Application of modified first order shear deformation theory to vibration analysis of SSSS and CCSS thick anisotropic rectangular plates

K. O. NJOKU *, O. M. IBEARUGBULEM, U. C. ANYA and A. U. IGBOJIAKU

Department of civil engineering federal university of technology, Owerri, Imo state, Nigeria.

World Journal of Advanced Research and Reviews, 2023, 20(01), 1268–1283

Publication history: Received on 17 September 2023; revised on 25 October 2023; accepted on 28 October 2023

Article DOI: https://doi.org/10.30574/wjarr.2023.20.1.2192

Abstract

This research presents a vibration analysis of a thick anisotropic rectangular plate using modified first shear deformation theory. Modified first shear deformation theory, which is not built upon the classical plate theory, was used to develop the kinematic equations and constitutive relations of a deformed section of thick anisotropic rectangular plate from which the generalized stress equations were determined. Using the assumptions of the theory, the strain energy and external work equations were formulated, and by employing the principles of total minimum potential energy, the total potential energy functional of a thick anisotropic plate was developed. Minimization of the total potential energy functional with respect to the deflection function (w) and with respect to the shear rotations ($\phi_x$) and ($\phi_y$), respectively, resulted in the governing differential equation and the two compatibility equations of the plate. The displacement functions that satisfy the governing and compatibility equations were obtained by solving the governing and compatibility equations. From the general displacement function, the peculiar deflection equations (shape functions) were obtained for the boundary conditions considered, which are simply supported on its four edges (SSSS), clamped on two adjacent edges, and simply supported on the other two (CCSS). Using the displacement equation and the equation for rotation in the x-direction ($\phi_x$) and equation for rotation in y-direction ($\phi_y$), the direct governing and two direct compatibility equations were obtained, from which coefficients that enable the formula for calculating fundamental natural frequencies to be obtained. For the boundary condition analyzed in this work, the stiffness coefficients ($k_{RR}$, $k_{Q}$, $k_{RQ}$, $k_{QQ}$, $k_{RRQ}$, $k_{NQ}$, $k_{NRQ}$ and $k_\lambda$) were computed and used in determining the fundamental natural frequency parameter values at various span to depth ratios (5, 10, 20, 25 and 100), aspect ratios (1 to 2 at the increment of 0.1) and angle of fibre orientation ($0^\circ$, $15^\circ$, $45^\circ$). The solutions of this study were compared with those from previous researchers. The fundamental natural frequency parameter values obtained in this study were compared with the work of Reddy (1984) for $0^\circ$ angle of fibre orientation at span to depth ratios of 5, 10, 20, 25 and 100 at an aspect ratio of 1. The percentage difference values were 6.073%, 3.197%, 1.132%, 0.788%, 0.255%, and 0.112%, respectively. These differences revealed the closeness of the results of this present study to the results of Reddy (1984). This shows that the present theory provides good and acceptable solutions to the vibration problems of thick anisotropic rectangular plates.

Keywords: Thick Plate; Anisotropic; Fundamental natural frequency; Governing Equations

1. Introduction

For vibration analysis of thick plates, shear deformation effects are very important; these effects are what the classical plate theory (CPT) did not take into account. The classical plate theory ignored the effect of through-thickness shear deformation and thus overrated the stiffness of the plate, which is of very considerable significance for thick plates. In order to define the correct behavior of thick plates, including shear deformation effects and the associated cross-
sectional warping are required and many theories have evolved to address this dearth. Refined traditional theories of plate analysis were advanced to overwhelm the shortcomings of classical plate theories (otherwise called Kirchhoff's plate theory). The first refined traditional theories are Mindlin theory and Reissner theory (Sadrnejad et al., 2009). These theories are called first-order shear deformation theories. They are called first-order theories because the straight line, which was normal to the middle surface before bending, remained straight but was no longer normal to the middle surface after bending. Hence, the profile equation of that line as used in the first-order theories is a coordinate. A major constraint of these first-order theories is the assumption of constant shear stress across the thickness of the plate. Correction factors are usually employed when these theories are used. To overcome the limitations of first-order theories, second, third and higher-order theories evolved (Sayyad and Ghugal, 2012; Sayyad, 2011). Popular among these theories is third-order theories (third-order shear deformation, exponential shear deformation, hyperbolic shear deformation, trigonometric shear deformation, etc.). Since the development of this theory, many researchers have used it in their respective studies on thick plates. These include the works of Qian et al. (2003), Batra and Vidoli (2002), Abdul-Razak and Haido (2002), Kank and Swaminathan (2001), and Kocak and Hassis (2002). These higher-order theories have a shear deformation profile line (usually called f(z)) that is not linear (not equal to the z coordinate). The underlying reason for the nonlinear shear deformation line is that shear stress was assumed to relate to the first derivative of the profile line equation. That is, shear stress is the product of the first derivative of F(z) and nominal stress (CPT shear stress and the first derivative of the profile line equation). The implication of this assumption is that the shear deformation profile line equation, F(z) is related to the shear stress profile equation, G(z). That is, G(z) is the first derivative of F(z) (Shimpi and Patel, 2006; Chikalthankar et al., 2013; Ibearugbulem et al., 2016a). A better assumption ought to have been that, whereas the vertical shear is a product of nominal shear stress and G(z), no relationship exists between G(z) and F(z), meaning that G(z) cannot be obtained by the first derivative of F(z). Another, better assumption ought to be that the rotation of the middle surface is not divided into a classical part and a shear deformation part. The same assumption goes for in-plane displacements, u and v. They are whole each and are not divided into classical and shear deformation parts (Ibearugbulem et al., 2016b; Ibearugbulem, 2016b). Ibearugbulem et al. (2016b) assumed that F(z) is equal to z (that the profile is a straight line after deformation) and that there is no relationship between F(z) and G(z). With these assumptions, Kirchhoff’s assumption of zero vertical shear strain or stress on the classical theory was avoided completely. In their opinion, the vertical shear strains and stresses can be very small to be ignored but can never be equal to zero (no matter how small). This erroneous assumption that, for classical theory, the vertical shear strain and stress are absolutely equal to zero was introduced in the third and higher-order theories. Ordinarily, in-plane displacements and stresses are not functions of span-to-depth ratios. It is only the out-of-plane displacement and stresses that are functions of the span-to-depth ratio. But this is not the case from the results obtained using third- and higher-order theories (Pagano, 1970; Pagano and Hatfield, 1972; Reddy, 1984; Idibi et al., 1997; Aydogdu, 2009; Daouadji et al., 2012). Ibearugbulem (2016b), in their work titled "Full shear deformation for analysis of thick plates," analyzed only isotropic plates. Another shortcoming of their work is the assumption of the displacement function. The present work tried to extend their work (Ibearugbulem, 2016b) to the case of vibration analysis of thick anisotropic plates. Theoretical formulation of natural frequency.

2. Theoretical formulation of natural frequency

![Figure 1 Deformation of a section of a thick anisotropic plate](image-url)
The displacement field consists of one out-of-plane displacement \( w \) and two in-plane displacements \( u \) and \( v \). From Figure 1, a fibre of the plate DF oriented in the \( z \) direction takes the positions \( D''F'' \) and \( D'F' \) as a result of bending and shear deformations in the \( x-z \) plane. Let the rotation in the \( x-z \) plane of a line initially normal to the middle surface before deformation be \( \phi_x \). The displacement of a point E having a distance \( z \) from the mid-plane in the line of action of the \( x \) axis is \( +z\phi_x \). Likewise, the displacement of point E along the \( y \) axis is \( +z\phi_y \). Where \( \phi_x \) is the rotation in the \( y-z \) plane of a line that was previously normal to the middle plane before the deformation of the plate.

The kinematic equations of the present theory are as follows:

The in-plane displacements of any point (like E) from Figure 1 is given by Equations (1) and (2) respectively.

\[
\begin{align*}
    u &= z\phi_x \\
    v &= z\phi_y
\end{align*}
\]

Equations (3) to (7) are the equations of the five engineering strain components

\[
\begin{align*}
    \varepsilon_x &= \frac{\partial u}{\partial x} = z \frac{\partial \phi_x}{\partial x} \\
    \varepsilon_y &= \frac{\partial v}{\partial y} = z \frac{\partial \phi_y}{\partial y} \\
    \gamma_{xy} &= 2\varepsilon_{xy} = 2\varepsilon_x = 2z \frac{\partial \phi_x}{\partial y} = 2z \frac{\partial \phi_y}{\partial x} \\
    \gamma_{xz} &= \varepsilon_{xz} + \varepsilon_{zx} = \phi_x + \frac{\partial w}{\partial x} \\
    \gamma_{yz} &= \varepsilon_{yz} + \varepsilon_{zy} = \phi_y + \frac{\partial w}{\partial y}
\end{align*}
\]

The equations of the vertical rotations \( \phi_x \) and \( \phi_y \) are expressed in Equations (8) and (9).

\[
\begin{align*}
    \phi_x &= \gamma_{xz} - \frac{\partial w}{\partial x} = c_x \frac{\partial w}{\partial x} \\
    \phi_y &= \gamma_{yz} - \frac{\partial w}{\partial y} = c_y \frac{\partial w}{\partial y}
\end{align*}
\]

The constitutive relationships are formulated as follows:

Applying Hooke's law, the anisotropic material engineering strains are expressed in terms of stress, Young's modulus, Poisson's ratios, and stress and are as given in Equations (10) to (14).

\[
\begin{align*}
    \varepsilon_{11} &= \frac{\sigma_{11}}{E_{11}} - \mu_{21} \frac{\sigma_{22}}{E_{22}} \\
    \varepsilon_{22} &= -\mu_{12} \frac{\sigma_{11}}{E_{11}} + \frac{\sigma_{22}}{E_{22}} \\
    \gamma_{12} &= \frac{1}{G_{12}} \tau_{12} \\
    \gamma_{13} &= \frac{1}{G_{13}} \tau_{13}
\end{align*}
\]
\[ \gamma_{23} = \frac{1}{G_{23}} \tau_{23} \quad \ldots \ldots \ldots \quad (14) \]

Note: \( E_{11} \) and \( E_{22} \) represents Young's moduli of elasticity of the anisotropic plate. \( \mu_{12} \) and \( \mu_{21} \) stands for Poisson's ratios of the anisotropic plate. \( G_{12}, G_{13} \) and \( G_{23} \) are the shear moduli of elasticity of the anisotropic plate. \( \sigma_{11} \) and \( \sigma_{22} \) are normal stresses.

By simultaneously solving Equations (10) and (11) and rearranging Equations (12), (13) and (14) respectively gave Equation (15a)

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\tau_{12} \\
\tau_{13} \\
\tau_{23}
\end{bmatrix} =
\begin{bmatrix}
e_{11} & e_{12} & 0 & 0 & 0 \\
e_{12} & e_{22} & 0 & 0 & 0 \\
0 & e_{33} & 0 & 0 & 0 \\
0 & 0 & e_{44} & 0 & 0 \\
0 & 0 & 0 & e_{55} & 0
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\gamma_{12} \\
\gamma_{13} \\
\gamma_{23}
\end{bmatrix}\quad \ldots \ldots \ldots \quad (15a)
\]

Where:

\[ E_{00} = \frac{E_{0}}{1 - \mu_{12}\mu_{21}} \quad \ldots \ldots \ldots \quad (15b) \]

\[ e_{11} = \frac{E_{11}}{E_{0}} \quad \ldots \ldots \ldots \quad (15c) \]

\[ e_{12} = \frac{\mu_{21}E_{11}}{E_{0}} = \frac{\mu_{12}E_{22}}{E_{0}} \quad \ldots \ldots \ldots \quad (15d) \]

\[ e_{22} = \frac{E_{22}}{E_{0}} \quad \ldots \ldots \ldots \quad (15e) \]

\[ e_{33} = \frac{G_{12}}{E_{0}} \cdot (1 - \mu_{12}\mu_{21}) \quad \ldots \ldots \ldots \quad (15f) \]

\[ e_{44} = \frac{G_{13}}{E_{0}} \cdot (1 - \mu_{12}\mu_{21}) \quad \ldots \ldots \ldots \quad (15i) \]

\[ e_{55} = \frac{G_{23}}{E_{0}} \cdot (1 - \mu_{12}\mu_{21}) \quad \ldots \ldots \ldots \quad (15j) \]

Note: \( E_{0} \) can be \( E_{11} \) or \( E_{22} \)

By arranging Equations (3), (4), (5), (6) and (7) in matrix form gives Equation (16).

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy} \\
\gamma_{yx} \\
\gamma_{yz} \\
\gamma_{zy}
\end{bmatrix} = 2 \begin{bmatrix}
\frac{\partial \phi_{x}}{\partial y} & \frac{\partial \phi_{y}}{\partial y} \\
\frac{\partial \phi_{y}}{\partial x} & \frac{\partial \phi_{x}}{\partial x} \\
\frac{\partial \phi_{x}}{\partial y} + \frac{\partial \phi_{y}}{\partial x} & \frac{\partial \phi_{y}}{\partial y} + \frac{\partial \phi_{y}}{\partial x}
\end{bmatrix} \quad \ldots \ldots \ldots \quad (16)
\]

Transforming Equation (15a) from local coordinate (1-2 coordinate) system to global coordinate (\( x-y \) coordinate) system using the transformation matrix \([T]\) yields Equation (17).
\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} = E_0 \begin{bmatrix}
e_{11} & e_{12} & 0 & 0 & 0 \\
e_{12} & e_{22} & 0 & 0 & 0 \\
0 & 0 & e_{33} & 0 & 0 \\
0 & 0 & 0 & e_{44} & 0 \\
0 & 0 & 0 & 0 & e_{55}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix} \quad .................................. (17)
\]

Where: the transformation matrix \([T]\) is defined as:
\[
[T] = \begin{bmatrix}
m^2 & n^2 & 2mn & 0 & 0 \\
\frac{m}{n} & \frac{n}{m} & -2mn & 0 & 0 \\
\frac{a}{m} & \frac{a}{n} & \frac{a}{m} & 0 & 0 \\
0 & 0 & \frac{a}{m} & \frac{a}{n} & \frac{a}{m} \\
0 & 0 & \frac{a}{m} & \frac{a}{n} & \frac{a}{m}
\end{bmatrix} \quad .................................. (18)
\]

Where: “m” and “n” are respectively \(\cos \theta\) and \(\sin \theta\), and \(\theta\) is the angle of orientation of the fibers.

When Equation (16) and (18) are substituted into Equation (17), Equation (19a) is obtained.

\[
\sigma = E_0 \begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 \\
a_{12} & a_{22} & a_{23} & 0 & 0 \\
a_{13} & a_{23} & a_{33} & 0 & 0 \\
0 & 0 & a_{44} & a_{45} & 0 \\
0 & 0 & a_{45} & a_{55} & 0
\end{bmatrix} \begin{bmatrix}
z \frac{\partial \Phi_x}{\partial x} \\
z \frac{\partial \Phi_y}{\partial y} \\
z \frac{\partial \Phi_z}{\partial z} \\
\frac{\partial \Phi_x}{\partial y} + \frac{\partial \Phi_y}{\partial x} \\
\frac{\partial \Phi_x}{\partial x} + \frac{\partial \Phi_y}{\partial y}
\end{bmatrix} \quad .................................. (19a)
\]

Where:
\[
a_{11} = m^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + n^4 e_{22} \quad .................................. (19b)
\]
\[
a_{12} = e_{12} (n^4 + m^4) + m^2 n^2 (e_{11} + e_{22} - 4e_{33}) \quad .................................. (19c)
\]
\[
a_{13} = m^3 n (e_{11} - e_{12} - 2e_{33}) + mn^3 (e_{12} - e_{22} + 2e_{33}) \quad .................................. (19d)
\]
\[
a_{22} = n^4 e_{11} + m^2 n^2 (e_{12} + 2e_{33}) + m^4 e_{22} \quad .................................. (19e)
\]
\[
a_{23} = mn^4 e_{11} - m^4 n e_{22} + (m^3 n - mn^3) (e_{12} + 2e_{33}) \quad .................................. (19f)
\]
\[
a_{33} = m^2 n^2 (e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33} (m^4 + n^4) \quad .................................. (19g)
\]
\[
a_{44} = m^2 e_{44} - 2mn e_{45} + n^2 e_{55} \quad .................................. (19h)
\]
\[
a_{45} = mn (e_{44} - e_{55}) + (m^2 - n^2) e_{45} \quad .................................. (19i)
\]
\[
a_{55} = n^2 e_{44} + 2mn e_{45} + m^2 e_{55} \quad .................................. (19j)
\]

The total potential energy functional is defined as Equation (20).
\[
\Pi = U + V \quad .................................. (20)
\]

Where: \(V\) is the work done on the thick plate and \(U\) is the internal energy of the thick rectangular anisotropic plate. 
strain energy of the plate is given as Equation (21).
World Journal of Advanced Research and Reviews, 2023, 20(01), 1268–1283

\[ U = \frac{1}{2} \iint \epsilon^T \sigma \, dx \, dy \, dz \]  \hspace{1cm} (21)

By filling in Equations (16) and (19a) into Equation (21) and rearranging the resulting equation, Equation (22a) is obtained. Equation (22a) is the strain energy equation of anisotropic rectangular plate based on modified first order shear deformation theory.

\[
U = \frac{D_{00}}{2} \iint \left( \frac{\partial \phi_x}{\partial x} \right)^2 a_{11} + 2a_{xy} \frac{\partial \phi_x}{\partial x} \frac{\partial \phi_y}{\partial y} + \left( \frac{\partial \phi_y}{\partial y} \right)^2 a_{22} \\
+ 2 \left[ \frac{\partial \phi_x}{\partial y} \frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial x} \frac{\partial \phi_y}{\partial y} \right] a_{13} + 2 \left[ \frac{\partial \phi_x}{\partial y} \frac{\partial \phi_y}{\partial x} + \frac{\partial \phi_y}{\partial y} \frac{\partial \phi_x}{\partial x} \right] a_{23} \\
+ \frac{12}{t^2} \left( \phi_x^2 + 2 \frac{\partial w}{\partial x} \phi_x + \left( \frac{\partial w}{\partial x} \right)^2 \right) a_{44} + \frac{24}{t^2} \left( \frac{\partial w}{\partial x} \frac{\partial \phi_x}{\partial y} + \frac{\partial w}{\partial y} \frac{\partial \phi_y}{\partial y} + \phi_y \phi_x + \frac{\partial w}{\partial y} \phi_y \right) a_{45} \\
+ \frac{12}{t^2} \left( \frac{\partial w}{\partial y} \phi_y + \left( \frac{\partial w}{\partial y} \right)^2 + \phi_y^2 \right) a_{55} \] \hspace{1cm} dx \, dy \hspace{1cm} (22a)

Where:

\[
D_{00} = \frac{E_0 t^3}{12} \hspace{1cm} (22b) \]

\[
a_{xy} = (a_{12} + 2a_{33}) \hspace{1cm} (22c) \]

The external work induced on a rectangular plate undergoing free vibration as given by Ibeabugsen et al. (2014) will be adopted in this study and is herein written as Equation (23).

\[
V_\lambda = \frac{M \cdot \lambda^2}{2} \int_0^a \int_0^b \int_0^1 w^2 \, dx \, dy \hspace{1cm} (23) \]

Where ‘M’ is the inertia mass of the plate while ‘\lambda’ is the fundamental natural frequency of the plate.

This inertia work when written in non-dimensional coordinate terms yields Equation (24).

\[
V_\lambda = \frac{M \cdot \lambda^2 ab}{2} \int_0^a \int_0^b \int_0^1 w^2 \, dR \, dQ \hspace{1cm} (24) \]

By filling in Equations (22a) and (24) into Equation (20), and rearranging the resulting equation in non-dimensional coordinate form, Equation (25a) is obtained.

\[
\Pi = \frac{BD_{00}}{2} \iint \left( \frac{\partial \phi_x}{\partial R} \right)^2 a_{11} + 2a_{xy} \frac{\partial \phi_x}{\partial R} \frac{\partial \phi_y}{\partial Q} + \frac{1}{\beta^2} \left( \frac{\partial \phi_y}{\partial Q} \right)^2 a_{22} + 2 \left[ \frac{1}{\beta} \frac{\partial \phi_x}{\partial R} \frac{\partial \phi_x}{\partial Q} + \frac{\partial \phi_y}{\partial R} \frac{\partial \phi_y}{\partial Q} \right] a_{13} \\
+ 2 \left[ \frac{1}{\beta^2} \frac{\partial \phi_x}{\partial R} \frac{\partial \phi_x}{\partial Q} + \frac{1}{\beta} \frac{\partial \phi_x}{\partial R} \frac{\partial \phi_y}{\partial Q} \right] a_{23} + \frac{12}{t^2} \left( a^2 \phi_x^2 + 2a \frac{\partial w}{\partial R} \phi_x + \left( \frac{\partial w}{\partial R} \right)^2 \right) a_{44} \\
+ \frac{24}{t^2} \left( a \frac{\partial w}{\partial R} \phi_x + \left( \frac{\partial w}{\partial R} \right)^2 + a^2 \phi_x \phi_x \right) a_{45} + \frac{12}{t^2} \left( 2a \frac{\partial w}{\partial Q} \phi_y + \frac{1}{\beta} \left( \frac{\partial w}{\partial Q} \right)^2 + a^2 \phi_y^2 \right) a_{55} \\
- \frac{M \cdot \lambda^2 w^2 a^2}{D_{00}} \] \hspace{1cm} dR \, dQ \hspace{1cm} (25a)

Where:

\[
\beta = \frac{b}{a}; \ \partial x = a \partial R \ and \ \partial y = b \partial Q \hspace{1cm} (25b) \]
The governing and two compatibility equations are obtained as follows:

By differentiating Equation (25a) with respect to the deflection \( w \) and equating the resulting equation to zero, the governing equation is obtained, which is given in Simplified form as Equation (26).

\[
\frac{12}{t^2} \left( a \frac{\partial \phi_R}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) a_{44} + \frac{12}{t^2} \left( a \frac{\partial \phi_Q}{\partial R} + a \frac{\partial \phi_R}{\partial \beta} + \frac{2}{\beta} \frac{\partial^2 w}{\partial R \partial \beta} \right) a_{45} + \frac{12}{t^2} \left( \frac{a}{\beta} \frac{\partial \phi_Q}{\partial \beta} + \frac{1}{\beta^2} \frac{\partial^2 w}{\partial \beta^2} \right) a_{55} - \frac{M \lambda \lambda^2 w a^2}{D_0} = 0 \quad \ldots \quad (26)
\]

Differentiating Equation (25a) with respect to \( \phi_R \) and equating the resulting equation to zero, the compatibility equation in \( x-z \) plane is obtained as Equation (27).

\[
\frac{\partial^2 \phi_R}{\partial R^2} a_{11} + \frac{a_{xx}}{\beta} \frac{\partial^2 \phi_Q}{\partial R \partial \beta} a_{12} + \left[ \frac{1}{\beta^2} \frac{\partial^2 \phi_Q}{\partial Q^2} \right] a_{13} + \frac{2}{\beta} \frac{\partial^2 \phi_Q}{\partial R \partial \beta} a_{13} + \frac{1}{\beta^2} \frac{\partial^2 \phi_Q}{\partial Q^2} a_{22} + \frac{\partial \phi_R}{\partial \beta} a_{23}, a_{44} + \frac{12}{t^2} \left( \frac{\partial^2 \phi_Q}{\partial \beta} \right) a_{45} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (27)
\]

Differentiating Equation (25a) with respect to \( \phi_Q \) gives compatibility equation in \( y-z \) plane as Equation (28).

\[
\frac{\partial^2 \phi_Q}{\partial R^2} a_{12} + \frac{\partial^2 \phi_R}{\partial R \partial \beta} a_{22} + \left[ \frac{1}{\beta^2} \frac{\partial^2 \phi_Q}{\partial Q^2} \right] a_{23} + \frac{2}{\beta} \frac{\partial^2 \phi_Q}{\partial R \partial \beta} a_{23} + \frac{\partial \phi_R}{\partial \beta} a_{22} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (28)
\]

The general displacement equations are determined as follows:

By working out equations (26), (27) and (28) and reducing the resulting equation, Equation (29a) is obtained.

\[
w = (a_{00} + a_{01} R + a_{02} R^2 + a_{03} R^3 + a_{04} R^4) (b_{00} + b_{01} Q + b_{02} Q^2 + b_{03} Q^3 + b_{04} Q^4) \quad \ldots \quad \ldots \quad \ldots \quad (29a)
\]

Equation (29a) can be represented as Equation (29b)

\[
w = A_1 h \quad \ldots \quad \ldots \quad \ldots \quad (29b)
\]

Where:

\[
h = [1 \quad R \quad R^2 \quad R^4 \quad 1 \quad Q \quad Q^2 \quad Q^4] \quad \ldots \quad \ldots \quad \ldots \quad (29c)
\]

\( A_1 \) is the coefficient of deflection; \( h \) is the shape function

By substituting Equation (29b) into the non-dimensional form of Equations (8) and (9) respectively gives equations (30) and (31).

\[
\phi_R = \frac{c_x}{a} A_1 \frac{\partial h}{\partial R} = \frac{A_2}{a} \frac{\partial h}{\partial R} \quad \ldots \quad \ldots \quad \ldots \quad (30)
\]

\[
\phi_Q = \frac{A_3}{a \beta} \frac{\partial h}{\partial Q} \quad \ldots \quad \ldots \quad \ldots \quad (31)
\]

Where: \( A_2 = A_1 \cdot c_x ; A_3 = A_1 \cdot c_y \) (\( A_2 \) and \( A_3 \) are the coefficients of vertical rotations \( \phi_x \) and \( \phi_y \) respectively)

In split deflection form, Equations (29a) can be written as Equation (32)

\[
w = w_R \times w_Q \quad \ldots \quad \ldots \quad \ldots \quad (32)
\]

\( w_R \) and \( w_Q \) are represented respectively as Equations (33a) and (33b).

\[
w_R = (a_{00} + a_{01} R + a_{02} R^2 + a_{03} R^3 + a_{04} R^4) \quad \ldots \quad \ldots \quad \ldots \quad (33a)
\]
\[ w_Q = (b_{00} + b_{01}Q + b_{02}Q^2 + b_{03}Q^3 + b_{04}Q^4) \]  \hspace{1cm} (33b) \]

Equations (33a) and (33b) are the deflection equations of a strip of the rectangular plate along the \( x \) and \( y \) axes respectively.

In generalized form, Equations (33a) and (33b) are given as Equation (34).

\[ w_\alpha = (\Delta_{00} + \Delta_{01}\alpha + \Delta_{02}\alpha^2 + \Delta_{03}\alpha^3 + \Delta_{04}\alpha^4) \]  \hspace{1cm} (34) \]

Equation (34) is the generalized split deflection equation and \( \alpha \) can be \( R \) or \( Q \) and \( \Delta \) can be \( a \) or \( b \) as the case may be.

By substituting the boundary conditions of a particular strip into the generalized deflection equation of the plate, the deflection equations of the plate along simply supported (S-S) strip and clamped at one end and simply supported at the other end (C-S) strip are given in Equations (35) and (36) respectively.

\[ S - S = \Delta_{04}(\alpha - 2\alpha^2 + \alpha^4) \]  \hspace{1cm} (35) \]

\[ C - S = \Delta_{04}(1.5\alpha^2 - 2.5\alpha^3 + \alpha^4) \]  \hspace{1cm} (36) \]

By combining the peculiar deflections along various strips, the peculiar deflection equations for plate of various support conditions are obtained as given in Equations (37) and (38).

\[ SSSS = A_1(R - 2R^2 + R^4)(Q - 2Q^3 + Q^4) \]  \hspace{1cm} (37) \]

\[ CCSS = A_1(1.5R^2 - 2.5R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4) \]  \hspace{1cm} (38) \]

The formula for calculating fundamental natural frequency is determined as follows:

By filling in Equations (29a), (30) and (31) into Equation (25a) and minimizing the resulting equation with respect to \( A_1 \) (coefficient of deflection) and \( A_2 \) (coefficient of \( x \)-\( z \) shear rotation along \( x \)-direction) and \( A_3 \) (coefficient of \( y \)-\( z \) shear rotation along \( y \)-direction) respectively, Equations (39), (40) and (41) are obtained.

\[ \frac{dIII}{dA_1} = \frac{\beta D_{00}}{2} \left(24a_{44} \left(\frac{a}{t}\right)^2 \left(A_1 + A_2\right)k_{NR} + \frac{24a_{45}}{\beta} \left(\frac{a}{t}\right)^2 \left(2A_1 + A_2 + A_3\right)k_{NRQ} + \frac{24a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 \left(A_1 + A_3\right)k_{NQ} - A_1 \frac{M_h^2}{\beta} \right) = 0 \]  \hspace{1cm} (39) \]

\[ \frac{dIII}{dA_2} = \frac{\beta D_{00}}{2} \left(2A_2 a_{11} + 2A_3 a_{13} \frac{a}{\beta^2} k_{RQ} + \frac{2a_{13}}{\beta} [2A_1 + A_3] k_{RRQ} + \frac{2a_{23}}{\beta^3} [A_2 + 2A_3] k_{RQQ} + 12a_{44} \left(\frac{a}{t}\right)^2 \left(2A_1 + A_2\right)k_{RQ} + \frac{24a_{45}}{\beta} \left(\frac{a}{t}\right)^2 \left(A_1 + A_3\right)k_{NRQ} + \frac{12a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 \left(A_1 + A_3\right)k_{NQ} \right) = 0 \]  \hspace{1cm} (40) \]

\[ \frac{dIII}{dA_3} = \frac{\beta D_{00}}{2} \left(2A_2 a_{xy} \frac{a}{\beta^2} k_{RQ} + 2A_3 a_{22} \frac{a}{\beta^4} k_0 + \frac{2a_{13}}{\beta} [A_2 + 2A_3] k_{RRQ} + \frac{2a_{23}}{\beta^3} [2A_1 + A_3] k_{RQQ} + \frac{24a_{45}}{\beta} \left(\frac{a}{t}\right)^2 \left(A_1 + A_2\right)k_{NRQ} + \frac{12a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 \left(A_1 + A_3\right)k_{NQ} \right) = 0 \]  \hspace{1cm} (41) \]

Note:

\[ k_R = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial x}\right)^2 dR dQ; \quad k_{RQ} = \int_0^1 \int_0^1 \frac{\partial h}{\partial x} \frac{\partial^2 h}{\partial x \partial Q} dR dQ; \quad k_{NR} = \int_0^1 \int_0^1 \frac{\partial h}{\partial x} \frac{\partial h}{\partial R} dR dQ; \quad k_{NQ} = \int_0^1 \int_0^1 \frac{\partial h}{\partial y} \frac{\partial h}{\partial Q} dR dQ; \quad k_\lambda = \int_0^1 \int_0^1 h^2 dR dQ \]
By solving simultaneously, the simplified form of Equations (40) and (41) gave Equations (42) and (43).

\[ A_2 = T_2 A_1 = A_1 \frac{(d_{12} d_{23} - d_{13} d_{22})}{(d_{12}^2 - d_{11} d_{22})} \]  \hspace{1cm} (42)

\[ A_3 = T_3 A_1 = A_1 \frac{(d_{12} d_{13} - d_{11} d_{23})}{(d_{12}^2 - d_{11} d_{22})} \]  \hspace{1cm} (43)

Substituting Equations (42) and (43) into Equation (39) and making square of fundamental natural frequency (\( \lambda^2 \)) the subject gives Equation (44).

\[ \lambda^2 = \frac{k_T}{k_\lambda} \frac{D_{00}}{M a^4} \]  \hspace{1cm} (44)

simplifying Equation (44) gives Equation (45).

\[ \lambda = \frac{1}{a^2} \sqrt{\frac{k_T}{k_\lambda} \frac{D_{00}}{M}} \]  \hspace{1cm} (45)

Equation (45) can be expressed in the form of fundamental natural frequency parameter as Equation (46)

\[ \lambda a^2 \sqrt{\frac{M}{D_{00}}} = \frac{k_T}{k_\lambda} \]  \hspace{1cm} (46)

Where:

\[ d_{11} = a_{11} k_R + 2 \frac{a_{13}}{\beta} k_{RRQ} + 12 a_{44} \left(\frac{a}{t}\right)^2 k_{NNR} \]

\[ d_{12} = \frac{a_{22}}{\beta^2} k_{QQ} + \frac{a_{42}}{\beta} k_{RRQ} + \frac{a_{44}}{\beta} k_{RRQ} + \frac{12 a_{45} \left(\frac{a}{t}\right)^2}{\beta} k_{RRQ} \]

\[ d_{13} = -12 \left(\frac{a}{t}\right)^2 \left[ a_{44} k_{NNR} + \frac{a_{45}}{\beta} k_{RRQ} \right] \]

\[ d_{22} = \frac{a_{22}}{\beta^2} k_{QQ} + 2 \frac{a_{23}}{\beta^2} k_{RRQ} + \frac{12 a_{25} \left(\frac{a}{t}\right)^2}{\beta^2} k_{RRQ} \]

\[ d_{23} = -12 \left(\frac{a}{t}\right)^2 \left[ \frac{a_{45}}{\beta} k_{NNR} + \frac{a_{55}}{\beta^2} k_{RRQ} \right] \]

\[ k_T = 12 a_{44} \left(\frac{a}{t}\right)^2 (1 + T_2) k_{NNR} + \frac{12 a_{45} \left(\frac{a}{t}\right)^2 (2 + T_2 + T_3) k_{NNR} + \frac{12 a_{55} \left(\frac{a}{t}\right)^2 (1 + T_3) k_{NNR}} \]

3. Numerical problems

Using the above described theory, rectangular and square anisotropic thick plates that are simply supported (SSSS) and clamped on two adjacent edges and simply supported on the other two edges (CCSS) are analysed for fundamental natural frequency (\( \lambda \)), at various span to depth ratios (\( \frac{a}{t} = 5, 10, 20, 25, 50 \)), aspect ratios (\( \beta = 1 \) to 2) at increments of 0.1 and fibre orientation angles (\( \theta = 0^0, 15^0, 45^0 \)) with the following material properties: \( \frac{E_2}{E_1} = 0.52500, \frac{G_{12}}{E_1} = 0.26293, \frac{G_{23}}{E_1} = 0.15991, \frac{G_{32}}{E_1} = 0.26681, \mu_{12} = 0.44049 \) and \( \mu_{21} = 0.23124 \).

4. Results

The numerical results for the vibration analysis of SSSS and CCSS thick anisotropic rectangular plates with the material properties given in the section above and for span-depth ratios (\( \frac{a}{t} = 5, 10, 20, 25, 50 \)), aspect ratios (\( \beta = 1 \) to 2), and grain fiber orientation angles (\( \theta = 0^0, 15^0, 45^0 \)) obtained from Equation (46) are presented on Tables 1 to 6. The present solution is compared with solutions obtained from previous works by Srinivas and Roa (1970), Shimpi and Patel (2006), and Reddy (1984). The fundamental natural frequency parameter values (\( \lambda \)) were calculated for SSSS plate having the same material properties as given in the section above using the following formula: \( \lambda = \)
\[ \lambda \sqrt{\frac{\rho(1 - \mu_{12}\mu_{21})}{E_0}} \] If \( \beta = 0.5 \) for previous work, \( \beta \) for this study is \( \frac{1}{\rho} = \frac{1}{0.5} = 2.0 \). If \( \beta = 1.0 \) for previous work, \( \beta \) for this study is \( \frac{1}{\rho} = \frac{1}{1} = 1.0 \) and if \( \beta = 2.0 \) for previous work; for this study \( \beta \) is \( \frac{1}{\rho} = \frac{1}{2} = 0.5 \). Also, If \( \frac{1}{\rho} = 0.05 \) for previous work; for this present study \( \frac{1}{\rho} = \frac{1}{0.05} = 20 \).

The present study solution was further validated by comparing its solutions with those obtained from Reddy (2004) for SSSS thick orthotropic rectangular plate with the following material properties: \( E_1 = 25E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \mu_{12} = 0.25 \). The fundamental natural frequency parameter values were calculated using the following formula:

\[ \bar{\lambda} = \lambda \sqrt{\frac{a^2}{\tau} \frac{\rho}{E}} \] The compared results are presented in Tables 7 and 8.

| Table 1 Fundamental natural frequency parameter values for SSSS plate at \( \theta = 0^\circ \) |
|---|---|---|---|---|---|---|
| \( \beta \) | \( a^2 \frac{M}{D_{00}} \) | \( \frac{a}{t} \) |
| 5 | 10 | 20 | 25 | 50 | 100 |
| 1.1 | 13.8287 | 15.1476 | 15.5410 | 15.5903 | 15.6568 | 15.6735 |
| 1.5 | 11.5701 | 12.5828 | 12.8805 | 12.9177 | 12.9677 | 12.9804 |
| 1.6 | 11.2493 | 12.2215 | 12.5067 | 12.5423 | 12.5902 | 12.6023 |
| 1.9 | 10.5704 | 11.4591 | 11.7187 | 11.7510 | 11.7945 | 11.8055 |
Table 2 Fundamental natural frequency parameter values for SSSS plate at $\theta = 15^\circ$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\frac{\lambda a^2}{\sqrt{\frac{M}{D_{00}}}}$</th>
<th>$\frac{a}{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>14.9711</td>
<td>16.4112</td>
</tr>
<tr>
<td>10</td>
<td>16.8412</td>
<td>16.8951</td>
</tr>
<tr>
<td>20</td>
<td>16.9677</td>
<td>16.9860</td>
</tr>
<tr>
<td>25</td>
<td>15.7011</td>
<td>15.7172</td>
</tr>
<tr>
<td>50</td>
<td>14.7377</td>
<td>14.7523</td>
</tr>
<tr>
<td>100</td>
<td>13.9880</td>
<td>14.0015</td>
</tr>
<tr>
<td>1</td>
<td>14.1438</td>
<td>15.2940</td>
</tr>
<tr>
<td>1.1</td>
<td>15.2940</td>
<td>15.6282</td>
</tr>
<tr>
<td>1.2</td>
<td>15.6698</td>
<td>15.7258</td>
</tr>
<tr>
<td>1.3</td>
<td>15.7399</td>
<td>15.7939</td>
</tr>
<tr>
<td>1.4</td>
<td>15.9126</td>
<td>15.9662</td>
</tr>
<tr>
<td>1.5</td>
<td>16.0851</td>
<td>16.1409</td>
</tr>
<tr>
<td>1.6</td>
<td>16.2535</td>
<td>16.3096</td>
</tr>
<tr>
<td>1.7</td>
<td>16.4219</td>
<td>16.4715</td>
</tr>
<tr>
<td>1.8</td>
<td>16.5887</td>
<td>16.6385</td>
</tr>
<tr>
<td>1.9</td>
<td>16.7555</td>
<td>16.8121</td>
</tr>
<tr>
<td>2</td>
<td>16.9222</td>
<td>16.9758</td>
</tr>
</tbody>
</table>

Table 3 Fundamental natural frequency parameter values for SSSS plate at $\theta = 45^\circ$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\frac{\lambda a^2}{\sqrt{\frac{M}{D_{00}}}}$</th>
<th>$\frac{a}{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15.3497</td>
<td>16.7053</td>
</tr>
<tr>
<td>10</td>
<td>17.1042</td>
<td>17.1540</td>
</tr>
<tr>
<td>20</td>
<td>17.2211</td>
<td>17.2380</td>
</tr>
<tr>
<td>25</td>
<td>17.1540</td>
<td>17.2211</td>
</tr>
<tr>
<td>50</td>
<td>15.7258</td>
<td>15.7399</td>
</tr>
<tr>
<td>100</td>
<td>15.7399</td>
<td>15.7939</td>
</tr>
<tr>
<td>1</td>
<td>14.1438</td>
<td>15.2940</td>
</tr>
<tr>
<td>1.1</td>
<td>15.2940</td>
<td>15.6282</td>
</tr>
<tr>
<td>1.2</td>
<td>15.6698</td>
<td>15.7258</td>
</tr>
<tr>
<td>1.3</td>
<td>15.7399</td>
<td>15.7939</td>
</tr>
<tr>
<td>1.4</td>
<td>15.9126</td>
<td>15.9662</td>
</tr>
<tr>
<td>1.5</td>
<td>16.0851</td>
<td>16.1409</td>
</tr>
<tr>
<td>1.6</td>
<td>16.2535</td>
<td>16.3096</td>
</tr>
<tr>
<td>1.7</td>
<td>16.4219</td>
<td>16.4715</td>
</tr>
<tr>
<td>1.8</td>
<td>16.5887</td>
<td>16.6385</td>
</tr>
<tr>
<td>1.9</td>
<td>16.7555</td>
<td>16.8121</td>
</tr>
<tr>
<td>2</td>
<td>16.9222</td>
<td>16.9758</td>
</tr>
</tbody>
</table>
Table 4 Fundamental natural frequency parameter values for CCSS plate at $\theta = 0^\circ$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\lambda a^2 \sqrt{\frac{M}{D_{00}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{a}{\bar{t}}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>19.0773</td>
</tr>
<tr>
<td>1.4</td>
<td>15.6314</td>
</tr>
<tr>
<td>1.5</td>
<td>15.1998</td>
</tr>
<tr>
<td>1.6</td>
<td>14.8549</td>
</tr>
<tr>
<td>1.7</td>
<td>14.5753</td>
</tr>
<tr>
<td>1.8</td>
<td>14.3459</td>
</tr>
<tr>
<td>1.9</td>
<td>14.1554</td>
</tr>
</tbody>
</table>

Table 5 Fundamental natural frequency parameter values for CCSS plate at $\theta = 15^\circ$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\lambda a^2 \sqrt{\frac{M}{D_{00}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{a}{\bar{t}}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>19.2429</td>
</tr>
<tr>
<td>1.4</td>
<td>15.6907</td>
</tr>
<tr>
<td>1.5</td>
<td>15.2421</td>
</tr>
<tr>
<td>1.6</td>
<td>14.8829</td>
</tr>
<tr>
<td>1.7</td>
<td>14.5912</td>
</tr>
<tr>
<td>1.8</td>
<td>14.3514</td>
</tr>
<tr>
<td>1.9</td>
<td>14.1521</td>
</tr>
</tbody>
</table>
### Table 6 Fundamental natural frequency parameter values for CCSS plate at $\theta = 45^\circ$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\lambda a^2 \sqrt{\frac{M}{D_{00}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{a}{t}$</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>19.7374</td>
</tr>
<tr>
<td>1.3</td>
<td>16.3046</td>
</tr>
<tr>
<td>1.4</td>
<td>15.6277</td>
</tr>
<tr>
<td>1.5</td>
<td>15.0874</td>
</tr>
<tr>
<td>1.8</td>
<td>13.9962</td>
</tr>
<tr>
<td>1.9</td>
<td>13.7483</td>
</tr>
<tr>
<td>2</td>
<td>13.5391</td>
</tr>
</tbody>
</table>

### Table 7 Fundamental natural frequency parameter results of present study compared with the results of previous research for SSSS thick orthotropic rectangular plate at $\theta = 0^\circ$.

<table>
<thead>
<tr>
<th>$\frac{a}{t}$</th>
<th>Theory</th>
<th>$\lambda t \sqrt{\frac{\rho (1-\mu_{12}\mu_{21})}{E_o}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta$</td>
</tr>
<tr>
<td>10</td>
<td>Present study (P)</td>
<td>0.0471</td>
</tr>
<tr>
<td></td>
<td>Shimpi and Patel (2006), (SP)</td>
<td>0.0477</td>
</tr>
<tr>
<td></td>
<td>Reddy (1984), (R)</td>
<td>0.0474</td>
</tr>
<tr>
<td></td>
<td>Srinivas and Roa (1970), (SR)</td>
<td>0.0474</td>
</tr>
<tr>
<td></td>
<td>% Difference between P and SP</td>
<td>1.27389</td>
</tr>
<tr>
<td></td>
<td>% Difference between P and R</td>
<td>0.63693</td>
</tr>
<tr>
<td></td>
<td>% Difference between P and SR</td>
<td>0.63693</td>
</tr>
</tbody>
</table>
Table 8: Fundamental natural frequency parameter numerical results of present study compared with the results of Reddy (2004) for SSSS thick orthotropic rectangular plate at β = 1 and θ = 0°

<table>
<thead>
<tr>
<th>Theory</th>
<th>[\lambda a^2 \sqrt{\frac{M}{D_{00}}}]</th>
<th>[\frac{a}{t}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9.485</td>
<td>12.8632</td>
</tr>
<tr>
<td>10</td>
<td>12.8632</td>
<td>14.5194</td>
</tr>
<tr>
<td>20</td>
<td>14.7659</td>
<td>15.1156</td>
</tr>
<tr>
<td>25</td>
<td>15.207</td>
<td>15.207</td>
</tr>
<tr>
<td>50</td>
<td>15.190</td>
<td>15.190</td>
</tr>
<tr>
<td>100</td>
<td>15.077</td>
<td>15.077</td>
</tr>
</tbody>
</table>

5. Discussion

The fundamental natural frequency parameter numerical results for CCSS and SSSS anisotropic rectangular plates are presented on Tables 1 to 6. From Tables 1 to 6, it is observed that the natural frequency parameter \(\lambda a^2 \sqrt{\frac{M}{D_{00}}}\) values increased as span to depth ratio \((\frac{a}{t})\) increases for 0°, 15° and 45° angles of fibre orientation (θ) at any given values of aspect ratio (1 to 2). Natural frequency parameter values also decreased as aspect ratio increases at any given value of span to depth ratio (5, 10, 20, 25 and 100) and angle of fibre orientation (0°, 15°, and 45°). For SSSS plate, highest value of \(\lambda a^2 \sqrt{\frac{M}{D_{00}}}\) occurred at aspect ratio of 1.0 for \(\theta = 45^o\) and \(\frac{a}{t} = 100\) while the lowest value of \(\lambda a^2 \sqrt{\frac{M}{D_{00}}}\) occurred at aspect ratio of 2.0 for \(\theta = 45^o\) and \(\frac{a}{t} = 5\). For CCSS plate, highest value of \(\lambda a^2 \sqrt{\frac{M}{D_{00}}}\) occurred at aspect ratio of 1.0 for \(\theta = 45^o\) and \(\frac{a}{t} = 100\) while the lowest value of \(\lambda a^2 \sqrt{\frac{M}{D_{00}}}\) occurred at aspect ratio of 2.0 for \(\theta = 45^o\) and \(\frac{a}{t} = 5\). This shows that the natural frequency parameter values have a more significant effect on SSSS and CCSS thin anisotropic rectangular plates than it has on SSSS and CCSS thick anisotropic plates.

5.1 Comparison of present study solution with those from previous researchers

From Table 7, vibration analysis result of thick orthotropic rectangular plate having all sides simply supported obtained using refined plate theory (RPT) result of Shimpi and Patel (2006), higher order shear deformation theory (HSDT) result of Reddy (1984) and three-dimensional elasticity theory (TDET) result of Srinivas and Roa (1970). It is seen that the present study yields an excellent value for the fundamental frequency; this is shown by the result of the percentage difference between the present solution and those of previous research works. Thus, the present theory gives good solutions for free-vibration analysis of SSSS thick orthotropic plates. From Table 8, the vibration analysis result of thick orthotropic rectangular plate having all sides simply supported obtained using modified first order shear deformation theory is presented and compared with the first order shear deformation theory (FSDT) results of Reddy (2004) for span to depth ratios of 5, 10, 20, 25, 50 and 100 at an aspect ratio of 1. It is observed from Table 8, that the solution of present theory gets closer to that of Reddy (2004) as the plate gets thinner (span to depth ratio increases) as can be seen from the results of the percentage difference for the various span to depth ratios. Table 8, shows that the present study solution is the correct solution since modified first order shear deformation theory does not require shear correction factor which first order shear deformation theory uses. From the table, it is seen that the percentage difference reduces in value as the plate becomes thin, this shows that Reddy (2004) does not give a better solution when used to analyse SSSS thick orthotropic plate. Thus, the present theory gives better and accurate solutions for free-vibration analysis of SSSS thick orthotropic plate.

6. Conclusion

In this study, a modified first order shear deformation theory is applied to free vibration analysis of anisotropic plate (SSSS and CCSS). The results obtained from this study were compared with those from previous authors. Observations show that the results of the fundamental natural frequency parameter predicted by the present theory are in close agreement with those of Reddy (2004) as the plate gets thinner (span to depth ratio increases) as can be seen from the results of the percentage difference between the present study and those of previous research works. Thus, the present theory gives good solutions for free-vibration analysis of SSSS thick orthotropic plate.
agreement with those of previous researchers. The present method is capable of calculating reasonably correct values for free vibration problems of thick and thin anisotropic rectangular plates and can be adopted by future scholars to solve thick and thin anisotropic rectangular plate problems.

Compliance with ethical standard

Acknowledgements

I wish to appreciate my supervisors, Engr. Dr. O. M. Ibearugbulem, Rev. Engr. Prof. L.O. Ettu and Engr. Dr. U.C Anya for their immense effort in initiating and supervising my Ph.D. thesis from which this article is developed. Above all, this study would not have been a reality without the Almighty God who provided life, wisdom and sound health for this work. May His name be praised, adored, glorified and worshiped forever and ever, Amen.

Disclosure of conflict of interest

No conflict of interest to be disclosed.

References


