	17.7870
1.8	14.3459	16.5285	17.2541	17.3479	17.4754	17.5077
1.9	14.1554	16.3100	17.0257	17.1182	17.2439	17.2758
2	13.9957	16.1266	16.8340	16.9253	17.0496	17.0810

Table 5 Fundamental natural frequency parameter values for CCSS plate at θ = $\mathbf{15^o}$

β	$\lambda a^2\sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	19.2429	22.1105	23.0671	23.1909	23.3592	23.4019
1.1	17.9540	20.5846	21.4573	21.5700	21.7233	21.7621
1.2	16.9920	19.4572	20.2718	20.3769	20.5197	20.5559
1.3	16.2592	18.6045	19.3771	19.4767	19.6120	19.6463
1.4	15.6907	17.9462	18.6874	18.7829	18.9125	18.9454
1.5	15.2421	17.4285	18.1456	18.2379	18.3632	18.3950
1.6	14.8829	17.0147	17.7128	17.8026	17.9246	17.9554
1.7	14.5912	16.6791	17.3619	17.4498	17.5690	17.5992
1.8	14.3514	16.4034	17.0738	17.1600	17.2770	17.3066
1.9	14.1521	16.1743	16.8343	16.9191	17.0343	17.0635
2	13.9847	15.9818	16.6332	16.7168	16.8305	16.8592

Table 6 Fundamental natural frequency parameter values for CCSS plate at θ = $\mathbf{45^o}$

β	$\lambda a^2\sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
1	19.7374	22.3982	23.2502	23.3591	23.5067	23.5440
1.1	18.2765	20.5618	21.2808	21.3722	21.4960	21.5273
1.2	17.1651	19.1896	19.8186	19.8983	20.0061	20.0333
1.3	16.3046	18.1423	18.7078	18.7793	18.8759	18.9003
1.4	15.6277	17.3275	17.8470	17.9125	18.0010	18.0234
1.5	15.0874	16.6830	17.1680	17.2291	17.3116	17.3324
1.6	14.6504	16.1654	16.6240	16.6817	16.7596	16.7792
1.7	14.2925	15.7441	16.1820	16.2371	16.3113	16.3301
1.8	13.9962	15.3969	15.8183	15.8713	15.9427	15.9607
1.9	13.7483	15.1077	15.5157	15.5670	15.6361	15.6535
2	13.5391	14.8643	15.2614	15.3112	15.3784	15.3953

Table 7 Fundamental natural frequency parameter results of present study compared with the results of previous research for SSSS thick orthotropic rectangular plate at θ = $\mathbf{0^o}$.

a/t	**Theory**	$\lambda t\sqrt{(\frac{\rho(1-\mu_{12}\mu_{21})}{E_o})}$
		β
		1
10	Present study (P)	0.0471
	Shimpi and Patel (2006), (SP)	0.0477
	Reddy (1984), (R)	0.0474
	Srinivas and Roa (1970), (SR)	0.0474
	% Difference between P and SP	1.27389
	% Difference between P and R	0.63693
	% Difference between P and SR	0.63693

Table 8 Fundamental natural frequency parameter results of present study compared with the results of Reddy (2004) for SSSS thick orthotropic rectangular plate at β = 1 and θ = $\mathbf{0^o}$

Theory	$\lambda a^2\sqrt{\frac{M}{D_{00}}}$					
	$\frac{a}{t}$					
	5	**10**	**20**	**25**	**50**	**100**
Present study (P)	9.485	12.8632	14.5194	14.7659	15.1156	15.207
Reddy (2004), (R)	8.909	12.452	14.355	14.651	15.077	15.190
% Difference between P and R	6.07275	3.197	1.1323	0.7781	0.2554	0.1118

5. Discussion

The fundamental natural frequency parameter numerical results for CCSS and SSSS anisotropic rectangular plates are presented on Tables 1 to 6. From Tables 1 to 6, it is observed that the natural frequency parameter ($\lambda a^2.\sqrt{\frac{M}{D_{00}}}$) values increased as span to depth ratio ($^a/_t$) increases for 0^o, 15^o and 45^o angles of fibre orientation (θ) at any given values of aspect ratio (1 to 2). Natural frequency parameter values also decreased as aspect ratio increases at any given value of span to depth ratio (5, 10, 20, 25 and 100) and angle of fibre orientation (0^o, 15^o, and 45^o). For SSSS plate, highest value of $\lambda a^2.\sqrt{\frac{M}{D_{00}}}$ = 17.238 occurred at aspect ratio of 1.0 for θ = 45^o and $^a/_t$ = 100 while the lowest value of $\lambda a^2.\sqrt{\frac{M}{D_{00}}}$ = 9.931 occurred at aspect ratio of 2.0 for θ = 45^o and $^a/_t$ = 5. For CCSS plate, highest value of $\lambda a^2.\sqrt{\frac{M}{D_{00}}}$ = 23.5440 occurred at aspect ratio of 1.0 for θ = 45^o and $^a/_t$ = 100 while the lowest value of $\lambda a^2.\sqrt{\frac{M}{D_{00}}}$ = 13.5391 occurred at aspect ratio of 2.0 for θ = 45^o and $^a/_t$ = 5. This shows that the natural frequency parameter values have a more significant effect on SSSS and CCSS thin anisotropic rectangular plates than it has on SSSS and CCSS thick anisotropic plates.

5.1 Comparison of present study solution with those from previous researchers

From Table 7, vibration analysis result of thick orthotropic rectangular plate having all sides simply supported obtained using modified first order shear deformation theory is presented and compared with refined plate theory (RPT) result of Shimpi and Patel (2006), higher order shear deformation theory (HSDT) result of Reddy (1984) and three-dimensional elasticity theory (TDET) result of Srinivas and Roa (1970). It is seen that the present study yields an excellent value for the fundamental frequency; this is shown by the result of the percentage difference between the present solution and those of previous research works. Thus, the present theory gives good solutions for free-vibration analysis of SSSS thick orthotropic plates. From Table 8, the vibration analysis result of thick orthotropic rectangular plate having all sides simply supported obtained using modified first order shear deformation theory is presented and compared with the first order shear deformation theory (FSDT) results of Reddy (2004) for span to depth ratios of 5,10,20,25, 50 and 100 at an aspect ratio of 1. It is observed from Table 8, that the solution of present theory gets closer to that of Reddy (2004) as the plate gets thinner (span to depth ratio increases) as can be seen from the results of the percentage difference for the various span to depth ratios. Table 8, shows that the present study solution is the correct solution since modified first order shear deformation theory does not require shear correction factor which first order shear deformation theory uses. From the table, it is seen that the percentage difference reduces in value as the plate becomes thin, this shows that Reddy (2004) does not give a better solution when used to analyse SSSS thick orthotropic plate. Thus, the present theory gives better and accurate solutions for free-vibration analysis of SSSS thick orthotropic plate

6. Conclusion

In this study, a modified first order shear deformation theory is applied to free vibration analysis of anisotropic plate (SSSS and CCSS). The results obtained from this study were compared with those from previous authors. Observations show that the results of the fundamental natural frequency parameter predicted by the present theory are in close agreement with those of previous researchers. The present method is capable of calculating reasonably correct values for free vibration problems of thick and thin anisotropic rectangular plates and can be adopted by future scholars to solve thick and thin anisotropic rectangular plate problems.

Compliance with ethical standard

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Disclosure of conflict of interest

No conflict of interest to be disclosed.

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