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Certain applications of *q*-sigmoid function to neighborhood of analytic functions

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Abstract

In the present investigation a *q*-sigmoid function will be introduced to improve some earlier results in geometric function theory. In particular modified *q*-sigmoid function *q*-Jack lemma will be applied to certain well defined neighborhood of analytic functions.

Keywords: Analytic functions; Sigmoid function; *q*-Sigmoid functions; Neighborhoods; Jack's lemma; q-Jack's lemma.

1. Introduction and Definitions

In recent research special functions like sigmoid functions have been generalized to produce *q*- sigmoid functions. Looking at it we have

Definition 1.1 *[6] [2] Sigmoid function which can be in the form*

$$G(s) = \frac{1}{1+e^{-s}} \quad , \qquad s \in \mathbb{R} \qquad (1.1)$$

$$G(s) = \frac{1}{1+e^{-s}} = \frac{1}{2} + \frac{s}{4} - \frac{s^3}{48} + \frac{s^5}{480} - \frac{17s^7}{80640} + \dots \qquad (1.2)$$

The modified sigmoid function is then defined as

$$\eta(s) = \frac{2}{1+e^{-s}} = 1 + \frac{s}{2} - \frac{s^3}{24} + \frac{s^5}{240} - \frac{17s^7}{40320} + \cdots \qquad (1.3)$$

In other to define q-sigmoid function consider some established result connection to this function.

Definition 1.2 *[3] Let q >* 0 *be any fixed real number and m a non-negative integer, the q -integer of r is of the form*

$$[m]_q \quad := \begin{cases} \dfrac{1-q^m}{1-q} & , q \neq 1 \\ m & , q = 1 \\ 0 & , m = 0 \end{cases} \qquad (1.4)$$

Definition 1.3 *[3] The q-fractional is defined as*

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$$[m]_q! := \begin{cases} [m]_q[m-1]_q...[1]_q \\ 1 \quad , m=0 \end{cases} \quad (1.5)$$

Definition 1.4 *[5], [7] A q-analogue of the ordinary exponential function* $e^s = \sum_{m=0}^{\infty} \frac{s^m}{m!}$

is of the form

$$e_q^s = \sum_{m=0}^{\infty} \frac{s^m}{[m]!} \quad (1.6)$$

Definition 1.5 *A q-Sigmoid function is defined as*

$$G_q(s) = \frac{1}{1+e_q^{-s}} \quad (1.7)$$

and the modified q-sigmoid function will be

$$\eta_q(s) = \frac{2}{1+e_q^{-s}} = 1 + \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} \left[\sum_{m=1}^{\infty} \frac{(-1)^m}{[m]_q!} s^m \right]^n \quad (1.8)$$

Definition 1.6

The q-difference operator:

For $q \in (0,1)$ *and* $f(z) \in$ A, *defined on a q-geometric set B , the of* $f(z)$ *is defined as q-differential* $D_{z,q}$

$$D_{z,q}f(z) := \begin{cases} \frac{f(z)-f(qz)}{z(1-q)} & (z \in \mathbb{C}/0) \\ f'(0), & (z=0) \end{cases} \quad (1.9)$$

that is $D_{z,q}f(z) = 1 + \sum_{m=2}^{\infty} [m]_q a_m z^{m-1}, \quad (z \neq 0)$ *where,* $[m]_q = \frac{1-q^m}{q-1}$ *and as q* $\to 1$, $[m]_q \to m$ *which gives* $f(z)$.

Let A be the class of functions defined by

$$f(z) = z + \sum_{m=1}^{\infty} a_m z^m \quad (1.10)$$

which is analytic in the open disk U = $\{z \in \mathbb{C}: |z| < 1\}$ satisfying the condition $f(0) = 0$ and $f'(0) = 1$.

Definition 1.7 *The function*

$$f_{\eta_q}(z) = z + \sum_{m=1}^{\infty} \eta_q(s) a_m z^m \quad (1.11)$$

is analytic and univalent in U *and belongs to the class* $A\eta_q$ *of the form (1.10) for* $\lim_{z\to\infty} \eta_q(z) = 2$.

Let $f_{\eta q}(z)$ *and* $h_{\eta q}(z) \in A\eta_q$, $h_{\eta q}(z)$ *is said to be* (ϑ,μ,η_q) *- neighbourhood for* $h_{\eta q}(z)$ *if*

it satisfies

$$\left|D_{z,q}f_{\eta_q}(z) - e^{i\vartheta}D_{z,q}h_{\eta_q}(z)\right| < \mu \quad , z \in \mathbb{U} \tag{1.12}$$

for some $-\pi < \vartheta < \pi$ *and* $\mu > \sqrt{2(1-\cos\vartheta)}$. *Denoted by* $(\vartheta,\mu,\eta_q) - N(h_{\eta q})$ *and also* $f_{\eta q} \in (\vartheta,\mu,\eta_q) - M(h_{\eta q})$ *if it satisfies*

$$\left|\frac{f_{\eta_q}(z)}{z} - \frac{e^{i\vartheta}h_{\eta_q}(z)}{z}\right| < \beta \quad , z \in \mathbb{U}$$

for some $-\pi < \alpha < \pi$ *and* $\mu > \sqrt{2(1-\cos\vartheta)}$.

Theorem 1.1 *If* $f_{\eta q} \in A\eta_q$ *satisfies*

$$\sum_{m=2}^{\infty}[m]_q\left|a_m - e^{i\vartheta}b_m\right| \le \frac{1}{\eta_q(s)}\left[\mu - \sqrt{2(1-cos\vartheta)}\right] \tag{1.14}$$

for some $-\pi < \vartheta < \pi$ *and* $\mu > \sqrt{2(1-\cos\vartheta)}$ *then* $f_{\eta q} \in (\vartheta,\mu,\eta_q) - N(h_{\eta q})$.

Proof 1 *Note that*

$$\begin{aligned}\left|D_{z,q}f_{\eta_q}(z) - e^{i\vartheta}D_{z,q}h_{\eta_q}(z)\right| &= \left|(1-e^{i\vartheta}) + \textstyle\sum_{m=2}^{\infty}\eta_q(s)[m]_q(a_m - e^{i\vartheta}b_m)z^{m-1}\right| \\ &\le \left|(1-e^{i\vartheta})\right| + \eta_q(s)\textstyle\sum_{m=2}^{\infty}[m]_q\left|a_m - e^{i\vartheta}b_m\right|\left|z^{m-1}\right| \\ &< \sqrt{2(1-cos\vartheta)} + \eta_q(s)\textstyle\sum_{m=2}^{\infty}[m]_q\left|a_m - e^{i\vartheta}b_m\right|\end{aligned}$$

from (1.12) we see that

$$\sum_{m=2}^{\infty}[m]_q\left|a_m - e^{i\vartheta}b_m\right| \le \frac{1}{\eta_q}\left[\mu - \sqrt{2(1-\cos\vartheta)}\right]$$

Thus $f_{\eta q} \in (\vartheta,\mu,\eta_q) - N(h_{\eta q})$.

Example

Given

$$h_{\eta_q}(z) = z + \sum_{m=1}^{\infty}\eta_q(s)b_m z^m \quad \in \mathbb{A}_{\eta_q}$$

we consider

$$f_{\eta_q}(z) = z + \sum_{m=1}^{\infty}\eta_q(s)a_m z^m \quad \in \mathbb{A}_{\eta_q}$$

with

$$a_m = \frac{\frac{1}{\eta_q(s)}\left[\mu - \sqrt{2(1-cos\vartheta)}e^{i\delta}q^{m-1}\right]}{[m]_q^2[m-1]_q} + e^{i\vartheta}b_m \qquad (-\pi \le \delta \le \pi,\ m = 2,3,4,\ldots)$$

It is useful to note that from (1.4) and 1.5)

$$\frac{1}{[m-1]_q} - \frac{1}{[m]_q} = \frac{[m]_q - [m-1]_q}{[m-1]_q[m]_q} = \frac{q^{m-1}}{[m-1]_q[m]_q}. \tag{1.15}$$

also without loss of generality,

$$\sum_{m=2}^{\infty}\left(\frac{1}{[m-1]_q}-\frac{1}{[m]_q}\right)=1 \quad . \quad (1.16)$$

Hence

$$\sum_{m=2}^{\infty}[m]_q\left|a_m-e^{i\vartheta}b_m\right| = \sum_{m=2}^{\infty}[m]_q\left|\frac{\frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]e^{i\delta}q^{m-1}}{[m]_q^2[m-1]_q}+e^{i\vartheta}b_m-e^{i\vartheta}b_m\right|$$
$$=\frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]\left(\sum_{m=2}^{\infty}\frac{q^{m-1}}{[m]_q[m-1]_q}\right)$$
$$=\frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]\left(\sum_{m=2}^{\infty}\frac{1}{[m-1]_q}-\frac{1}{[m]_q}\right)$$

therefore $f_{\eta q} \in (\alpha,\beta,\eta) - N(l_{\eta q}(z))$

Corollary 1.1 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$\sum_{m=2}^{\infty}\ [m]_q\left||a_m|-|b_m|\right| \ \leq\ \frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]$$

for some $(-\pi \leq \vartheta \leq \pi)$, $\mu > \sqrt{2(1-\cos\vartheta)}$ *and some* $\arg a_m - \arg b_m = \vartheta$ $(m = 2,3,4,\ldots)$

then $f_{\eta q}(z) \in ((\vartheta,\mu,\eta) - N(h_{\eta q}(z))$.

Proof: From (1.1) we see that

$$\sum_{m=2}^{\infty}\ [m]_q\left|a_m-e^{i\vartheta}b_m\right| \ \leq\ \frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]$$

implies that $f_{\eta q}(z) \in (\vartheta,\mu,\eta_q) - N(l_{\eta q}(z))$

suppose $arg\ a_m = \varrho_m$ then $arg\ b_m = \varrho_m - \vartheta$

Therefore $|a_m - e^{i\vartheta}b_m| \leq |a_m|e^{i\varrho m} - |b_m|e^{i\varrho m+i\vartheta} = ||a_m| - |b_m||$

implies $\left|a_m-e^{i\vartheta}b_m\right| \leq \left||a_m|-|b_m|\right|$

hence, $\sum_{m=2}^{\infty}\ [m]_q\left||a_m|-|b_m|\right| \ \leq\ \frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right].$

Theorem 1.2 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$\sum_{m=2}^{\infty}\ [m]_q\left|a_m-e^{i\vartheta}b_m\right| \ \leq\ \frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right] \quad (z\in\mathbf{U}) \quad , (1.17)$$

for some $-\pi \leq \vartheta \leq \pi$ *and* $\mu > \sqrt{2(1-\cos\vartheta)}$ *then* $f_{\eta q}(z) \in ((\vartheta,\mu,\eta_q) - M(h_{\eta q}(z))$.

Corollary 1.2 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$\sum_{m=2}^{\infty}[m]_q\left||a_m|-|b_m|\right| \leq \frac{1}{\eta_q(s)}\left[\mu-\sqrt{2(1-cos\vartheta)}\right]$$

for some $(-\pi \leq \vartheta \leq \pi)$ *and* $\mu > \sqrt{2(1-\cos\vartheta)}$ *and some* $\arg a_m - \arg b_m = \vartheta(m = 2,3,4,\ldots)$ *then* $f_{\eta q}(z) \in (\vartheta,\mu,\eta_q) - M(l_{\eta q}(z))$.

We will now give necessary conditions for neighbourhoods.

$- e^{i\vartheta}b_m) = (m-1)\varrho,$

Theorem 1.3 *If* $f_{\eta q}(z) \in (\vartheta,\mu,\eta_q) - N(h_{\eta q}(z))$ *and* $arg(a_m\ (m = 2,3,4,...)$ *then,*

$$\sum_{m=2}^{\infty} [m]_q \left|a_m - e^{i\vartheta} b_m\right| \le \frac{1}{\eta_q(s)} [\mu + cos\vartheta - 1].$$

Proof 2 *For* $f_{\eta q}(z) \in (\vartheta,\mu,\eta_q) - N(h_{\eta q})$*,we have*

$$\begin{aligned}\left|D_{z,q} f_{\eta_q}(z) - e^{i\vartheta} D_{z,q} h_{\eta_q}(z)\right| &= \left|(1 - e^{i\vartheta}) + \eta_q(s) \sum_{m=2}^{\infty} [m]_q (a_m - e^{i\vartheta} b_m) z^{m-1}\right| \\ &= \left|(1 - e^{i\vartheta}) + \sum_{m=2}^{\infty} \eta_q(s) [m]_q \left|a_m - e^{i\vartheta} b_m\right| e^{i(m-1)\varrho} z^{m-1}\right|\end{aligned}$$

$< \mu$ *for all* $z \in$ E.

Consider z *such that* $arg\ z = -\varrho$. *Then,* $z^{m-1} = |z|^{m-1} e^{-i(m-1)\varrho}$.

so from above $|a_m - ei\varrho b_m|ei(m-1)\varrho zm-1 = |a_m - ei\varrho b_m|ei(m-1)\varrho zm-1e-i(m-1)\varrho$

also $-e^{i\vartheta} = -\cos\vartheta - \sin\vartheta$ *and* $|z| = |x + iy| = \sqrt{x^2 + y^2}$. *For a point* $z \in$ U, *we see that*

$$\left|D_{z,q} f_{\eta_q}(z) - e^{i\vartheta} D_{z,q} h_{\eta_q}(z)\right| = \left|(1 - e^{i\vartheta}) + \sum_{m=2}^{\infty} \eta_q(s) [m]_q \left|a_m - e^{i\vartheta} b_m\right| |z|^{m-1}\right|$$

$$= |1 - \cos\vartheta - i \sin\vartheta + \sum_{m=2}^{\infty} \eta_q(s) [m]_q |a_m - e^{i\vartheta} b_m| |z|^{m-1}|$$

$$= \left(\left[1 + \eta_q(s) \sum_{m=2}^{\infty} [m]_q \left|a_m - e^{i\vartheta} b_m\right| |z|^{m-1} - cos\vartheta\right]^2 + sin^2\vartheta\right)^{\frac{1}{2}}$$

$$< \mu$$

for $z \in$ U.

Which implies that $(1 - cos\vartheta) + \eta_q(s) \sum_{m=2}^{\infty} [m]_q \left|a_m - e^{i\vartheta} b_m\right| |z|^{m-1} < \mu$ *for* $z \in$ U.

Letting $|z| \to 1$ *we have that*

$$\sum_{m=2}^{\infty} [m]_q \left|a_m - e^{i\vartheta} b_m\right| \le \frac{1}{\eta_q(s)} [\mu + cos\vartheta - 1].$$

Theorem 1.4 *Also if*

$f_{\eta q}(z) \in ((\vartheta, \mu, \eta_q) - N(h_{\eta q}(z))$

and $arg(a_m - e^{i\vartheta} b_m) = (m - 1)\varrho$, $(m = 2,3,4,...)$ *then*

$$\sum_{m=2}^{\infty} \left|a_m - e^{i\vartheta} b_m\right| \le \frac{1}{\eta_q(s)} [\mu + cos\vartheta - 1].$$

Application of q-Jack's lemma

Lemma 2.1 *[8] Let the function* $f(z)$ *be analytic in* U *with* $f(0) = 0$ *if a point* $z_o \in$ U *such that*

$max|z|\le|zo|\ |f(z)| = |f(zo)|$

then $z_o D_{z,q} f(z) = sf(z_o)$ *where s is real and* $s \ge 1$

Theorem 2.1 *If* $f_{\eta q}(z) \in$ A$_{\eta q}$ *satisfies*

$$\left|D_{z,q}f_{\eta_q}(z) - e^{i\vartheta}D_{z,q}h_{\eta_q}(z)\right| < 2\mu\eta_q(s) - \sqrt{2(1-cos\vartheta)} \quad , z \in \mathrm{U} \qquad (2.1)$$

for some $(-\pi \le \vartheta \le \pi)$ *and* $\mu > \frac{\sqrt{2(1-cos\vartheta)}}{2\eta_q(s)}$, *then*

$$\left|\frac{f_{\eta_q}(z)}{z} - e^{i\vartheta}\frac{h_{\eta_q}(s)}{z}\right| < \mu\eta_q(s) + \sqrt{2(1-cos\vartheta)}, z \in \mathbf{U}.$$

Proof 3 *We define* $w(z)$ *as*

$$\frac{1}{\eta_q(s)}\left(\frac{f_{\eta_q}(z)}{z} - e^{i\vartheta}\frac{h_{\eta_q}(z)}{z} - (1-e^{i\vartheta})\right) = \mu w(z).$$

which implies that $w(z)$ *is analytic in* U *and* $w(0) = 0$. *So,*

$$|D_{z,q}f_{\eta q}(z) - e^{i\vartheta} D_{z,q}h_{\eta q}(z)| = |(1 - e^{i\vartheta}) + \mu\, \eta_q(s)w(z)(1 + z\frac{D_{z,q}w(z)}{w(z)})|. \qquad (2.2)$$

Let $z_0 \in$ U *be such point that* $\max_{|z|\le|z1|} |w(z)| = |w(z_0)| = 1$ *by Lemma (2.1) and from equation (2.2) we have*

$w(z_0) = e^{i\vartheta}$ *and* $z_0\frac{D_{z,q}w(z_0)}{w(z_0)} = k \ge 1$

$$\begin{aligned}\Rightarrow \left|D_{z,q}f_{\eta_q}(z_0) - e^{i\vartheta}D_{z,q}h_{\eta_q}(z_0)\right| &= |(1-e^{i\vartheta}) + \mu\eta_q(s)e^{i\vartheta}(1+k)|\\ &\ge |e^{i\vartheta}|\ \mu\eta_q(s)(1+k) - |1-e^{i\vartheta}|\\ &\ge \mu\eta_q(s)(1+k) - \left|1-e^{i\vartheta}\right|.\end{aligned}$$

In particular, when $k = 1$

$$|D_{z,q}f_{\eta q}(z_0) - e^{i\vartheta}D_{z,q}h_{\eta q}(z_0)| \ge 2\mu\eta_q(s) - \sqrt{2(1-\cos\vartheta)}.$$

This is in contradiction to the condition in Theorem 2.1 hence we do not have $z_0 \in$ U *such*

that $|w(z_0)| = 1$. *It implies that* $|w(z)| < 1$ *for all* $z \in \mathbf{U}$. *So we have that*

$$\begin{aligned}\left|\frac{f\eta_q(z)}{z} - \frac{e^{i\vartheta}h_{\eta_q}(z)}{z}\right| &= \left|(1-e^{i\vartheta}) + \mu\eta_q(s)w(z)\right|\\ &\le |1-e^{i\vartheta}| + \mu\eta_q(s)|w(z)|\\ &< \mu\eta_q(s) + \sqrt{2(1-cos\vartheta)}.\end{aligned}$$

If we make $\vartheta = \frac{\pi}{2}$ in Theorem 2.1, we obtain the corollary below.

Corollary 2.2 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$\left|D_{z,q}f_{\eta_q}(z) - iD_{z,q}h_{\eta_q}(z)\right| < 2\mu\eta_q(s) - \sqrt{2}, \quad z \in \mathbf{U}, \qquad (2.3)$$

for some $\mu > \frac{1}{\eta_q(s)\sqrt{2}}$

then, $\left|\frac{f_{\eta_q}(z)}{z} - \frac{ih_{\eta_q}(z)}{z}\right| < \mu\eta_q(s) + \sqrt{,2} \qquad z \in \mathbf{U}.$

Theorem 2.2 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$Re\left(\left(D_{z,q}f_{\eta_q}(z) - e^{i\alpha}D_{z,q}h_{\eta_q}(z)\right)\right) > \frac{1}{\eta\ (z)}(1 - cos\alpha) - \frac{3\beta}{4}\ , z \in \mathbb{E}$$

for some $-\pi \leq \alpha \leq \pi$, *then*

$$Re\left(\frac{f_{\eta q(z)}}{z} - \frac{e^{i\alpha}h_{\eta q}(z)}{z}\right) > \frac{1}{\eta_q(z)}\ (1 - \cos\alpha) - \frac{\beta}{2}\ , \quad z \in \mathbb{E}\,.$$

Cor[illegible]y 2.3 *If* $f_{\eta q}(z) \in A_{\eta q}$ *satisfies*

$$Re\left(D_{z,q}f_{\eta_q}(z) - iD_{z,q}h_{\eta_q}(z)\right) > \frac{1}{\eta_q(z)} - \frac{3\beta}{4}$$

for some $\beta > 0$ *then,*

$$Re\left(\frac{h_{\eta_q}(z)}{z} - \frac{e^{i\alpha}l_{\eta_q}(z)}{z}\right) > \frac{1}{\eta_q(z)} - \frac{\beta}{2}.$$

Furthermore, if $\beta = 2(1 - \tau)$ $(0 \leq \tau \leq 1)$ *then,*

$$Re\left(D_{z,q}f_{\eta_q}(z) - iD_{z,q}h_{\eta_q}(z)\right) > \frac{1}{\eta_q(z)} - \frac{3}{2}(1 - \tau)$$

implies that

$$Re\left(\frac{f_{\eta_q}(z)}{z} - \frac{ih_{\eta_q}(z)}{z}\right) > \frac{1}{\eta_q(z)} + \tau - 1, \qquad z \in \mathbb{E}$$

Conclusion

In this article, we showed that if $f_{\eta_q}(z)$ satisfies $\sum_{m=2}^{\infty}[m]_q\ |a_m - e^{i\vartheta}\ b_m\ z^{m-1} \leq \frac{1}{\eta_q(s)}\left[\mu - \sqrt{2(\ 1 - cos\vartheta)}\right]$ and $\sum_{m=2}^{\infty}[m]_q \mid |a_m| - \ |b_m| \mid\ \leq \frac{1}{\eta_q(s)}\left[\mu - \sqrt{2(\ 1 - cos\vartheta)}\right]$ then it belongs to the neighborhoods $(\vartheta,\mu,\eta_q) - N\left(h_{\eta_q}(z)\right)$ *and* $(\vartheta,\mu,\eta_q) - M\left(h_{\eta_q}(z)\right)$ hence $\sum_{m=2}^{\infty}[m]_q\ \left||a_m| - |b_m|\right| \leq \frac{1}{\eta_q(s)}\left[\mu - \sqrt{2(\ 1 - cos\vartheta)}\right]$. We then applied q-Jack lemma and showed that $Re\left\{\frac{f_{\eta_q}(z)}{z} - \frac{ih_{\eta_q}(z)}{z}\right\} >\ 1 +\ \eta_q(s)(1 - \tau)$.

Compliance with ethical standards

Disclosure of Conflict of interest.

Author declares no competing interest.

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