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Bessel functions of first kind and their Anuj transforms

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Abstract

Various problems of Statistics, Mathematics, Radio Physics, Nuclear Physics, Atomic Physics, Fluid Mechanics, Engineering and Science can easily handle by applying integral transform techniques on their mathematical models. Problems of heat equation, Schrodinger equation, Laplace equation, Helmholtz equation and wave equation have solutions in terms of Bessel functions. To solve such equations by integral transform methods, we need to know the integral transform of Bessel functions. In this paper, authors discuss Bessel functions of first kind and determine their Anuj transforms.

Keywords: Anuj Transform; Bessel Function; Inverse Anuj Transform; Laplace Transform; Fundamental Functions

Mathematics subject classification: 35A22; 44A05; 44A20

1. Introduction

Nowadays integral transform methods have various applications to solve the problems of Engineering and Science [1-2]. Researchers used different integral transform methods and solved various ordinary differential equations [3-4]; partial differential equations [5]; Volterra integral equations [6-21] and Volterra integro differential equations [22-37]. Aggarwal with different scholars [38-43] determined the Kamal; Mahgoub; Mohand; Aboodh; Elzaki and Sawi transforms of Bessel's functions. Priyanka and Aggarwal [44] used Rishi transform and determined the solution of the bacteria growth problem by developing its model using differential equation. Kumar et al. [45] determined the concentrations of the reactants of first order consecutive chemical reaction using Anuj transform. Kumar et al. [46] considered Anuj transformto determine the blood glucose concentration of a patient during continuous intravenous injection. The motive of the present paper is to determine the Anuj transform of Bessel functions of first kind of orders zero, one and two.

2. Nomenclature of symbols

- $\mathcal{F}$, family of piecewise continuous and exponential order function;
- $\mathcal{A}$, Anuj transform operator;
- $\mathcal{A}^{-1}$, inverse Anuj transform operator;
- $\in$, belongs to;
- !, the usual factorial notation;
- Γ, the classical Gamma function;

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- $\mathcal{L}$, Laplace transform operator;
- N, the set of natural numbers;
- R, the set of reals;
- $J_n(t)$, Bessel function of first kind of order n;
- $J_0(t)$, Bessel function of first kind of order zero;
- $J_1(t)$, Bessel function of first kind of order one;
- $J_2(t)$, Bessel function of first kind of order two

3. Definition of Anuj transform

If $H(t) \in \mathcal{F}, t \geq 0$ then the Anuj transform of $H(t)$ is defined as [21]

$$\mathcal{A}\{H(t)\} = r^2 \int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t}dt = h(r),\ r > 0 \tag{1}$$

4. Inverse Anuj Transform

The inverse Anuj transform of $h(r)$, denoted by $\mathcal{A}^{-1}\{h(r)\}$, is another function $H(t)$ having the characteristic that $\mathcal{A}\{H(t)\} = h(r)$.

5. Relation between Laplace and Anuj transforms

If $\mathcal{L}\{H(t)\} = \int_0^\infty H(t)e^{-rt}dt = \Psi(r)$, (2)

then $h(r) = r^2\Psi\left(\frac{1}{r}\right)$ (3)

and $\Psi(r) = r^2 h\left(\frac{1}{r}\right)$ (4)

Proof: Equation (1) gives

$$h(r) = r^2 \int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t}dt = r^2\left\{\int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t}dt\right\} = r^2\Psi\left(\frac{1}{r}\right)$$

Now equation (2) gives

$$\Psi(r) = \int_0^\infty H(t)e^{-rt}dt = r^2\left\{\frac{1}{r^2}\int_0^\infty H(t)e^{-rt}dt\right\} = r^2 h\left(\frac{1}{r}\right).$$

6. Properties of Anuj transform

In this part, we will describe the properties of Anuj transform that will be used in later section of this manuscript.

6.1. Linearity [46]

If $H_j(t) \in \mathcal{F},\ t \geq 0, j = 1,2,3,\dots\dots,n$ with $\mathcal{A}\{H_j(t)\} = h_j(r), j = 1,2,3,\dots\dots n$ then $\mathcal{A}\{\sum_{j=1}^n \ell_j H_j(t)\} = \sum_{j=1}^n \ell_j\, \mathcal{A}\{H_j(t)\} = \sum_{j=1}^n \ell_j\, h_j(r)$, where ℓ_j are arbitrary constants.

6.2. Change of Scale [46]

If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then $\mathcal{A}\{H(\ell t)\} = \frac{1}{\ell^3}h(\ell r)$, where ℓ is arbitrary constant.

6.3. Translation [46]

If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then

$\mathcal{A}\{e^{\ell t}H(t)\} = (1-\ell r)^2 h\left(\frac{r}{1-\ell r}\right)$, where ℓ is arbitrary constant.

6.4. Remark 1

Equations (3) and (4) can be use for establishing the further properties of Anuj transform.

7. Anuj transforms of the derivatives of a function [45]

If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then

$\mathcal{A}\{H'(t)\} = \frac{1}{r}h(r) - r^2H(0).$

$\mathcal{A}\{H''(t)\} = \frac{1}{r^2}h(r) - rH(0) - r^2H'(0).$

$\mathcal{A}\{H'''(t)\} = \frac{1}{r^3}h(r) - H(0) - rH'(0) - r^2H''(0).$

Remark 2: Tables 1-2 visualized the Anuj transforms and inverse Anuj transforms of fundamental functions respectively.

Table 1 Anuj transforms of fundamental functions [45]

S.N.	$\boldsymbol{H(t)} \in \boldsymbol{\mathcal{F}}, \boldsymbol{t} > 0$	$\boldsymbol{\mathcal{A}\{H(t)\} = h(r)}$
1	1	r^3
2	$e^{\ell t}$	$\left(\frac{r^3}{1-\ell r}\right)$
3	$t^\lambda, \lambda \in N$	$\lambda!\, r^{\lambda+3}$
4	$t^\lambda, \lambda > -1, \lambda \in R$	$r^{\lambda+3}\Gamma(\lambda+1)$
5	$sin(\ell t)$	$\left(\frac{\ell r^4}{1+r^2\ell^2}\right)$
6	$cos(\ell t)$	$\left(\frac{r^3}{1+r^2\ell^2}\right)$
7	$sinh(\ell t)$	$\left(\frac{\ell r^4}{1-r^2\ell^2}\right)$
8	$cosh(\ell t)$	$\left(\frac{r^3}{1-r^2\ell^2}\right)$

Table 2 Inverse Anuj transforms of fundamental functions [45]

S.N.	$h(r)$	$H(t) = \mathcal{A}^{-1}\{h(r)\}$
1	r^3	1
2	$\left(\frac{r^3}{1-\ell r}\right)$	$e^{\ell t}$
3	$r^{\lambda+3}, \lambda \in N$	$\frac{t^\lambda}{\lambda!}$
4	$r^{\lambda+3}, \lambda > -1, \lambda \in R$	$\frac{t^\lambda}{\Gamma(\lambda+1)}$

5	$\left(\dfrac{r^4}{1+r^2\ell^2}\right)$	$\dfrac{sin(\ell t)}{\ell}$
6	$\left(\dfrac{r^3}{1+r^2\ell^2}\right)$	$cos(\ell t)$
7	$\left(\dfrac{r^4}{1-r^2\ell^2}\right)$	$\dfrac{sinh(\ell t)}{\ell}$
8	$\left(\dfrac{r^3}{1-r^2\ell^2}\right)$	$cosh(\ell t)$

8. Bessel functions of first kind [38-43]

Bessel's function of first kind of order n, where $n \epsilon N$ is given by

$$J_n(t) = \frac{t^n}{2^n n!}\left\{1 - \frac{t^2}{2.(2n+2)} + \frac{t^4}{2.4.(2n+2)(2n+4)} - \frac{t^6}{2.4.6.(2n+2)(2n+4)(2n+6)} + \cdots \ldots\right\} \tag{5}$$

Bessel's function of first kind of zero order is given by

$$J_0(t) = \left\{1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots\right\} \tag{6}$$

Bessel's function of first kind of order one is given by

$$J_1(t) = \left\{\frac{t}{2} - \frac{t^3}{2^2.4} + \frac{t^5}{2^2.4^2.6} - \frac{t^7}{2^2.4^2.6^2.8} + \cdots ..\right\} \tag{7}$$

Equation (7) can also be written as

$$J_1(t) = \left\{\frac{t}{2} - \frac{t^3}{2^3.2!} + \frac{t^5}{2^5.2!.3!} - \frac{t^7}{2^7.3!.4!} + \cdots\right\} \tag{8}$$

Bessel's function of first kind of order is given by

$$J_2(t) = \left\{\frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} - \frac{t^8}{2^2.4^2.6^2.8.10} + \cdots \ldots\right\} \tag{9}$$

9. Relation between $J_0(t)$ and $J_1(t)$ [38]

$$\frac{d}{dt}[J_0(t)] = -J_1(t) \tag{10}$$

10. Relation between $J_0(t)$ and $J_2(t)$ [42]:

$$J_2(t) = J_0(t) + 2J_0''(t) \tag{11}$$

11. Anuj transform of Bessel functions of first kind

We find the Anuj transforms of $J_0(t), J_1(t), J_3(t), e^{kt}J_0(t), e^{kt}J_1(t), e^{kt}J_2(t), J_0(kt), J_1(kt)$ and $J_2(kt)$ in this section.

11.1. Anuj transform of Bessel function of first kind of order zero $J_0(t)$:

Operating Anuj transform on both sides of (6), we have

$$\mathcal{A}\{J_0(t)\} = \mathcal{A}\left\{1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots\right\} \tag{12}$$

Use of linearity property of Anuj transform on (12) gives

$$\mathcal{A}\{J_0(t)\} = \mathcal{A}\{1\} - \mathcal{A}\left\{\frac{t^2}{2^2}\right\} + \mathcal{A}\left\{\frac{t^4}{2^2.4^2}\right\} - \mathcal{A}\left\{\frac{t^6}{2^2.4^2.6^2}\right\} + \cdots.$$

$$\Rightarrow \mathcal{A}\{J_0(t)\} = r^3 - \frac{1}{2^2}2!\,r^5 + \frac{1}{2^2.4^2}4!\,r^7 - \frac{1}{2^2.4^2.6^2}6!\,r^9 + \cdots.$$

$$\Rightarrow \mathcal{A}\{J_0(t)\} = r^3\left[1 - \frac{1}{2}(r)^2 + \frac{1.3}{2.4}(r)^4 - \frac{1.3.5}{2.4.6}(r)^6 + \cdots.\right]$$

$$\Rightarrow \mathcal{A}\{J_0(t)\} = r^3\left[\frac{1}{\sqrt{1+r^2}}\right] = \left[\frac{r^3}{\sqrt{1+r^2}}\right] \tag{13}$$

11.2. Anuj transform of Bessel function of first kind of order one$J_1(t)$:

Operating Anuj transform on both sides of (10), we have

$$\mathcal{A}\left\{\frac{d}{dt}[J_0(t)]\right\} = -\mathcal{A}\{J_1(t)\} \tag{14}$$

Use of Anuj transform of derivatives of a function property in (14) gives

$$\left(\frac{1}{r}\right)\mathcal{A}\{J_0(t)\} - r^2J_0(0) = -\mathcal{A}\{J_1(t)\} \tag{15}$$

Use of (6) and (13) in (15) provides

$$\left(\frac{1}{r}\right)\left[\frac{r^3}{\sqrt{1+r^2}}\right] - r^2 = -\mathcal{A}\{J_1(t)\}$$

$$\Rightarrow \mathcal{A}\{J_1(t)\} = r^2 - \left(\frac{1}{r}\right)\left[\frac{r^3}{\sqrt{1+r^2}}\right]$$

$$\Rightarrow \mathcal{A}\{J_1(t)\} = r^2 - \left[\frac{r^2}{\sqrt{1+r^2}}\right]$$

$$\Rightarrow \mathcal{A}\{J_1(t)\} = \left[\frac{r^2\left(\sqrt{1+r^2}-1\right)}{\sqrt{1+r^2}}\right] \tag{16}$$

11.3. Anuj transform of Bessel function of first kind of order two$J_2(t)$:

Operating Anuj transform on both sides of (11), we have

$$\mathcal{A}\{J_2(t)\} = \mathcal{A}\{J_0(t) + 2J_0{''}(t)\} \tag{17}$$

Use of linearity property of Anuj transform on (17) gives

$$\mathcal{A}\{J_2(t)\} = \mathcal{A}\{J_0(t)\} + 2\mathcal{A}\{J_0{''}(t)\} \tag{18}$$

Use of Anuj transform of derivatives of a function property in (18) gives

$$\mathcal{A}\{J_2(t)\} = \mathcal{A}\{J_0(t)\} + 2\left[\frac{1}{r^2}\mathcal{A}\{J_0(t)\} - rJ_0(0) - r^2J_0{'}(0)\right] \tag{19}$$

Use of (6), (10) and (13) in (19) provides

$$\mathcal{A}\{J_2(t)\} = \left[\frac{r^3}{\sqrt{1+r^2}}\right] + 2\left[\frac{1}{r^2}\left[\frac{r^3}{\sqrt{1+r^2}}\right] - r + r^2J_1(0)\right] \tag{20}$$

Using (7) in (20), we have

$$\mathcal{A}\{J_2(t)\} = \left[\frac{r^3}{\sqrt{1+r^2}}\right] + 2\left[\left[\frac{r}{\sqrt{1+r^2}}\right] - r\right]$$

$$\Rightarrow \mathcal{A}\{J_2(t)\} = \left[\frac{r^3+2r\left(1-\sqrt{1+r^2}\right)}{\sqrt{1+r^2}}\right] \tag{21}$$

11.4. Anuj transform of $e^{kt}J_0(t)$:

From (13), we have

$$\mathcal{A}\{J_0(t)\} = \left[\frac{r^3}{\sqrt{1+r^2}}\right]$$

Using translation property of Anuj transform on above equation, we have

$$\mathcal{A}\{e^{kt}J_0(t)\} = (1-kr)^2\left[\frac{\left(\frac{r}{1-kr}\right)^3}{\sqrt{1+\left(\frac{r}{1-kr}\right)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{e^{kt}J_0(t)\} = \left[\frac{r^3}{\sqrt{(1-kr)^2+r^2}}\right] \tag{22}$$

11.5. Anuj transform of $e^{kt}J_1(t)$:

From (16), we have

$$\mathcal{A}\{J_1(t)\} = \left[\frac{r^2\left(\sqrt{1+r^2}-1\right)}{\sqrt{1+r^2}}\right]$$

Using translation property of Anuj transform on above equation, we have

$$\mathcal{A}\{e^{kt}J_1(t)\} = (1-kr)^2\left[\frac{\left(\frac{r}{1-kr}\right)^2\left(\sqrt{1+\left(\frac{r}{1-kr}\right)^2}-1\right)}{\sqrt{1+\left(\frac{r}{1-kr}\right)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{e^{kt}J_1(t)\} = \left[\frac{r^2\left(\sqrt{(1-kr)^2+r^2}-(1-kr)\right)}{\sqrt{(1-kr)^2+r^2}}\right] \tag{23}$$

11.6. Anuj transform of $e^{kt}J_2(t)$:

From (21), we have

$$\mathcal{A}\{J_2(t)\} = \left[\frac{r^3+2r\left(1-\sqrt{1+r^2}\right)}{\sqrt{1+r^2}}\right]$$

Using translation property of Anuj transform on above equation, we have

$$\mathcal{A}\{e^{kt}J_2(t)\} = (1-kr)^2\left[\frac{\left(\frac{r}{1-kr}\right)^3+2\left(\frac{r}{1-kr}\right)\left(1-\sqrt{1+\left(\frac{r}{1-kr}\right)^2}\right)}{\sqrt{1+\left(\frac{r}{1-kr}\right)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{e^{kt}J_2(t)\} = \left[\frac{r^3+2r(1-kr)\left\{(1-kr)-\sqrt{(1-kr)^2+r^2}\right\}}{\sqrt{(1-kr)^2+r^2}}\right] \quad (24)$$

11.7. Anuj transform of $J_0(kt)$:

From (13), we have

$$\mathcal{A}\{J_0(t)\} = \left[\frac{r^3}{\sqrt{1+r^2}}\right]$$

Using change of scale property of Anuj transform on above equation, we have

$$\mathcal{A}\{J_0(kt)\} = \frac{1}{k^3}\left[\frac{(kr)^3}{\sqrt{1+(kr)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{J_0(kt)\} = \left[\frac{r^3}{\sqrt{1+k^2r^2}}\right] \quad (25)$$

11.8. Anuj transform of $J_1(kt)$:

From (16), we have

$$\mathcal{A}\{J_1(t)\} = \left[\frac{r^2\left(\sqrt{1+r^2}-1\right)}{\sqrt{1+r^2}}\right]$$

Using change of scale property of Anuj transform on above equation, we have

$$\mathcal{A}\{J_1(kt)\} = \frac{1}{k^3}\left[\frac{(kr)^2\left(\sqrt{1+(kr)^2}-1\right)}{\sqrt{1+(kr)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{J_1(kt)\} = \frac{1}{k}\left[\frac{r^2\left(\sqrt{1+k^2r^2}-1\right)}{\sqrt{1+k^2r^2}}\right] \quad (26)$$

11.9. Anuj transform of $J_2(kt)$:

From (21), we have

$$\mathcal{A}\{J_2(t)\} = \left[\frac{r^3+2r\left(1-\sqrt{1+r^2}\right)}{\sqrt{1+r^2}}\right]$$

Using change of scale property of Anuj transform on above equation, we have

$$\mathcal{A}\{J_2(kt)\} = \frac{1}{k^3}\left[\frac{(kr)^3+2(kr)\left(1-\sqrt{1+(kr)^2}\right)}{\sqrt{1+(kr)^2}}\right]$$

$$\Rightarrow \mathcal{A}\{J_2(kt)\} = \frac{1}{k^2}\left[\frac{k^2r^2+2kr\left(1-\sqrt{1+k^2r^2}\right)}{\sqrt{1+k^2r^2}}\right] \quad (27)$$

12. Conclusion

In this paper, authors fruitfully obtained the Anuj transform of Bessel's functions of first kind of order zero, one and two i.e. $J_0(t)$, $J_1(t)$ and $J_3(t)$. Authors also obtained the Anuj transform of $e^{kt}J_0(t)$, $e^{kt}J_1(t)$, $e^{kt}J_2(t)$, $J_0(kt)$, $J_1(kt)$ and $J_2(kt)$ using translation and change of scale properties of Anuj transform. These results are important for determining the values of improper integrals containing Bessel's functions in integrand. Results of this paper can use in future study for determining the solutions of Bessel's equations.

Compliance with ethical standards

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Disclosure of conflict of interest

No conflict of interest to disclosed.

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