- Multi Objective Optimization Ratio Analysis (MOORA)
- Analytical Hierarchy Method (AHP)
- Analytical Network Method ANP etc.

2.2. Multi objective optimization ratio analysis (MOORA)

The MOORA method which was introduced by Brauers (Brauers, 2006) is such a multi objective optimization technique that can be successfully applied to solve various types of MCDM problems.

2.2.1. Algorithm of MOORA method under MCDM

The MOORA method starts with a matrix of responses (performance measures) of different alternatives on different criteria (objectives or attributes). The matrix is shown below (Equation 1).

$$\begin{array}{cc} & \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_n \end{array} \\ X = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} & \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where x_{ij} is the performance rating (response) to the *i*th alternative (A_i) under *j*th criterion (C_j). *m* is the number of alternatives and *n* is the number of criteria.

The MOORA method employs a ratio system in which each response of an alternative on an attribute (criterion) is compared to a denominator. The denominator is a representative for all alternatives concerning that attribute (Brauers et al. 2007; Kalibatas and Turskis, 2008).

Brauers et al. (2008) considered various ratios such as the square root of the sum of squares of each alternative per objective, total ratios, Scharlig ratios, Weitendorf ratios, Jutter ratios, Stop ratios, Van Delft and Nijkamp ratios of maximum value, Korth ratios, Peldschus *et al*. and Peldschus ratios for nonlinear normalization. They concluded that the square root of the sum of squares of each alternative per objective is the best one for the denominator which is given below.

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}\left(x_{ij}^{2}\right)}} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

x_{ij}^{*} is normalized value of response *i* with respect to attribute *j*. In the current research work, the maximum score under each attribute has also been used as the denominator of the ratio system and an effort has been made to exhibit that this ratio system is also suitable for finding the optimal solution. The following ratio system is the second best for normalization process in MOORA.

$$x_{ij}^{*} = \frac{x_{ij}}{\max\limits_{i}\left(x_{ij}\right)} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

For the computation of normalized response using the above Eq. (2b), first the maximum score under each attribute is found. Then all the scores under certain attribute irrespective of benefit or non-benefit are divided by the concerned maximum score using Eq. (2b). x_{ij}^{*} is a dimensionless quantity in the interval [0,1] representing the normalized score of alternative *i* on attribute *j*. However, sometimes the interval could be [-1; 1]. For example, in the case of productivity growth of some factories, industries, sectors, regions or countries may be negative instead of positive thus the interval becomes [-1;1] (Brauers *et al.*, 2008).

For multi-objective optimization these normalized performances are added in case of maximization and subtracted in case of minimization. Then the optimization problem becomes

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^* \quad \text{......................(4)}$$

Where g is the number of benefit criteria to be maximized and $(n-g)$ is the number of non-benefit criteria to be minimized. y_i^* is final score of i^{th} alternatives with respect to all the attributes. In the above case it is assumed that all the attributes are of same importance.

In most of the real-life problems different weights are given to the attributes as per their relative importance. When the weights of attributes are taken into consideration then the Eq. 5 can be expressed as follows.

$$y_i^* = \sum_{j=1}^{g} w_j * x_{ij}^* - \sum_{j=g+1}^{n} w_j * x_{ij}^* \quad \text{...........(5)}$$

Where w_j^* is the weight of jth attribute (criterion), which can be evaluated using any well-known approach either AHP or Entropy method. The value of y_i^* may be positive, negative or zero. These y_i^* values are arranged in descending order. The best alternative is one which is associated with highest y_i^* value and the worst alternative is one which is associated with the lowest y_i^* value.

2.3. Simple additive weighting (saw)

2.3.1. Step 1 Formation of decision matrix

Criterion outcomes of decision alternatives can be collected in a table called Decision Matrix comprised of a set of columns and rows. The matrix rows represent decision alternatives, with matrix columns representing criteria. A value found at the intersection of row and column in the matrix represents a criterion outcome - a measured or predicted performance of a decision alternative on a criterion. The decision matrix is a central structure of the MCDA/MCDM since it contains the data for comparison of decision alternatives.

$$\begin{array}{cc} & \begin{array}{ccccc} C1 & & CJ & & Cn \end{array} \\ X = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} & \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad \text{..................(6)}$$

x_{ij} is the performance rating of alternative ***i*** with respect to criterion ***j***,

A_j is i^{th} alternative, C_j is the j^{th} criterion

2.3.2. Step 2 Formation of Weight Matrix

Different importance weights to various criteria may be awarded by the decision makers. These importance weights forms the weight as follows.

$$W = [W_1 \cdots \quad W_j \ \ldots \quad W_n] \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

2.3.3. Step 3 Normalization of performance rating

Units and dimensions of performance ratings of columns under criteria differ. For the purpose of comparison, these performance ratings are converted into dimensionless units by normalization using following equations

$$\bar{x}_{ij} = \frac{x_{ij}}{\max\limits_{i}(x_{ij})} \text{ for benefit criteria } \boldsymbol{j} \ \ldots\ldots\ldots\ldots\ldots (8)$$

$$\bar{x}_{ij} = \frac{\min\limits_{i}(x_{ij})}{x_{ij}} \text{ for non-benefit criteria } \boldsymbol{j} \ \ldots\ldots\ldots\ldots (9)$$

Normalized decision matrix

$$\bar{X} = \begin{array}{c} A_1 \\ \vdots \\ A_2 \\ \vdots \\ A_m \end{array} \begin{bmatrix} \bar{x}_{11} \cdots & \ldots \bar{x}_{1j} \ldots & \bar{x}_{1n} \\ \vdots & \vdots & \vdots \\ \bar{x}_{i1} \ldots & \ldots \bar{x}_{ij} \ldots & \bar{x}_{in} \\ \vdots & \vdots & \vdots \\ \bar{x}_{m1} & \bar{x}_{mj} & \bar{x}_{mn} \end{bmatrix}_{m \times n} \ \ldots\ldots\ldots\ldots (10)$$

Step 4 composite score

Computation of composite score (CS_i) for alternative *i*

$$CS_i = \sum_{j=1}^{n} \left(\overline{w}_j * \bar{x}_{ij} \right)$$

2.3.4. Step 5 Ranking and selection of best alternative

Ranking of products in descending order of composite scores (CS_i).

2.4. Entropy

Entropy was originally a thermodynamic concept, first introduced into information theory by Shannon (see Shannon, 1948 [21]). It has been widely used in the engineering, socioeconomic and other fields. According to the basic principles of information theory, information is a measure of system's ordered degree, and the entropy is a measure of system's disorder degree.

2.4.1. Step1 Calculate p_{ij} (the ith scheme's jth indicator value's proportion).

$p_{ij} = r_{ij} / \sum_{j=1}^{m} r_{ij}$, r_{ij} is the ith scheme's jth indicator value.

2.4.2. Step2 Calculate the jth indicator's entropy value

e_j. $e_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij}$, $k = 1/\ln m$, m is the number of assessment schemes.

2.4.3. Step3 Calculate weight wj (jth indicator's weight).

$$w_j = (1 - e_j) / \sum_{j=1}^{n} (1 - e_j), \ n \text{ is the number of indicators,}$$

$$\text{and } 0 \le w_j^{\prime} \le 1, \textstyle\sum_{j=1}^{n} w_j = 1.$$

In entropy method, the smaller the indicator's entropy value ej is, the bigger the variation extent of assessment value of indicators is, the more the amount of information provided, the greater the role of the indicator in the comprehensive evaluation, the higher its weight should be. [Table:2]

2.5. Sensitivity analysis

In actual situation decision-making is rather dynamic process not static. It varies in the continuous changing environment. In reality the value of decision-making attitude depends upon decision maker's personal choice. Under such circumstances decision making attitude behaves as a variable that may yield different results. Keeping it in mind, the proposed model for the selection of magnesium alloy has been enhanced by sensitivity analysis [Fig:2] to provide a readymade solution of the current problem under variable decision-making attitude. The governing equation of the material measure (AM) is given by

$$AM_i = \alpha(OFM_i - SFM_i) + SFM_i \text{ …………… (11)}$$

where, i = 1, 2…m.
OFM_i = Objective factor measure for the alternative i
SFM_i = Subjective factor measure for the alternative i
α = Objective factor decision weight/Coefficient of attitude

3. Material

The selection of piping elements of stainless steel on the functional requirement, manufacturing capabilities, cost and customer requirement. The problem involves identification of different stainless steel [Table: 1] that are used in the manufacturing of pipe and to select the best among them. Similar properties of all materials are tabulated in Tab. 1. Five stainless steels with seven important properties are considered. The decision maker has to compare all the materials regarding each aspect and has to judge the best one, and this is difficult decision-making problem. So, these MCDM methods is applied to select optimal piping material in this section.

Table 1 Piping materials and its properties [L. Anojkumar et. all (2014)]

Material	Yield strength {B}	Ultimate tensile strength {B}	% Of elongation	Hardness {B}	Cost	Corrosion rate	Wear rate
J4	382	728	48	98	112	0.16	2.75
JSLAUS	420	790	58	97	210	0.31	2.63
204Cu	415	795	55	96	120	0.05	2.5
409 M	270	455	32	78	184	0.40	4
AISI 304	256	610	60	86	89	0.01	2.59

3.1. Problem formulation

In practical manufacturing environment, piping materials are made of stainless Steel. Among these seven properties- Yield strength, Ultimate tensile strength, Hardness are beneficiary, and others are non- beneficiary.

An organization has got 5 different materials with different specifications for pipe. The decision maker considered 7 selection criteria.

4. Result

In entropy method, the smaller the indicator's entropy value ej is, the bigger the variation extent of assessment value of indicators is, the more the amount of information provided, the greater the role of the indicator in the comprehensive evaluation, the higher its weight should be.

Table 2 The weighted values are

	Yield strength	Ultimate tensile strength	% Of elongation	Hardness	Cost	Corrosion rate	Wear rate
Waighted values	0.1283	0.1247	0.1120	0.1052	0.1503	0.2885	0.0911

4.1. In the MOORA method

Table 3 Step 1 Determination of normalized decision matrix

Material	Yield strength {B}	Ultimate tensile strength {B}	% Of elongation	Hardness {B}	Cost	Corrosion rate	Wear rate
J4	0.4801	0.4734	0.4159	0.4799	0.3334	0.3001	0.4173
JSLAUS	0.5279	0.5137	0.5026	0.4750	0.6252	0.5814	0.3990
204Cu	0.5216	0.5169	0.4766	0.4701	0.3573	0.0938	0.3793
409 M	0.3394	0.2959	0.2773	0.3819	0.5478	0.7502	0.6069
AISI 304	0.3218	0.3966	0.5199	0.4211	0.2650	0.0188	0.3930

Table 4 Step 2 Determination of weighted normalized decision matrix

Material	Yield strength {B}	Ultimate tensile strength {B}	% Of elongation	Hardness {B}	Cost	Corrosion rate	Wear rate
J4	0.0616	0.0590	0.0466	0.0505	0.0501	0.0866	0.0380
JSLAUS	0.0677	0.0640	0.0563	0.0500	0.0940	0.1677	0.0364
204Cu	0.0669	0.0644	0.0534	0.0494	0.0537	0.0271	0.0346
409 M	0.0435	0.0369	0.0310	0.0402	0.0823	0.2164	0.0553
AISI 304	0.0413	0.0494	0.0582	0.0443	0.0398	0.0054	0.0358

The value of a ......sum of all weighted normalized values for all beneficial column

Table 5 Step 3 Determination of weighted multi objective optimization

Material	J4	JSLAUS	204Cu	409 M	AISI 304
	0.1672	0.1880	0.1847	0.1115	0.1489

Table 6 The value of b ......sum of all weighted normalized values for all non-beneficial column

Material	J4	JSLAUS	204Cu	409 M	AISI 304
	0.2252	0.3480	0.1648	0.3942	0.1253

Table 7 Step 4 the value of a-b

Material	J4	JSLAUS	204Cu	409 M	AISI 304
	-0.0580	-0.1600	0.0200	-0.2828	0.0236

4.1.1. STEP 5

Arranging the final value(a-b) in descending order: --------->>> M5 > M3 > M1 > M2 > M4>>

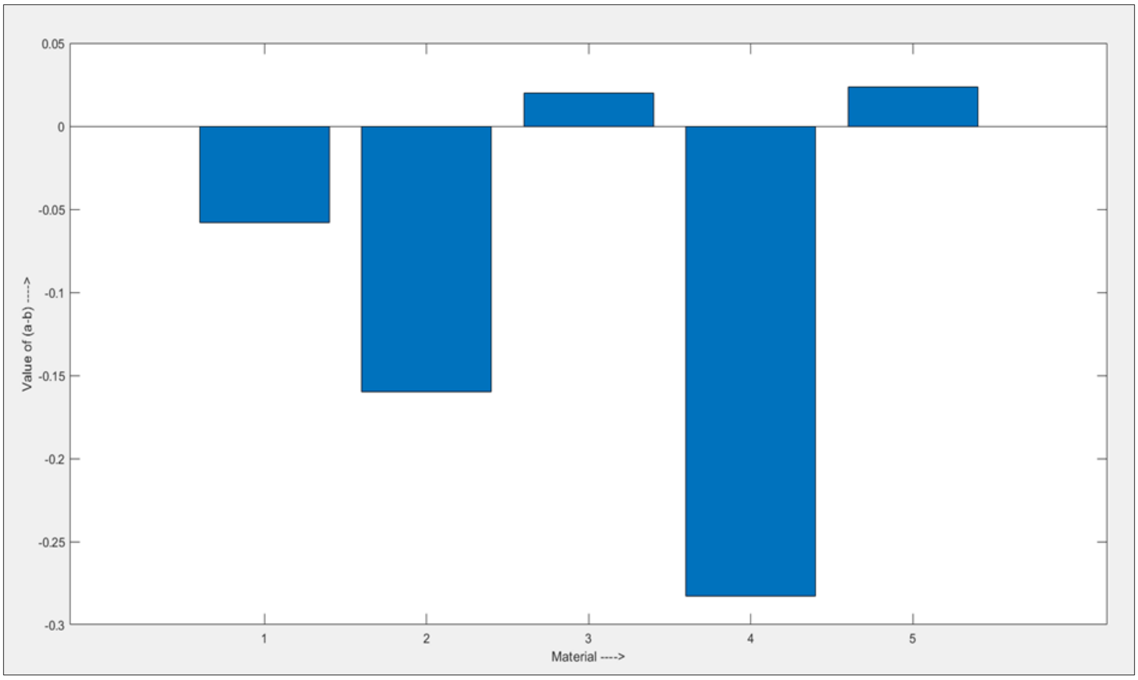


Figure 1 Arranging the final value of material

4.2. Sensitivity Analysis

Table 8 The value of closeness co-efficient in MOORA method

Material	when alpha=0	when alpha=1
J4	-0.2252	0.1672
JSLAUS	-0.3480	0.1880
204Cu	-0.1648	0.1847
409 M	-0.3942	0.1115
AISI 304	-0.1253	0.1489

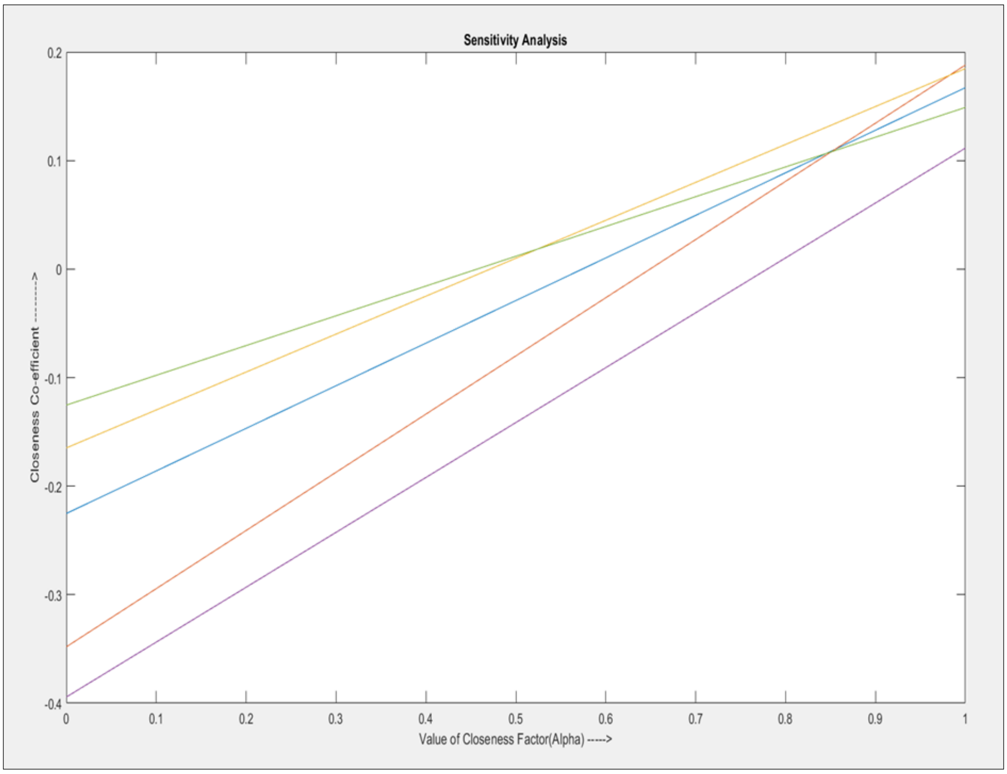


Figure 2 Sensitivity Analysis in MOORA

4.3. In the SAW method

The weighted values got from entropy method

Table 9 Step 1 Determination of normalized decision matrix

Material	Yield strength {B}	Ultimate tensile strength {B}	% Of elongation	Hardness {B}	Cost	Corrosion rate	Wear rate
J4	0.9095	0.9157	0.8000	0.7959	0.7946	0.0625	0.9091
JSLAUS	1.0000	0.9937	0.9667	0.8041	0.4238	0.0323	0.9506
204Cu	0.9881	1.0000	0.9167	0.8125	0.7417	0.2000	1.0000
409 M	0.6429	0.5723	0.5333	1.0000	0.4837	0.0250	0.6250
AISI 304	0.6095	0.7673	1.0000	0.9070	1.0000	1.0000	0.9653

Table 10 Step 2 Determination of weighted normalized decision matrix

Material	Yield strength {B}	Ultimate tensile strength {B}	% Of elongation	Hardness {B}	Cost	Corrosion rate	Wear rate
J4	0.1166	0.1142	0.0896	0.0837	0.1194	0.0180	0.0828
JSLAUS	0.1283	0.1239	0.1082	0.0846	0.0637	0.0093	0.0866
204Cu	0.1267	0.1247	0.1026	0.0855	0.1115	0.0577	0.0911
409 M	0.0824	0.0714	0.0597	0.1052	0.0727	0.0072	0.0570
AISI 304	0.0782	0.0957	0.1120	0.0954	0.1503	0.2885	0.0880

The values of (s) are:

Table 11 Step 3 Computation of composite score s......by sum of all weighted normalized rows

Material	J4	JSLAUS	204Cu	409 M	AISI 304
	0.6244	0.6046	0.6998	0.4556	0.9080

4.3.1. STEP 4

Arranging the final value (s) in descending order: --------->>> M5 > M3 > M1 > M2 > M4....in SAW method

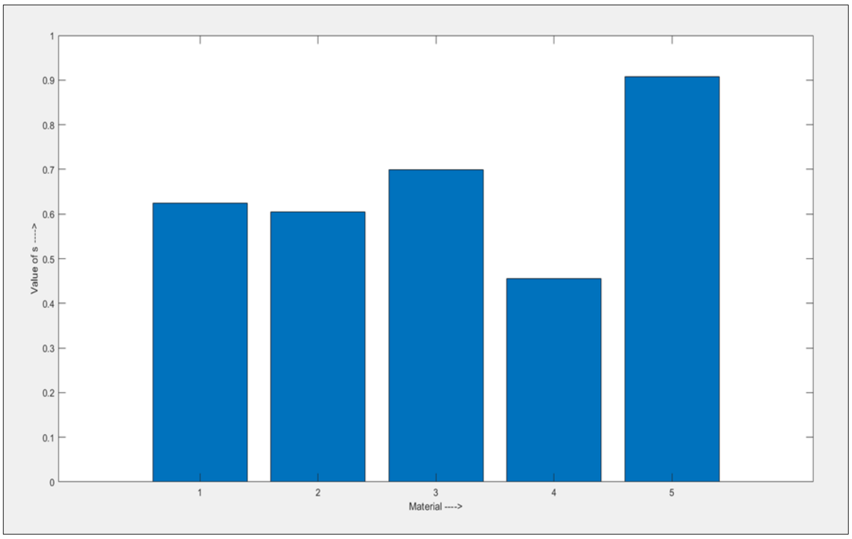
>> 

Figure 3 Arranging the final value (s) in descending order in SAW method

4.4. Sensitivity Analysis

Table 12 The value of closeness co-efficient in SAW method

Material	when alpha=0	when alpha=1
J4	0.3040	0.3204
JSLAUS	0.2442	0.3604
204Cu	0.3457	0.3540
409 M	0.2420	0.2135
AISI 304	0.6222	0.2858

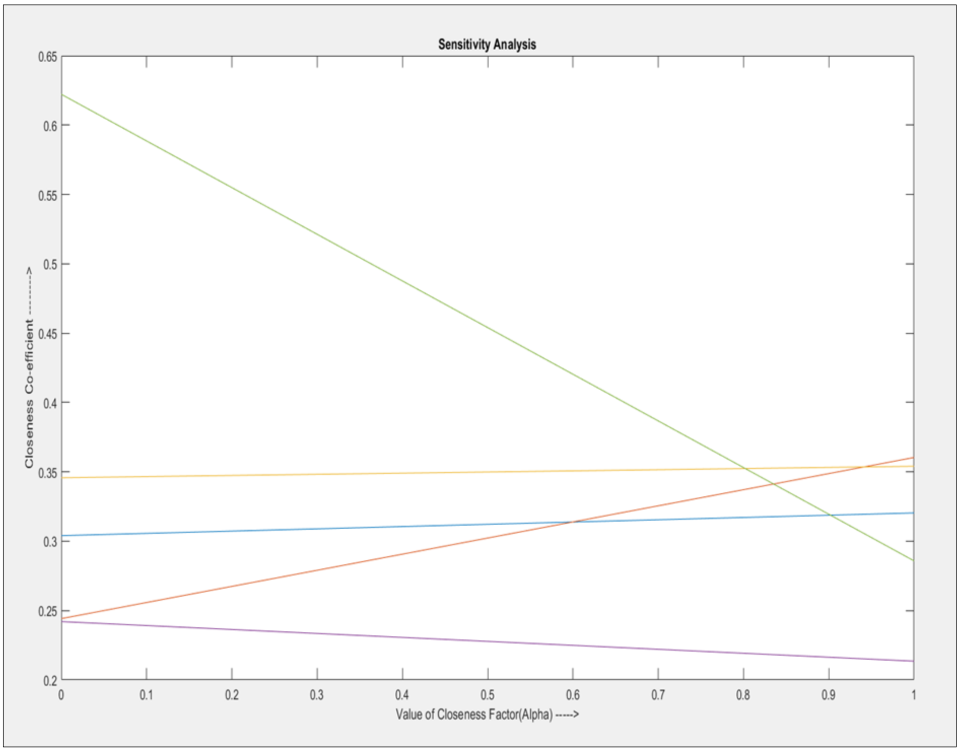


Figure 4 Sensitivity Analysis in MOORA

In SAW and MOORA methods, ranks of alternatives are given in descending order of their respective composite score. So, the ranking of alternatives of materials are as follows: M5 > M3 > M1 > M2 > M4. It means that Material 5 is the best as it maximizes the benefit criteria and minimizes the cost criteria that is Material 5 is the nearest to be optimal solution [Table 13].

4.5. Comparative analysis of ranking of materials using MCDM methods

Table 13 Comparative analysis

Material	Saw (rank)	Moora (rank)
J4	3	3
JSLAUS	4	4
204Cu	2	2
409 M	5	5
AISI 304	1	1

5. Discussion

From the result we see that for the three different processes of MCDM, the result is same. The ranking of 1st to 5th materials are same for those two different processes. For the simplicity, prompt result getting the accurate value and also getting the best ranking we have used the MATLAB software. By this software we can also make rank of any system for any number of alternatives and criteria within a fraction of second with accuracy.

We have also made the sensitivity analysis with graphical representation in which we see that both in SAW and MOORA method. From the sensitivity analysis graph, we also get the rank of the lathes for any alpha value by drawing a vertical line from that alpha value to the straight line of the lathe in the graph. That's why for doing the sensitivity analysis our result does not depends any different decision makers with their different weighted values.

6. Conclusion

It is quite clear that selection of a proper material for a given manufacturing application involves a large number of considerations. The use of SAW and MOORA methods are observed to be quite capable and computationally easy to evaluate and select the proper machine from a given set of alternatives. These methods use the measures of the considered criteria with their relative importance in order to arrive at the final ranking of the alternative material. Thus, these popular MCDM methods can be successfully employed for solving any type of decision-making problems having any number of criteria and alternatives in the manufacturing domain. Use of MATLAB software makes MCDM problem simple and gives prompt results which is very essential in today's decision-making environment. As a future scope, a fuzzy MOORA, fuzzy SAW based methodology may be developed to aid the decision makers to take decisions in presence of imprecise and incomplete data.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declare that there is no conflict of interest within this research.

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