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Advanced structural health monitoring and damage detection using inverse methods and perturbation stiff-ness matrix analysis

Arvin Fardipour [1,*], Norman Herl [2], Sarmad Waqas [3] and Mehdi Nodeh [4]

[1] *Department of Civil Engineering, Islamic Azad University, Rudehen, Tehran, Iran.*
[2] *Federal Board of Intermediate and Secondary Education, Islamabad, Pakistan.*
[3] *Osmani & Company Pvt. Ltd, Karachi, Pakistan.*
[4] *Institute of sanitary engineering and waste management Leibniz university of Hanover, Hanover, Germany.*

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Abstract

Sensor technology advancements have the health monitoring task for modern intricate structures introduce many challenges, primarily centered around the augmentation of safety margins and operational dependabil-ity. The demand for diverse systems or methodologies is, therefore, increasing. These systems or methodolo-gies should be capable of detecting damage, monitoring remotely, and conducting continuous evaluations in real-time. As part of this study, inverse techniques rooted in wave propagation analysis within structural frameworks are utilized to identify damage response vectors. Additionally, mathematical formulations grounded in minimal rank perturbation theory facilitate determining a perturbed stiffness matrix for a com-promised structure. Various excitations are included in the analysis, which is based on intricate non-linear structural modeling. Using streamlined procedures, these findings confirm the inherent versatility of a range of methodologies capable of meeting both rapid monitoring needs and exhaustive analytical needs. In addition, the proposed approach promotes the use of intelligent structures embedded with a variety of sensors that are not restricted by limitations related to sensor type or spatial deployment. Monitoring structural health and detecting damage could be significantly improved by this strategic deployment.

Keywords: Machine Learning; Structural Health Monitoring; Damage Detection; Inverse Methods; Perturbation Stiffness Matrix Analysis; Wave Propagation Analysis

1. Introduction

Construction projects are becoming more complicated in scale and design, new materials and ideas are used more frequently, and operational and environmental conditions are deteriorating [1]. Interconnected transportation systems, such as highway bridges, must be continuously evaluated and, in some cases, in real-time. It is now possible to integrate many embedded sensors into innovative structures with various sensing capabilities [2]. A comprehensive understanding of structural behavior is now possible with the help of these sensors, which have revolutionized measurement, evaluation, and control options for engineering systems. Understanding existing structures' health status, coupled with advancements in sensor technology, poses challenges that require innovative theoretical and experimental approaches [3]. The recent study undertook a collaborative research initiative that comprehensively explores construction defects within reinforced concrete corbels. Their work seamlessly integrates both experimental and numerical investigations, resulting in a thorough assessment that substantially contributes to the field of structural engineering. Through their collective efforts, this team plays a pivotal role in identifying and analyzing defects present in concrete corbels, advancing our understanding of structural integrity in these critical components [4]. A non-linear model parameter has also been explored for reinforced concrete columns in new construction [5]. Recent research

* Corresponding author: Arvin Fardipour.

delves into the significant realm of concrete recycling, shedding light on sustainable construction practices. Their collective efforts yield valuable insights into the challenges and opportunities inherent in concrete recycling, thus making substantial contributions to advancing environmentally friendly construction methodologies.[6]

Ahmad A. et al. introduce a comprehensive framework for the development of artificial neural networks aimed at predicting the load-carrying capacity of reinforced concrete members. This innovative approach significantly enhances structural assessment methodologies. Their research, published in S.N. Applied Sciences, marks a pivotal advancement in the application of AI within structural engineering, particularly in the accurate prediction of load capacity.[7] One study employed a cutting-edge approach that combines image processing and artificial neural networks (ANN) to predict the compressive strength of Concrete Repair Mortar (CRM) samples. Their study, featured in the IOP Conference Series, illuminates the considerable potential of ANN and image processing techniques in accurately predicting material strength, thereby advancing the field of material science and construction technology[8]. Ahmad A. et al., through collaborative efforts, advance the field of structural safety by rigorously assessing the reliability of reinforced concrete code predictions with the application of artificial neural networks. Their research, presented at the 1st International Conference on Structural Safety, emphasizes the paramount importance of precise code predictions in ensuring and enhancing structural integrity, making a substantial contribution to the field [9]. In a complementary study, Ahmad A. et al. conducted a rigorous reliability analysis of models predicting T-beam responses at the ultimate limit. This research, prominently featured in the Proceedings of the Institution of Civil Engineers, significantly enhances our understanding of T-beam performance, offering valuable insights into structural behavior under extreme conditions and contributing significantly to structural engineering [10]. Their collaborative efforts extend further to demonstrate the effectiveness of neural network-based prediction in assessing the behavior of reinforced concrete members across a spectrum of loading scenarios encompassing both simple and complex conditions. This research, prominently featured in Applied Sciences, represents a significant stride in the application of artificial intelligence to predict and comprehend the intricate behavior of concrete structures, providing invaluable contributions to the field of structural engineering [11]. In a pioneering endeavor, Ahmad A. et al. present groundbreaking research focused on assessing the load-carrying capacity of Reinforced Concrete (R.C.) members, harnessing the capabilities of artificial neural networks. Unveiled at the 11th HSTAM International Congress on Mechanics in Athens, Greece, in May 2016, their work significantly advances the state-of-the-art in structural engineering. This research not only underscores the potential of AI-driven methodologies but also contributes to enhancing our ability to predict and optimize the load-bearing capabilities of R.C. members, fostering innovation in structural design and safety practices [12].

Health monitoring systems integrate non-destructive testing methods, but many methods are localized, so comprehensive global structural assessments are impossible. Vibration and modal analysis techniques can provide a comprehensive understanding of structural health due to the need for a holistic approach [13,14]. Natural frequencies, mode shapes, and damping ratios determine vibration signatures for health monitoring. By employing numerous algorithms categorizing according to different criteria, a refined simulation typically compares the behavior of undamaged and damaged structures. This study investigated wave behavior as it propagated through structures in an inverse methodology to identify damage [15,16].

Meanwhile, Joushideh, N. et al. utilize Ground-Penetrating Radar (GPR) to comprehensively characterize scour-induced subsurface deformations in port structures, providing valuable insights into cutting-edge structural health monitoring techniques. Their collective efforts contribute significantly to the assessment of subsurface damage, emphasizing the pivotal role of remote sensing technologies in enhancing structural evaluation practices [17]. In their research, Joushideh N. et al. delve into the investigation of rubble mound breakwater stability under challenging hydrodynamic conditions, offering invaluable insights that bolster our understanding of coastal structure integrity. Their collective contributions in assessing slope stability and conducting numerical settlement analyses for breakwaters significantly advance the field of coastal engineering [18].

Following this section is an overview of the essential theories relevant to the study to ensure comprehensive coverage. Subsequent sections analyze simulation models comparatively. Using MATLAB software, we begin by establishing the building's characteristics in an undamaged state. In addition to these structural details, the replication of the building also features reduced stiffness due to damage. Second, simulation output data is used for subsequent analysis as observed data.

2. Simulation of undamaged structure

The three degrees of freedom (3DOF) simulated system that was examined to determine damage using the least squares method is depicted in Figure 1.

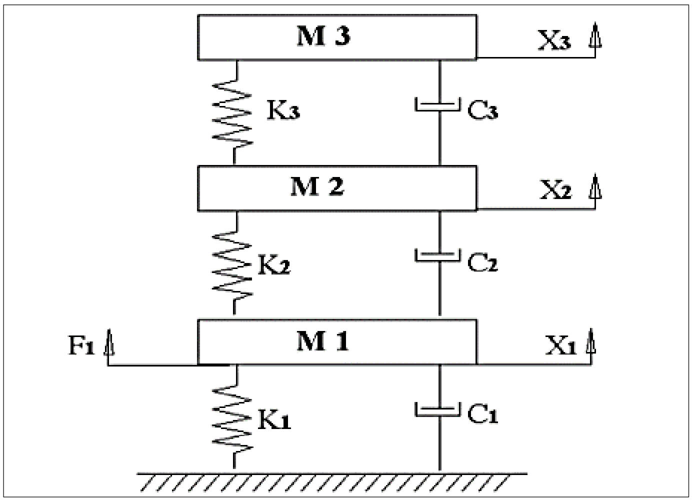

Figure 1 Three degrees of freedom (3DOF) simulated system.

Figure 1 illustrates a schematic representation of a 3-degree-of-freedom (DOF) system. A degree of freedom refers to an independent motion that a structure can undergo. The three DOFs represented in the figure correspond to three possible types of motion that can occur in a single structural element or in a building's entire story. The first DOF depicted in the image is translational motion along the horizontal X-axis. This DOF represents the side-to-side motion of the building or structural element. The second DOF is translational motion along the horizontal Y-axis, representing the forward and backward motion of the structure. The third DOF corresponds to rotational motion around the vertical Z-axis, indicating the possibility of twisting or rotating motion of the structure around its central axis. Together, these three DOFs provide a simplified model for analyzing the behavior of a structure under different loading conditions. The motions of a structure in each of these DOFs can be calculated using various techniques, such as modal analysis, to evaluate the structure's response to seismic or other dynamic loads. The results of such analyses are crucial for designing safe and stable structures that can withstand potentially destructive external forces.

The stiffness matrix is a crucial component in structural analysis, as it allows us to establish a relationship between the forces applied to a structure and its resulting displacements. In this study, we will calculate the stiffness matrix for a 3-degree-of-freedom (DOF) system, represented by a 3 by 3 square matrix denoted by [K]. To determine [K], the first step involves selecting a coordinate system for the system under consideration, the horizontal plane defined by the X and Y axes and the Z axis representing the vertical axis. For the building under consideration, only the X-axis will be considered. Next, we determine the stiffness of each of the three DOFs individually, with k11 representing the stiffness of the first DOF, which accounts for horizontal motion in the X direction. The second and third DOFs are represented by k22 and k33, respectively. As a prerequisite to calculating K11, K22, K33, we need to know the stiffness of each individual element in the structure, such as columns. These individual stiffness values are then used to calculate the global stiffness of the structure, which is represented by k11, k22, and k33.

Finally, we assemble the global stiffness matrix [K] by placing the individual stiffness values in their respective positions in the matrix. This matrix can then be used to solve the equations of motion for the structure, providing valuable insights into its behavior and response under different loading conditions.

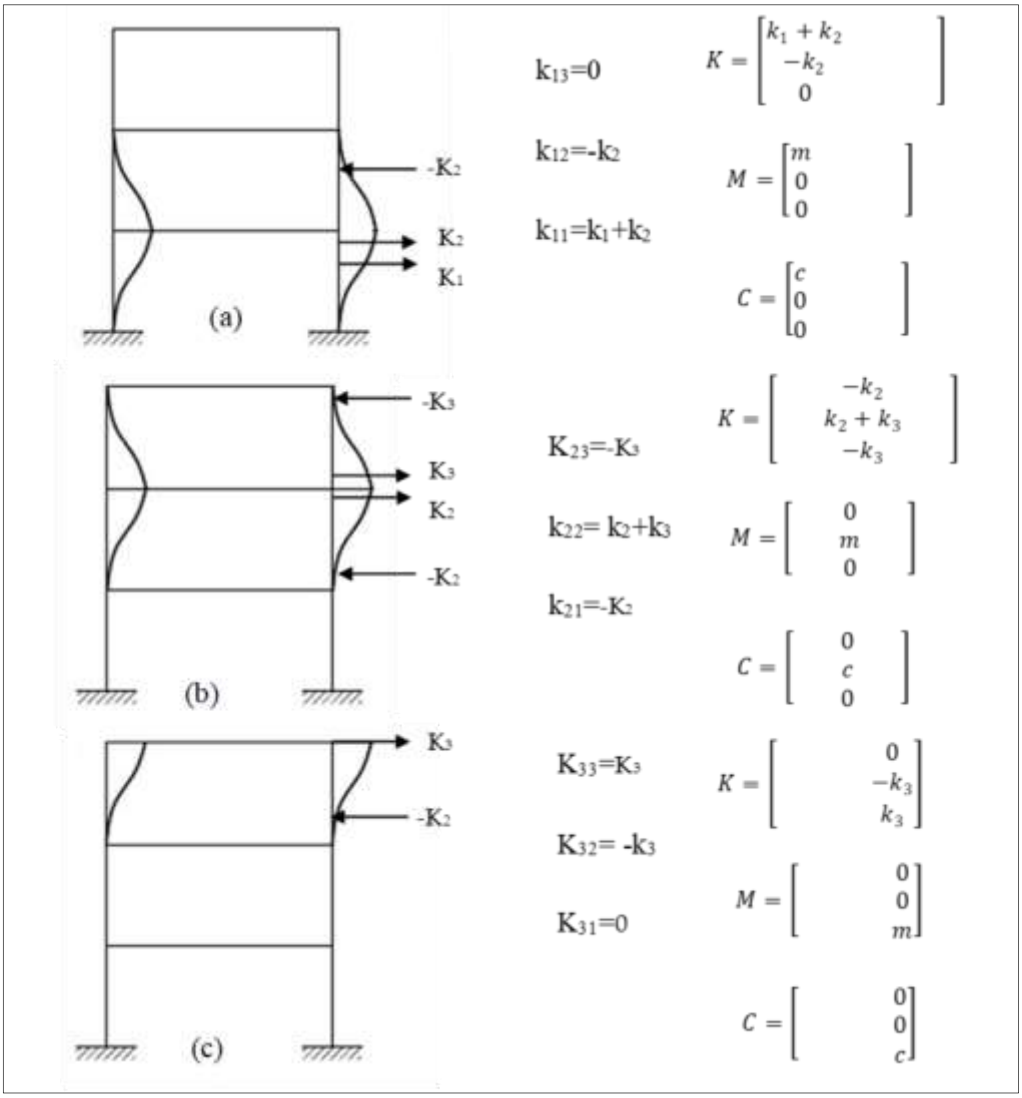

Figure 2 (a) Apply $u_1=1$, u_2, $u_3=0$, and determine K_{i1} (b) Apply $u_2=1$, u_1, $u_3=0$, and determine K_{i2} (c) Apply $u_3=1$, u_1, $u_2=0$, and determine K_{i3}.

The system's equations of motion are as follows:

$$m\ddot{x}_1 + c\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = f_1(t)$$

$$m\ddot{x}_2 + c \times \dot{x}_2 - k_2x_1 + (k_2 + k_3)x_2 - k_3x_3 = f_2(t)$$

$$m\ddot{x}_3 + c\dot{x}_3 - k_3x_2 + k_3x_3 = f_3(t)$$

The undamaged condition was represented by the values m = 1.5, c = 0.9, and $k_1 = 1\times10^2$,

$K_2 = 1\times10^2$, $k_3 = 1\times10^2$.

The structure's changes in displacement and frequency were the feature that was utilized for the detecting method, and it was calculated by simulating the response to a harmonic excitation of, $f(t) = \cos(\omega t)$. To predict data, the Newmark method is employed as follows:

2.1. Initial calculation

- Form static stiffness matrix K, mass matrix M, and damping matrix C
- Specify integration parameters β and γ

- Calculate integration constants

$$b_1 = \frac{1}{\beta \Delta t^2} \qquad b_2 = \frac{1}{\beta \Delta t} \qquad b_3 = \beta - \frac{1}{2} \qquad b_4 = \gamma \Delta t b_1$$

$$b_5 = 1 + \gamma \Delta t b_2 \qquad b_6 = \Delta t(1 + \gamma b_3 - \gamma)$$

- From effective stiffness matrix $\overline{K} = K + b_1 M + b_4 C$
- Triangularines effective stiffness matrix $\overline{K} = LDL^T$
- Specify initial conditions $u_0\,,\ \dot{u}_0\,,\ \ddot{u}_0$

2.2. For each time step

2.2.1. Calculate effective load vector

$$\bar{F}_t = F_t + M(b_1 u_{t-\Delta t} - b_2 \dot{u}_{t-\Delta t} - b_3 \ddot{u}_{t-\Delta t}) + C(b_4 u_{t-\Delta t} - b_5 \dot{u}_{t-\Delta t} - b_6 \ddot{u}_{t-\Delta t})$$

*2.2.2. Solve for node displacement vector at time **t***

$LDL^T u_t = \bar{F}_t$ Forward and back-substitution only

*2.2.3. Calculate node velocities and acceleration at time **t***

$$\dot{u}_t = b_4(u_t - u_{t-\Delta t}) + b_5 \dot{u}_{t-\Delta t} + b_6 \ddot{u}_{t-\Delta t}$$

$$\ddot{u}_t = b_1(u_t - u_{t-\Delta t}) + b_2 \dot{u}_{t-\Delta t} + b_3 \ddot{u}_{t-\Delta t}$$

2.2.4. Go to step II.A with $t = t + \Delta t$

2.3. Outliers and errors

Based on Figure 2, a comparison between the observed data, which represents the measured displacement, and the calculated data is shown through the residuals. The residuals are the differences between the observed and calculated data at each time point. By observing Figure 2, it is evident that the residuals have much higher values during times 1.5, 2.5, and 3.5 (sec) in comparison to other time points. Therefore, it can be concluded that these three critical times should be selected for the subsequent section to assess and compare the simulated and calculated models.

$e = U_{obs} - U_{pre}$

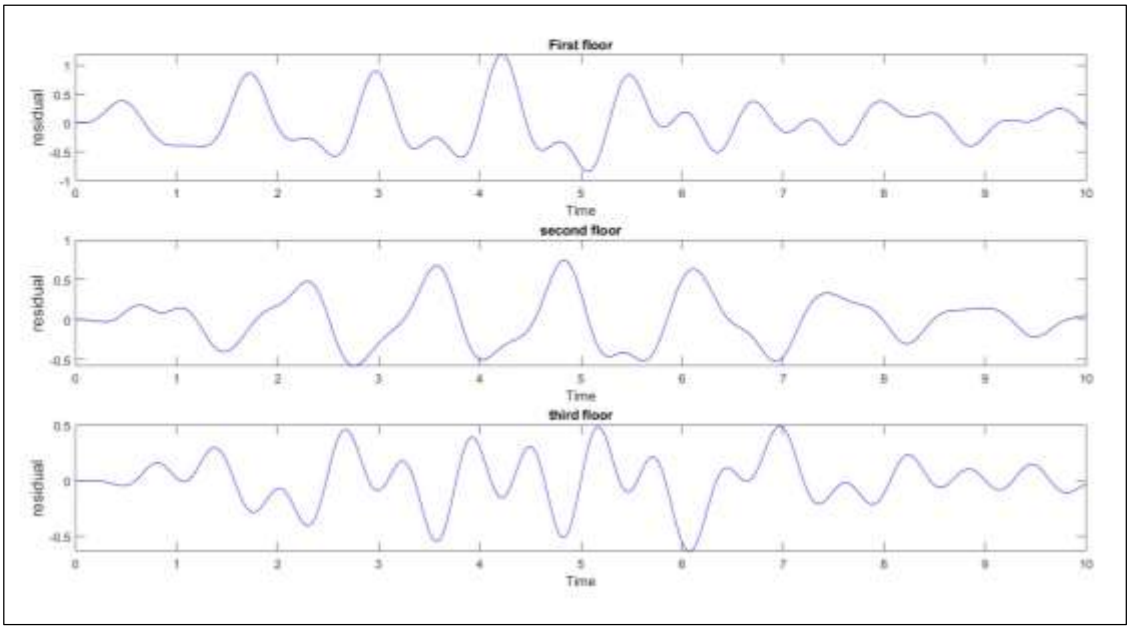

Figure 3 Difference between observed and predicted data (residuals)

2.4. Creating the matrix of changes

Fracture, erosion, bolt loosening, and other structural phenomena manifest as a decrease in stiffness. Changes in stiffness have an impact on how a structure's stress waves propagate, making them excellent messengers of this information. We define the change in stiffness as damage in this context and identify it using the response of the wave propagation to the damage.

The following equations control how a structure reacts to a stress wave:

$$[Md]\{\ddot{u}\} + [Cd]\{\dot{u}\} + [Kd]\{u\} = \{P\}.$$

The stiffness, damping, and mass matrices are perturbed to reflect the altered or damaged condition and represented by $[Cd] = [C_0]-[C]$, $[Md] = [M_0]-[M]$, $[Kd] = [K_0]-[K]$, and the underlying equations are as follows:

$$[M0 - \Delta M]\{\ddot{u}\} + [C0 - \Delta C]\{\dot{u}\} + [K0 - \Delta K]\{u\} = \{P\}.$$

$$[M0]\{\ddot{u}\} + [C0]\{\dot{u}\} + [K0]\{u\} = \{P\} + [\Delta K]\{u\} + [\Delta C]\{\dot{u}\} + [\Delta M]\{\ddot{u}\}$$

$$\{D\} = [M0]\{\ddot{u}\} + [C0]\{\dot{u}\} - \{P\}$$

We refer to {D} which contains damage information as the damage response vector. It is a simple technique to extract the needed damage information if we can somehow figure out D. Consider the situation when the stiffness is the only thing that changes:

$$[K_{damage}][u(t)] = [D(t)]$$

The information of {u(t)} and {D(t)} for three chosen times were used to convert them to the square matrix [u] and [D], respectively. How to choose the suitable time for this process was explained in the previous part.

To obtain [ΔK] that has an ill-conditioning, we need to calculate the equation based on the least square theory.

$$[K_{damage}] = [u(t)^T u(t)]^{-1} [u(t)]^T [D(t)]$$

The variable [D(t)] is a 3 by 3 dimensional matrix that corresponds to the observed data obtained from the simulated damaged model. Meanwhile, the variable [u(t)] is a 3 by 3 dimensional matrix that serves as the G matrix in this context. Moreover, the variable $[K_{damage}]$ represents a 3 by 3 dimensional matrix that encompasses the stiffness changes in the structural model. $[K_{damage}]$ is considered as a set of model parameters that will be predicted via a least square solution method to identify the amount of structural damage. Upon analyzing the matrix of stiffness changes, as shown in table1, it is evident that the largest changes occurred on the second floor of the structure. Based on this observation, it can be inferred that there is a significantly higher likelihood of damage on this particular floor. By selecting the second column as the comparison benchmark, it is ensured that all values being compared will have a non-zero reference point. This helps to eliminate the possibility of erroneous comparisons and ensures that the results obtained are scientifically valid.

Table 1 Stiffness changes details

$K_{Undamaged}$	$K_{Damaged}$	ΔK
100	33	67
200	16	184
100	32	68

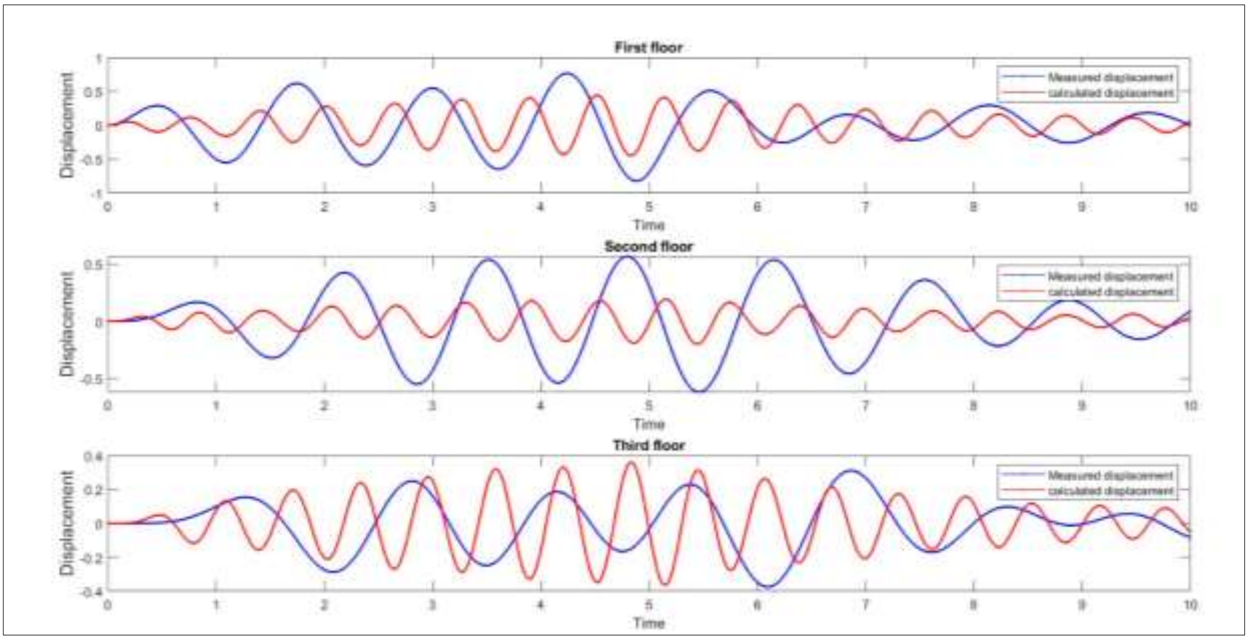

Figure 4 Measured and calculated displacements

According to Figure 4, it appears that the undamaged structure (red line) has a smaller amplitude of displacement compared to the damaged structure (blue line) under the same harmonic excitation. Additionally, the undamaged structure has a shorter period and higher frequency compared to the damaged structure. The reduced stiffness of a damaged structure can cause it to have a lower natural frequency and larger displacement under the same harmonic excitation. This is because the stiffness of a structure affects its natural frequency and response to external loads. A reduction in stiffness means that the structure is less able to resist deformation and is more easily displaced by external loads. As a result, the damaged structure will have a larger amplitude of displacement at the same frequency compared to the undamaged structure. Additionally, the damaged structure will have a longer period and lower frequency, as its reduced stiffness causes its natural frequency to decrease. The relationship between reduced stiffness, natural frequency, and displacement is important to consider when assessing the performance of damaged structures and designing strategies for their repair or replacement.

3. Discussion

Monitoring the health of contemporary complex structures is a highly challenging task that requires advanced techniques and tools. It focuses on the inverse technique based on wave propagation to identify damage response vectors using the inverse method for structural health monitoring. As highlighted by Kim et al. [19], the utilization of wireless sensor networks for health monitoring showcases the potential of modern technologies to address these challenges effectively.

It has been shown in the study that modal and vibration analysis methods can provide reliable insights into structural integrity. Using techniques such as Particle Swarm Optimization, Nanda et al. [20] illustrate the importance of vibration-based damage detection techniques.

Led by Mahmoudabadi, N.S., innovative research explores the application of viscous dampers for retrofitting reinforced concrete frames, offering novel solutions for structural enhancement. This work, featured in the Turkish Journal of Computer and Mathematics Education, underscores the crucial role dampers play in bolstering the seismic performance of concrete structures [21]. Additionally, Mahmoudabadi, N.S., conducted an exhaustive study on cable behavior, investigating the impact of spring dampers and viscous dampers on the dynamic response of structural elements. This research significantly advances our understanding of cable systems and their dynamic behavior under various loading conditions, contributing substantially to structural engineering knowledge [22]. For structural health assessment, engineers and researchers now have more tools at their disposal. Doubling et al. [23] summarize the need for combining various techniques to ensure accurate and robust vibration-based damage identification results. A major contribution to structural health monitoring made by Karbhari and Ansari [24] emphasizes the need for continuing research and innovation in this field. In line with Schommer et al. [25], this study explored the potential of model updating techniques in refining structural health assessment by combining static and dynamic measurements. It is crucial to establish a holistic view of structural conditions in order to be able to detect damage more accurately. Such multidimensional approaches help to establish a holistic view of structural conditions.

In their review, Avci et al. [26] point out that traditional damage detection methods are being replaced by machine learning and deep learning approaches, reaffirming the evolution of damage detection methodologies. As a result, health monitoring systems are shifting towards more sophisticated techniques that can potentially improve accuracy and efficiency. According to Kim et al. [19] and Avci et al. [26], sensors embedded within complex structures emerge as essential tools for real-time structural health monitoring. By providing continuous data streams, these sensors allow for the early detection of structural anomalies, resulting in proactive maintenance.

4. Conclusions

To demonstrate the application of discussed methods for health monitoring and damage detection in complex structures, this study has presented a mathematical simulation of a three-story building using MATLAB software. An example of how the approaches discussed can be applied in real-life scenarios can be seen in this simulation, which demonstrates their potential in the real world. By providing insights into various methodologies for monitoring structural health and detecting damage, this study contributes to the field of structural engineering. A diverse range of references spanning traditional techniques to modern advancements are incorporated into this study in order to emphasize the importance of multidisciplinary approaches. By making informed decisions and intervening in time, these insights promise to improve complex structures' safety, reliability, and longevity.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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