

## Assessment of Slope Stability by the Fellenius Slice Method: Analytical and numerical approach

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### Abstract

Slope stability is a topic of great importance in civil engineering, as slope failures can cause considerable damage to infrastructure and downstream properties. In this study, we applied the Fellenius slice method, using both the analytical method, as well as the numerical method, which was performed using the SLOPE/W module of the Geostudio software. The results obtained by both methods showed that the increase in soil cohesion improves the stability of the slope. The safety coefficients obtained by the analytical method vary between 0.534 and 1.086, while those obtained by the numerical method vary between 0.539 and 1.096, for cohesion values ranging from 4 kPa to 20 kPa. The safety coefficients obtained by the analytical and numerical methods follow straight lines of equation  $y=0.0344x+0.4092$  and  $y=0.0345x+0.4169$ , respectively. The results of the analytical method show that a safety factor of 1.5 is achieved at a value of 32 kPa for the cohesion, while the numerical method shows a safety factor of 1.5 achieved at a value of 32 kPa for the cohesion. The difference between the two cohesions is explained by the smaller number of slices used in the analytical method. Nevertheless, the equation from the analytical method can be used as a general guide to evaluate the evolution of the safety coefficient of a slope with surcharge under long-term behavior with incremental cohesion, but this does not exclude verification by dedicated software based on the finite element method.

**Keywords:** Slope; Stability; Safety Factor; Cohesion

### 1. Introduction

Slope stability is a crucial topic in civil engineering because, as pointed out by (1),(2),(3),(4),(5) Slope failures can cause considerable damage to downstream infrastructure and properties. Slope-stability analysis is one of the parameters in the design of road embankments that the designer must consider in order to ensure stable and safe construction (6). To improve the understanding of embankment stability, studies have been carried out, such as the analysis of embankment stability by the finite element method (7),(8) (Zhang et al., 2021), research on slope stability in geotechnical engineering and a GIS-based study for landslide susceptibility mapping (9).

In addition, several other sources confirm the importance for engineers to have slope stability analysis tools. For example, (Slopes, 2014),(10) highlight that slope stability analyses are a key element in ensuring the safety of people and infrastructure. Similarly, (11),(12) indicate that slope stability analysis tools are essential for assessing the risk of ground movement and designing appropriate protective measures. These tools include analytical and numerical methods, such as limit analysis and finite element method.

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Analytical methods are a key element in assessing slope stability. These methods are based on mathematical equations derived from soil mechanics and are particularly suitable for the analysis of simple, homogeneous slopes. However, other sources emphasize the importance of numerical methods for dealing with more complex situations, such as the effects of runoff and erosion on slope stability(13),(14).

Analytical methods are often used for simple and homogeneous slopes, but have limitations for complex and heterogeneous slopes. According to Bishop et al (2000), numerical methods are preferred in these cases as they use finite element models to simulate the behavior of soils and rocks under real loading conditions, thus allowing a more accurate analysis of slope stability.

It is important to note that numerical methods can be costly in terms of time and resources and their reliability can be affected by uncertainties in the modelling parameters and site conditions. Indeed, numerical methods often require a significant amount of data and computational time to be implemented in a meaningful way. Therefore, it is important for engineers to understand the limitations and assumptions of these methods when analyzing slope stability.

The use of simulation software for slope stability analysis can introduce potential errors that need to be taken into account. Errors can be caused by incorrect data entry, inappropriate parameter selection or misinterpretation of results. Therefore, users should be aware of these errors and take precautions to minimize their impact on the final results.

It is very important to compare the results of simulation software with those of analytical and numerical methods to ensure the reliability of the results. This comparison allows the detection of manipulation errors such as incorrect data input, incorrect parameter settings or misinterpretations of results, which can lead to incorrect conclusions. By ensuring the reliability of the results, engineers can make more informed decisions to ensure the stability of slopes and the safety of people and property downstream.

### *Objectives*

The objective of this study is to improve the safety of infrastructure and downstream properties by developing reliable and accurate slope stability analysis methods, taking into account both analytical and numerical methods, as well as potential sources of error in the use of modelling software.

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## **2. Material and methods**

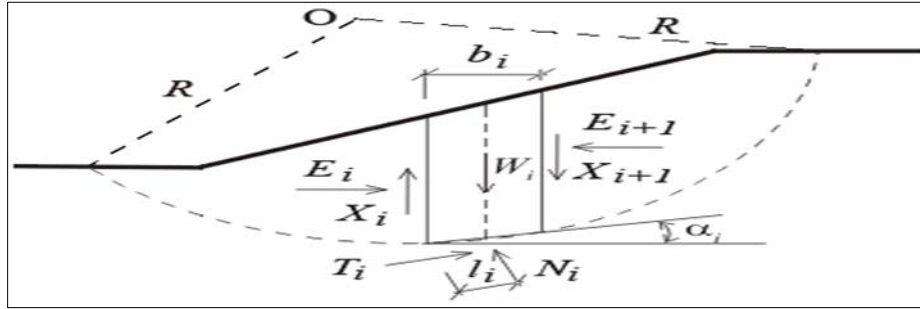
To conduct our study on slope stability, we chose to use the Limit Equilibrium Method, which is considered reliable and easy to use by many engineers(15),(16),(17),(9),(18),(19) . In this method, we applied the slice method due to the heterogeneity of the layers in our study case, which is composed of two materials: the embankment and the foundation soil. The slice method consists of dividing the embankment into several slices and then analysing the forces acting on each of these slices. The Fellenius slice method(20),(21)is one of the most commonly used analytical methods for slope stability analysis. It consists of dividing the slope into several vertical slices, whose equilibrium is analysed using the equations of soil mechanics. This method is particularly useful for heterogeneous slopes(22). This method is particularly useful for heterogeneous slopes (Imanzadeh et al., 2015), as it allows for variations in soil properties along the surface of the slope to be taken into account. Limit equilibrium method divides the soil in potential sliding surface into several blocks, and calculates stability coefficient by establishing static equilibrium equation and torque equilibrium equation of each block and whole sliding surface based on Mohr-Coulomb yield criterion(23) . For this purpose, we used the Fellenius method, both analytically using MS Excel, and numerically using the specialised software Geostudio(24) and more specifically its Slope/W module.

The slice method is widely used to analyse slope stability, especially for complex and heterogeneous slopes, as it takes into account the variations in soil properties along the slope surface by dividing it into several slices.

### **2.1. Fellenius / Petterson method**

#### *2.1.1. Assumptions (25):*

- 1) For each slice, the resultant of the interslice forces is zero.
- 2) The resultants of  $E_i$  and  $X_i$  are equal to the resultants of  $E_{i+1}$  and  $X_{i+1}$  , also their lines of actions coincide.



**Figure 1** The slice with applied forces

According to 1) and 2):  $E_i = E_{i+1} = X_i = X_{i+1} = 0$

$$N_i = W_i \cos \alpha_i ; T_i = W_i \sin \alpha_i$$

The normal stress resulting from the normal force  $N_i$  is:

$$\sigma_i = \frac{N_i}{l_i}$$

$$\sigma_i = \frac{W_i \cos \alpha_i}{l_i} \quad \text{And the tangential stress arising from } T_i \text{ is } \tau_i = \frac{W_i \sin \alpha_i}{l_i}$$

$$\tau_i = c_i + (\sigma_i - u_i) \tan \varphi_i \quad (\text{Long-term behavior: effective stress})$$

$$\tau_i = c_i + (\sigma_i - u_i) \tan \varphi_i$$

$$\tau_i = c_i + \left( \frac{W_i \cos \alpha_i}{l_i} - u_i \right) \tan \varphi_i$$

$$\tau_i = c_i + \frac{(W_i \cos \alpha_i - u_i l_i) \tan \varphi_i}{l_i}$$

$$\tau_i = \frac{c_i l_i + (W_i \cos \alpha_i - u_i l_i) \tan \varphi_i}{l_i}$$

The expression for the safety factor is as follows:

$$FS = \frac{\frac{c_i l_i + (W_i \cos \alpha_i - u_i l_i) \tan \varphi_i}{l_i}}{\frac{W_i \sin \alpha_i}{l_i}}$$

$$FS = \frac{c_i l_i + (W_i \cos \alpha_i - u_i l_i) \tan \varphi_i}{W_i \sin \alpha_i}$$

The general expression for the safety factor for all bands is:

$$FS = \frac{\sum c_i l_i + (W_i \cos \alpha_i - u_i l_i) \tan \varphi_i}{\sum W_i \sin \alpha_i} \quad \text{where } i \text{ is the number of the slice.}$$

The previously obtained safety coefficient does not take into account the effect of surcharges on the slope. In our case study, there is a surcharge  $\Delta\sigma_i$  on the slope. This surcharge has been converted into a volume load and considered as a weight surcharge on all the sections where surcharges are present.

The expression of the safety coefficient becomes under this condition:

$$FS = \frac{\sum c_i l_i + (W_i + \Delta\sigma_i)(\cos \alpha_i - u_i l_i) \tan \varphi_i}{\sum W_i \sin \alpha_i}$$

Taking into account geotextiles and surcharges, the safety coefficient of the slope gives:

$$FS = \frac{\sum c_i l_i + (W_i + \Delta \sigma_i)(\cos \alpha_i - u_i l_i) \tan \varphi_i}{\sum (W_i \sin \alpha_i + E_i')}$$

$E_i'$  being the load of the geotextile on the slope

### 2.2. Case study

Analytical and numerical analysis techniques were used to assess the stability taking into account the additional loads. The geotechnical characteristics as well as the geometrical configuration of the slope will be presented later.

**Table 1** Geotechnical parameters

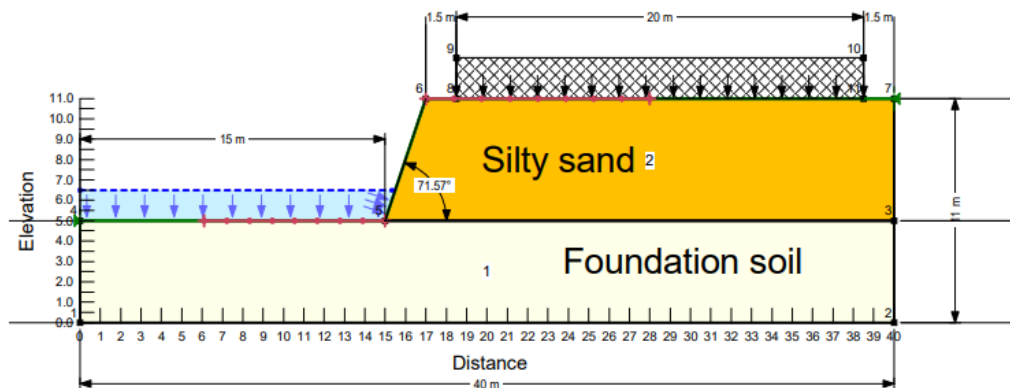
Soil type	Weight by volume	Cohesion	Friction angle
Silty sand	$\gamma = 18 \text{KN/m}^3$	$C = 04 \text{kpa}$	$\varphi = 25^\circ$
Foundation soil	$\gamma = 19 \text{KN/m}^3$	$C = 09 \text{kpa}$	$\varphi = 24.5^\circ$

With the following calculation assumptions:

- Stable foundation soil 5m deep;
- Addition of 6m high silty sand backfill;
- The inclination of the slope is  $71.57^\circ$ ;
- The water level is 1.5m above the ground level

Uniformly distributed load  $Q = 25 \text{ kpa}$  at the head of the slope with an offset of 1.5m from the edge

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion' (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
	Silty sand	Mohr-Coulomb	18	20	25	0	1



**Figure 2** Geometry of slope system

### 3. Results and discussion

#### 3.1. Analytical method

**Table 2** Summary of preliminary calculations

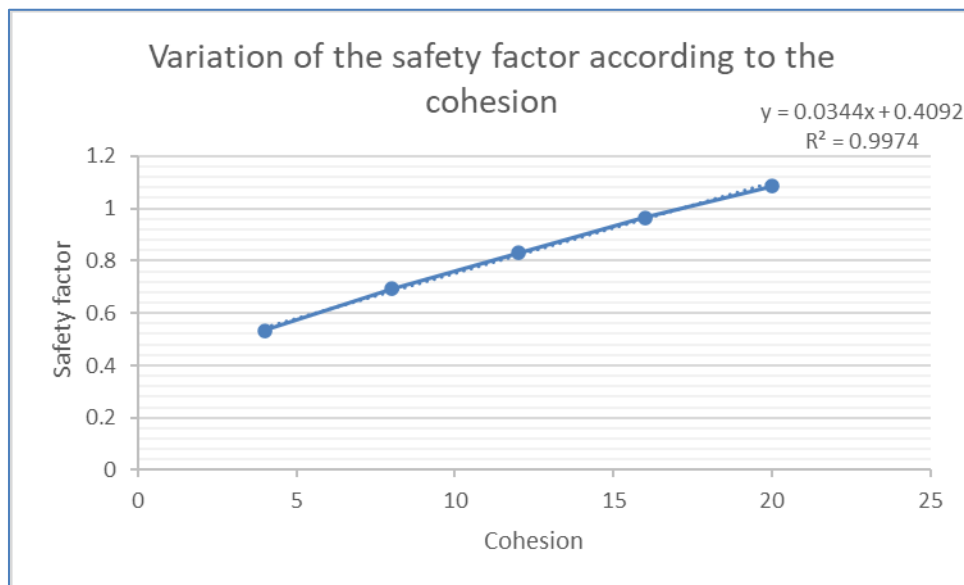
No. Slice	$N= W \cos \alpha$	$T=W \sin \alpha$	$c*b$	$N \cos \alpha -ul$	Tan(phi)	
1	-18,9277	21,146311	4,8	-18,9277	-0,733322	18,680094
2	-69,94578	-33,14134	4,8	-69,94578	-0,733322	56,092751
3	-83,04335	-0,735071	4,8	-94,81129	-0,733322	74,327167
	-5,6934	0,050394	4,8	2,075	-0,733322	3,2783576
4	22,181095	-25,22686	4,8	25,951966	-0,733322	-14,23114
5	1,639698	11,878967	4,8	-21,25112	-0,733322	20,383909
6	-4,2084	0	4,8	-16,94	-0,733322	17,222469
7	6,4789224	5,5459148	4,8	-2,461389	-0,733322	6,6049897
		20,481685				10,941516

The table below shows the variation of the safety coefficient as a function of cohesion using the analytical method

**Table 3** Variation of the safety coefficient as a function of cohesion by the analytical method

Cohesion	C= 4kPa	C= 8kPa	C= 12kPa	C=16kPa	C=20 kPa
Safety factor according to Fellenius	0,534	0,694	0,83	0,966	1,086

The curve below shows the evolution of the safety factor as a function of cohesion



**Figure 3** Curve showing the variation of the safety coefficient as a function of cohesion using the analytical method

Analyzing the results obtained by the analytical method, it can be seen that the function obtained for the variation of the safety coefficient as a function of the cohesion of the slope soil is a linear and increasing function, with a value of

$R2=0.9974$ . This function is expressed by the equation  $y=0.0344x+0.4092$ . According to this function, to achieve a safety coefficient of 1.5(3) for this slope, a cohesion of 31.71, or 32 kPa, would be required.

In general, if the factor of safety of a slope is within the interval between 0 and 1.0, the slope is actively unstable. The value over 1.0 indicates that the slope is considered stable (26),(12).

### 3.2. Numerical method

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion' (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
□	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
■	Silty sand	Mohr-Coulomb	18	4	25	0	1

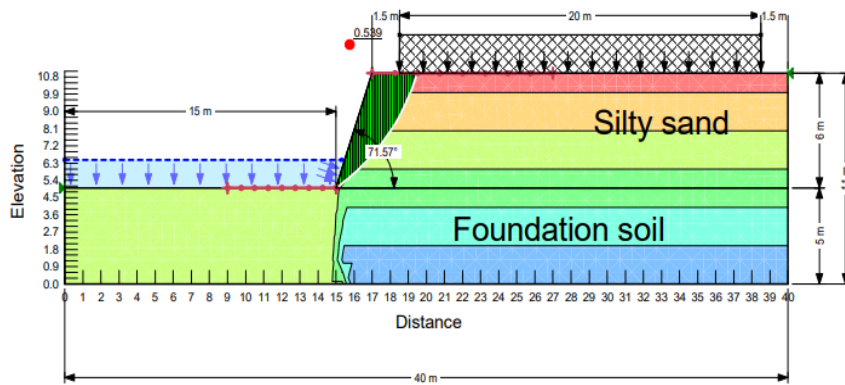


Figure 4 Safety factor FS =0.539 of the slope with cohesion C=4 kPa

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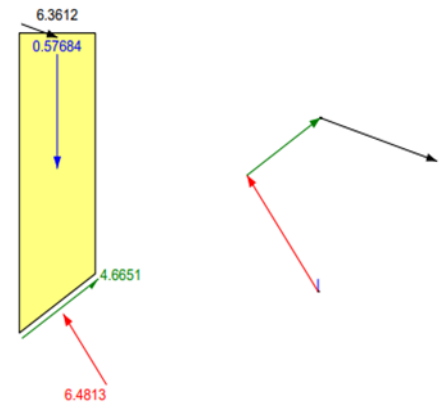


Figure 5 Free-body diagram of Unit 1 with cohesion C=4kPa

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion' (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
□	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
■	Silty sand	Mohr-Coulomb	18	8	25	0	1

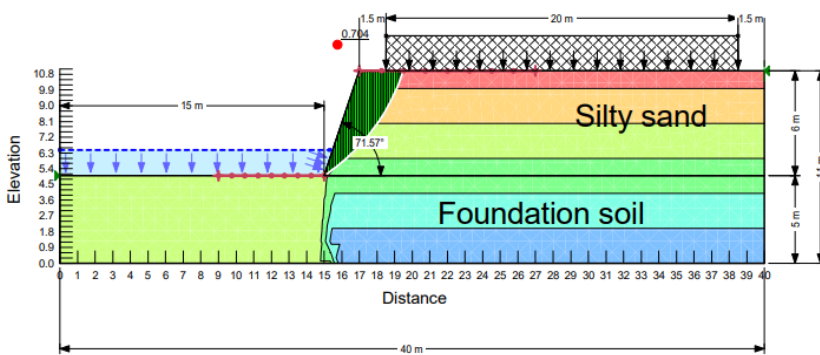


Figure 6 Safety factor FS =0.704 of the slope with cohesion C=8 kPa

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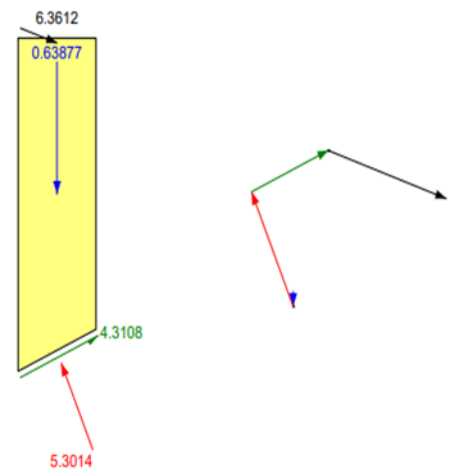


Figure 7 Free-body diagram of Unit 1 with cohesion C=8kPa

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion' (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
□	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
■	Silty sand	Mohr-Coulomb	18	12	25	0	1

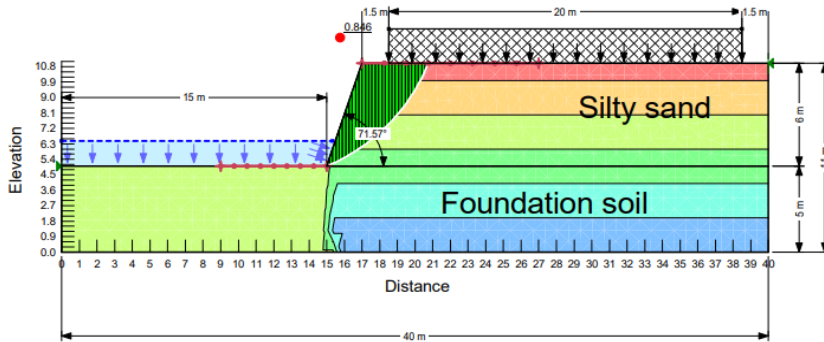


Figure 8 Safety factor FS =0.846 of the slope with cohesion C=12 kPa

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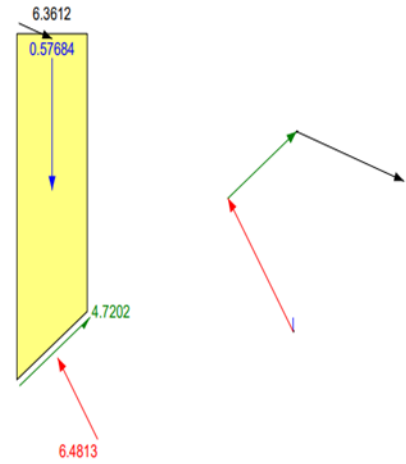


Figure 9 Free-body diagram of Unit 1 with cohesion C=12 kPa

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion' (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
□	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
■	Silty sand	Mohr-Coulomb	18	16	25	0	1

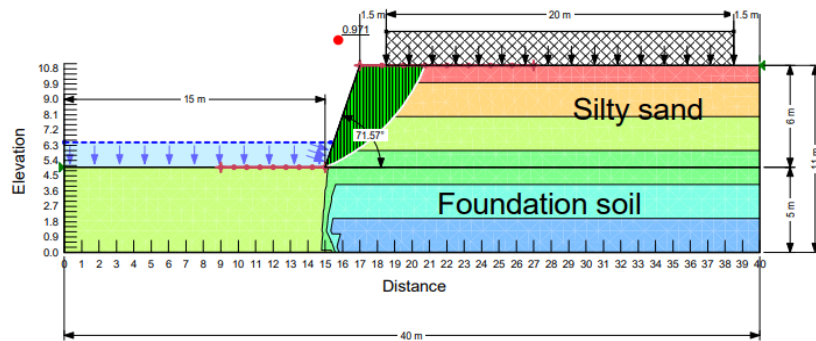


Figure 9 Safety factor FS =0.971 of the slope with cohesion C=16 kPa

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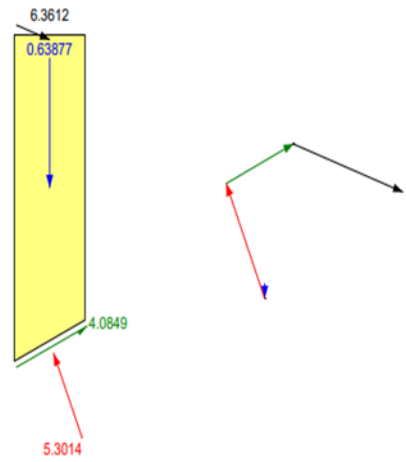


Figure 10 Free-body diagram of Unit 1 with cohesion C=16 kPa

Couleur	Nom	Modèle	Poids volumique (kN/m <sup>3</sup> )	Cohésion (kPa)	Phi' (°)	Phi-B (°)	Ligne piézométrique
	Foundation soil	Mohr-Coulomb	19	9	24.5	0	1
	Silty sand	Mohr-Coulomb	18	20	25	0	1

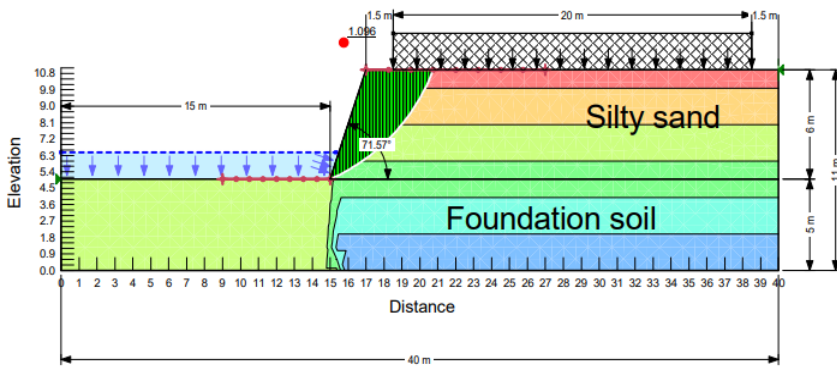


Figure 11 Safety factor FS =1.096 of the slope with cohesion C=20 kPa

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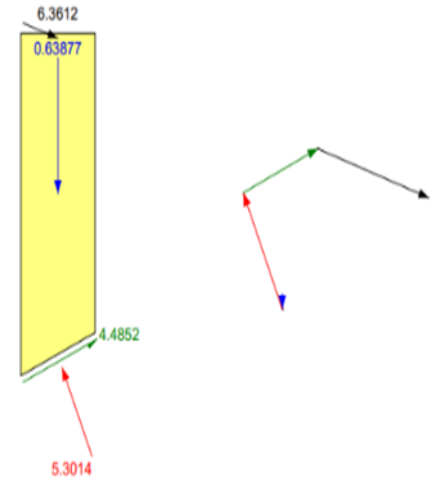


Figure 12 Free-body diagram of Unit 1 with cohesion C=20 kPa



Figure 12 Variation of the safety coefficient as a function of cohesion by the numerical method

The function  $y=0.0345x+0.4169$  obtained from the numerical analysis of the slope shows an increasing linear relationship between the safety coefficient and the soil cohesion. This relationship is confirmed by the studies of (27) ,(12) which show that the safety coefficient increases with increasing soil cohesion. Finally, the study (28) reviewed the effect of vegetation on slope stability and pointed out that vegetation can improve soil cohesion and thus contribute to slope stability.

Based on this function, it can be deduced that a safety factor of 1.5 for this slope would require a cohesion of 31.39 or 32 kPa, which is consistent with the results obtained by the analytical method.

The results obtained in this study highlight the crucial importance of soil cohesion in the stability of slopes



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#### 4. Conclusion

In conclusion, this study verified the stability of an artificial slope with surcharge in the presence of a water table using analytical and numerical methods. The safety coefficients obtained showed that soil cohesion has a significant impact on the stability of the slope. The results of the analytical and numerical method showed a difference due to the number of slices used in the analytical method. The results obtained can be useful for the design and construction of artificial slopes taking into account the surcharge conditions and the water table.

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#### Compliance with ethical standards

##### *Disclosure of conflict of interest*

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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