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The universality of Rydberg constant is investigated with concern for ethics

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Abstract

Bearing in mind the unintended restriction of the Rydberg constant to atomic properties, there is a need to defend the universality of the Rydberg constant beyond atomic physics against the backdrop of the demands of ethics in research and publishing. The objectives of the research were:1) to show that the Rydberg constant is not just a composite fundamental physical constant applicable to the field of atomic physics but also nuclear matter physics; and 2) to examine the burning issues of the need for ethics in research, critique, or evaluation, and publishing. The methods were theoretical and calculational. The derivational and calculational results gave equations and values that were positive and verifiable; the calculation of the Rydberg constant using the derived equation after substituting the calculated mass radius of proton in this research and other literature values and the well-known physical constants reproduced the Rydberg constant to between 99.88 and 104.26 % of the 2016 CODATA value. Calculations based on charge radii from electron-proton scattering experiments produced results ranging from 72.53 to 75.34% of the CODATA value. Specifically, the calculation using the calculated mass radius of the proton in this research yielded approximately 1.096049145 exp. (+7)/m. Unethical behaviors in research, critique/evaluation review, and publishing were identified and discussed, and pieces of advice given accordingly. In conclusion, the universality of the Rydberg constant as a composite constant, applicable to both atomic and nuclear physics remains undisputable. The role of the Rydberg constant in the determination of periodic parameters of multi-electron atom is recommended for future research.

Keywords: Universality of Rydberg constant; Mass radii of the elementary particles; Charge radius of the proton; Ethics

1. Introduction

Over a century ago, the Swedish scientist Johannes Rydberg discovered a fundamental constant restricted to atomic physics. The constant is a physical constant relating to the electromagnetic spectra of an atom. The Rydberg constant is a fundamental constant of atomic physics that first appeared in formulas developed in 1890 by Johannes Rydberg, describing the wavelengths or frequencies of light in various series of related spectral lines, most notably those attributed to hydrogen [1]. According to Bohr [1] a new thinking outcome (yet to be fully comprehended in this research) referred to as the Ritz-Rydberg combination principle, a general law, enunciated by Ritz in 1908, has come to stay as one of the principles of atomic physics [1]. In accordance with this general law, the wave number (s) of any line of a spectrum can be expressed as:

$$\sigma = T_1 - T_2 \quad (1)$$

where T_1 and T_2 are two members of the set of terms characteristic of the element. The combination principle is seen as an empirical generalization proposed by Ritz in 1908 to describe the relationship between the spectral lines for "all

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atoms". This may imply both hydrogenic and nonhydrogenic atoms. If so, one would wonder why Bohr's equation could not be applied to multi-electron atoms. The reason, as stated elsewhere, is that no one knew what to do with average ionization energies until recently [2-4].

The reason as to why Bohr's equation could not be applied to multi-electron atoms, though it looks plausible, does not give a stronger reason that precludes doubt. Why is it that, despite the fact that human systems are never perfect, it is not permissible to allow errors by accident, and yet one or more scholars may have their hard-earned effort thrown out even before a third-party examination? This comment ushers in the issue of ethics in research, evaluation (critique), and publishing. It is possible that someone knew, but ego and a desire to maintain the status quo prevented any attempt to apply the knowledge in violation of the very important ethics of academics (an academician appreciates the contribution of old and new scholars); intellectualism (an intellectual recognizes other people as potential intellectuals with something to offer); and pedagogy (this requires the realization that no one is completely independent of any other knowledge and that no one is a *tabula rasa*). An earlier rejection of the equation [2] $a_i = n\,h/\text{p}(8\,m_e\,E_i)^{1/2}$ (where a_i n, h, m_e, and E_i are the radius of the atom of any kind, principal quantum number (pqn), Planck constant, rest mass of the electron or, if preferred, the reduced mass, and averaged ionization energy) by an unanimous reader or reviewer shows that such a person lacks pedagogical principles and gives the impression that the reader lacks Piaget's principle of decentration and that such a lack is not restricted to infants. The equation [2] has however, been published. Unlike the well-known and well-deserved Bohr's equation, the radius of an atom in its ground state as a neutral atom or as an ion is directly proportional to the pqn and inversely proportional to the square root of the averaged ionization energy. If a paper contains an exact equation as above, purported to give exactly the same answer as the old mathematical model, all that needs to be done by a true scientist is to apply the electronic calculator without bias so as not to discredit the author. Indeed, if n is equal to one and E_i is the averaged ionization energy for hydrogen, the value of a_i should be equal to approximately 5.29 exp. (-11) m. If it is right for hydrogen, why must it not be right for multi-electron atoms or ions? The wrong answer is always an attempt to deny the author the benefits of hard work and possible reward, except if such an effort has its origin in a preferred nationality. This is very unethical.

In continuation of due regard and recognition for previous work, one has to recall the work of Ritz. The Ritz combination principle states that the spectral lines of any element include frequencies that are either the sum or the difference of the frequencies of two other lines [1]. Another related version is that the wavenumber or reciprocal of the wave length (λ) of each spectral line can be written as the difference of "two terms". A typical example is the hydrogen atom, with the equation:

$$1/\lambda = R_\infty\left(\frac{1}{n_1^2} + \frac{1}{n_2^2}\right) \tag{2}$$

where the principal quantum numbers n_1 and n_2 compare in the following order: $n_2 > n_1$. Derived from this (Eq. (2)) is the equation of Ritz combination principle (en.wikipedia.org/wiki/Rydberg-Ritz-combination principle):

$$1/\lambda = \text{A} - \frac{N}{\left(m+\alpha + \beta\,(\alpha - \tilde{v})\right)^2} \tag{3}$$

where $\tilde{v}$, A, N, α, β and, m represent the wavenumber, series limit, universal constant, now known as R, constants, and numeral, now known as n. For the time being, it is unclear how Eq. (3) can be applied to the study of atomic spectra, let alone related to nuclear matter properties. Nevertheless, it seems the Rydberg constant has become a household name in atomic physics to the point that no further investigation needs to be carried out on it, either for the purpose of greater precision or some other applications. One of the earliest studies demonstrated how three successive ionization potentials, I_p, I_{p+1}, and I_{p+2} in an isoelectronic series of atoms resulted in a constant 2nd difference equal to twice the R_∞ divided by the square of the principal quantum number, n, of the electron being "ionized" [5]. The result is summarized in the equation as follows:

$$I_p - 2I_{p+1} + I_{p+2} = 2\,R_\infty/n^2 \tag{4}$$

where I_p, is the ionization potential. Nonetheless, how Eq. (4) can be applied for calculation is not clear yet. Besides, the left and right sides of the equation do not possess the same dimension.

The extent of advancement in high resolution spectroscopy with unfamiliar tunable lasers has enabled the determination of a new value of the Rydberg constant (R_∞) with an almost 10-fold improvement in accuracy over experimental values in the 1970s [6]. R_∞ values for 2016 [7] and 2012 [8] (CODATA reports) were 10973731.568508 and 10973731.568539/metres, respectively. These results notwithstanding, there is no attempt as a matter of policy to adopt the preceding equations where applicable for the determination of the Rydberg constant. The equations are reminders and open to those who may wish to investigate. Bearing in mind the unintended restriction of the Rydberg constant to atomic properties, there is a need to defend the universality of the Rydberg constant beyond atomic physics against the backdrop of the demands of ethics in research and publishing with the following objectives: 1) to show that the Rydberg constant is not just a composite fundamental physical constant [9] applicable to the field of atomic physics but also nuclear matter physics; and 2) to examine the burning issues of the need for ethics in research, critique, or evaluation, and publishing.

2. Further theoretical development

The equations for the calculation of the mass radii of the nucleons can be found in the literature [10]. A recent publication [11] shows how atomic property can be linked to purely nuclear property. This research would have no basis if there is no interest in mass radii or the so-called charge radius of the proton; unlike the calculated mass radius of the proton in the literature [10, 11] based on classical and mass-energy equivalent principles, the electron-scattering methods gave different values that in one case, resulted in "puzzling sensation" [12-18]. The atomic property and the nuclear property in question are, respectively, the average ionization energy of hydrogen and any other hydrogenic atom or ion and the mass radii of the nucleons. However, it was argued that such linkage might not be limited to hydrogenic atoms alone because there is a workable relationship between the average ionization energy of hydrogen and that of nonhydrogenic atoms [3, 11]. Such a relationship is given as follows:

$$\xi_{\mathbf{H}} = n_{\mathrm{i}}^2 \xi_{\mathrm{i}} / Z_{\mathrm{eff}}^2 \quad (5)$$

where ξ_{H}, n_{i}, ξ_{i}, and Z_{eff} are the average ionization energies of hydrogen atoms, the principal quantum number, the average ionization energy of any atom, and the corresponding effective nuclear charge respectively. The equation linking the atomic property to nuclear property, *i.e.* the mass radius is given as [11]:

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\, \mu_0^2\, e^{10}}{32\, h^4\, \varepsilon_0^3\, \pi\, m_{\mathrm{e}}\, \xi_{\mathrm{H}}} \quad (6)$$

where $\mathcal{J}$, m_{p}, μ_0, e, ε_0 and h are the mass radius of any elementary particle whose mass is larger than the mass (m_{e}) of an electron, magnetic constant, charge of an electron, permittivity in free space, and Planck's constant, respectively. If you read a standard physics textbook on atomic subfields, you might get the impression that the Rydberg constant only applies to hydrogenic atoms. However, based on basic mathematics and common sense, it can be shown that the Rydberg constant is an integral component of the mathematical equation for the determination of nuclear property. This can be shown by derivation, in a step-by-step approach, showing subtle, minor, but significant details. This can support the view that whenever there was a development in a new application of quantum theory, it was required to repeat trial-and-error experiments to determine which technique of calculation produced the correct results [19]. This amounts to showing details, unlike what is frequently observed in textbooks and some journals. In those text materials, the authors just discuss the effective approach as if it were followed from first principles and leave out the actual method that was used to discover it [19]. Recent concern was expressed in a recent publication [11]; a complex fraction, with advanced integrands in the nominator and denominator, leading to a solution without any intermediate steps, must be swallowed hook, line, and sinker.

Meanwhile, ε_0 is = $1/\mu_0 c^2$ such that substitution into Eq. (6) gives:

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\, \mu_0^5\, c^6 e^{10}}{32\, h^4\, \pi\, m_{\mathrm{e}}\, \xi_{\mathrm{H}}} \quad (7)$$

Given that the equation of Rydberg constant is [7, 8]:

$$R_\infty = \frac{m_e\, e^4}{8\, \varepsilon_0^2\, h^3\, c} \quad (8)$$

where c is the velocity of light in a vacuum it can be shown that in Eq. (7),

$$\frac{e^4}{h^3} = \frac{8 R_\infty\, \varepsilon_0^2\, c}{m_{\mathrm{e}}}. \quad (9)$$

Therefore, following substitution of Eq. (9) into Eq. (7) after rearrangement of the latter one can re-write as follows:

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\, \mu_0^5\, c^6 e^6}{32\, \pi\, m_{\mathrm{e}}\, \xi_{\mathrm{H}}\, h} \times \frac{8 R_\infty\, \varepsilon_0^2\, c}{m_{\mathrm{e}}} \tag{10a}$$

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\, \mu_0^5\, c^7 e^6}{4\, \pi\, m_e^2\, \xi_{\mathrm{H}}} \times \frac{R_\infty\, \varepsilon_0^2}{h} \tag{10b}$$

Once again, given that, ε_0 is = $1/\mu_0 c^2$, Eq. (10b) becomes:

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\, (\mu_0 c\, e^2)^3}{4\, \pi\, m_e^2\, \xi_{\mathrm{H}}\, h} R_\infty \tag{11}$$

Equation (11) shows the first instance of how the Rydberg constant could be a fundamental part of the equation for the determination of nuclear properties besides well-known atomic properties such as the average ionization energy of hydrogenic atoms and ions. Let it be known, however, that this is not restricted to hydrogenic atoms or ions, but rather, as Eq. (5) shows, a substitution into Eq. (11), gives a general equation for any such relationship between nuclear property and atomic property via the Rydberg constant, as long as experimental values of average ionization energies of different multi-electron atoms are known regardless of energy levels.

The validity of Eq. (11) can be established by first making R_∞ subject of the formula and then solving for it after substituting all the fundamental physical constants into the resulting equation. The resulting equation is:

$$R_\infty = \frac{4\, \pi\, m_e^2\, \xi_{\mathrm{H}}\, h\, \mathcal{J}_{\mathrm{P}}}{m_{\mathrm{P}}\, (\mu_0 c\, e^2)^3} \tag{12}$$

Knowing Eq. (8) as one of the Rydberg constant equations derived when the fine structure constant is substituted into the equation $\alpha^2 m_{\mathrm{e}}\, c/2h$ [7, 8], it means that there are two equations for the time being; the second equation is hereby given equation number for the purpose of discussion.

$$R_\infty = \frac{\alpha^2 m_e c}{2\, h} \tag{13}$$

Bearing in mind that the main focus is on the Rydberg constant, there is a need to compare the two equations available in the literature [7, 8]; Equation (8) is more complex, with higher exponents and six parameters, whereas Eq. (13) has one exponent and five parameters. Yet, as a corollary, another equation can be derived which can validate the procedure in this research and the equation in the literature [10] given as:

$$\mathcal{J}_{\mathrm{P}} = \frac{\mathrm{e}^6 m_{\mathrm{P}}}{4\, \pi\, m_e^2\, h^2 \varepsilon_0^3\, c^4} \tag{14}$$

Substitution of Eq. (14) into Eq. (12) gives:

$$R_\infty = \frac{4\, \pi\, m_e^2\, \xi_{\mathrm{H}}\, h}{m_{\mathrm{P}}\, (\mu_0 c\, e^2)^3} \frac{\mathrm{e}^6 m_{\mathrm{P}}}{4\, \pi\, m_e^2\, h^2 \varepsilon_0^3\, c^4} \tag{15a}$$

Simplification and substitution of $1/\mu_0 c^2$ (which is = ε_0) into Eq. (15a) gives:

$$R_\infty = \frac{\xi_{\mathrm{H}}}{h\, c} \tag{15b}$$

Therefore, Eq. (15b) represents the simplest equation of the Rydberg constant derived clearly in this research for hydrogen only. However, another form of it for larger hydrogenic atoms in the literature [20] is given as follows:

$$E = -\frac{2\, \pi^2 m_e\, Z^2 e^4}{n^2\, h^2} = -\, hcRZ^2/n^2 \tag{16a}$$

where R, n, E, and Z are the Rydberg constant, principal quantum number, total energy, and atomic number (as provided in the literature [20]); reduced mass (μ) of an electron = 9.104425137 exp. (–31) kg. This is calculated from the equation: $\mu = m_{\mathrm{e}}\, m_{\mathrm{P}}/(m_{\mathrm{e}} + m_{\mathrm{P}})$; For the purpose of analysis and discussion, there are two aspects of Eq. (16a) re-stated below:

$$E = -\frac{2\,\pi^2 m_e\, Z^2 e^4}{n^2\, h^2} = -2.699363193 \exp\ (-38)\ \text{J (If } n \text{ and } Z = 1). \tag{16b}$$

$$E = -\,h\,c\,R\,Z^2/n^2 = -\,2.179872323\ (-\,18)\ \text{J (If } n \text{ and } Z = 1) \tag{16c}$$

In line with conservative field force and cognate energy, the total energy is equal to the kinetic energy of the electron but opposite in sign. Unlike Eq. (16c), precluding any mistake, Eq. (16b) cannot give the total energy of either hydrogenic multi-electron atoms of the kind, given as ${}^{A}_{Z}X^{(Z-1)+}$or hydrogen atom if the n and Z in the latter are equal to 1. In line with ethics, it is for the scientific community to find a suitable name for Eq. (12) that for all time has given unshakable validity to the Rydberg constant as a universal constant.

The second instance of how the Rydberg constant could be a fundamental part of the equation for the determination of nuclear properties is derived as follows: If the average ionization energy of hydrogen atom is given as follows: $E_{\mathrm{H}} = e^4 m_{\mathrm{e}}/8\,\varepsilon_0^2\,h^2$, one can substitute the latter into Eq. (11) to give the following:

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\,(\mu_0 c\, e^2)^3}{4\,\pi\, m_e^2\, h} R_\infty \frac{8\,\varepsilon_0^2\, h^2}{e^4 m_{\mathrm{e}}} \tag{17a}$$

Rearranging and substituting $1/\mu_0\, c^2$ (which is = ε_0) into the resulting equation gives after simplifying:

$$\mathcal{J}_{\mathrm{P}} = \frac{2\, m_{\mathrm{P}}\, h\, e^2\, (\mu_0\, c)^3}{\pi\, m_e^3\, \mu_0^2\, c^4} R_\infty \tag{17b}$$

Simplification of Eq. (17b) gives:

$$\mathcal{J}_{\mathrm{P}} = \frac{2\, m_{\mathrm{P}}\, h\, e^2\, \mu_0}{\pi\, m_e^3\, \mathrm{c}} R_\infty \tag{18}$$

Equation (18) can also be validated by solving for R_∞, after substituting the value of $\mathcal{J}_{\mathrm{P}}$ in the literature [10, 11] and other fundamental constants into the equation below (derived from Eq. (18)).

$$R_\infty = \frac{\pi\, m_e^3\, \mathrm{c}\, \mathcal{J}_{\mathrm{P}}}{2\, m_{\mathrm{P}}\, h\, e^2\, \mu_0} \tag{19}$$

Another equation of mass radius of the proton can be calculated if Eq. (8) is substituted into Eq. (18) to give after simplification the following:

$$\mathcal{J}_{\mathrm{P}} = \frac{2\, m_{\mathrm{P}}\, h\, e^2\, \mu_0}{\pi\, m_e^3\, \mathrm{c}} \frac{m_e\, e^4}{8\,\varepsilon_0^2\, h^3\, c} \tag{20a}$$

Further simplification gives finally,

$$\mathcal{J}_{\mathrm{P}} = \frac{m_{\mathrm{P}} \mu_0^3}{4\,\pi} \left(\frac{e^3 c}{m_e\, h}\right)^2 \tag{20b}$$

Equation (20b) represents as a corollary, a simpler equation compared to other equations in the literature [10, 11], for the calculation of the mass radius of the nucleon. The equation is however, applicable to any other subatomic particle whose mass is > the mass of an electron.

3. Methods and materials

There was no experiment. As such, no materials and equipment were necessary; the methods are theoretical and calculational.

4. Results and discussion

The result expected from calculating R_∞ (Table 1) should thus confirm not just the universality of the latter as a fundamental physical constant but also the validity of the mass radii of the nucleons that have been calculated in different ways [10, 11, 25, 26, 27]. There is, however, no agreement with the literature value of the R_∞ value where charge radii in the literature [12–18] and indeed similar values of electromagnetic radii [21] are substituted into the derived equation for calculation (Table 1). The multiplicity of the charge radii of the proton and the negative radius of the neutron cannot give the value of R_∞ if substituted into Eq. (12); surprising, it seems this era of "COVID-19 and emerging variants has overshadowed the scientific community, disabling them from observing freely available literature materials showing the charge radius of the proton that are shorter than what they claim is an improvement from longer radii reported before their result". "The most lethal and rapidly spreading diseases that are destroying the progress in science and the value of humanity are intellectual pomposity". It gives the impression that no one from other backgrounds can give ideas that are valid except from their leading position, even if it has to do with the highest number of deaths. It is pointless to make reference to any country so as to be more academic. It is incorrect to allude to being puzzled. It is high time we brought morality and ethics into intellectualism to be defended as follows: To begin with, let it be known that this comment is not about the author of this research, but there is an absolute need to conform to international publishing standards which require researchers not to discriminate against other authors for whatever reason. They must cite other authors if, in particular, such works are freely available. Now the issue:

More recent estimates for the length of nucleon radii given in the literature [10] differ from those reported in earlier literature [13, 22]. The proton charge radii in the literature are 0.856 fm. [13], 0.840±0004 fm. [16], 0.84087 fm. [13], and lastly, for the purposes of this study, 0.8310±012 fm. [14]. The argument that 0.84 fm. may represent the proton charge radius is disproved by the latter figure; yet again, other reports based on an unfamiliar chiral bag model [23], which views the value to be longer than 0.8 fm., and that of Bochkarev *et al.* [24], which is also 0.8 fm., are shorter than 0.831±0.012 fm. [14]. If, without being personal, this paper excludes any ethical consideration, then it cannot be published because any false claim in research causes damage elsewhere if such a claim is to be applied in further studies in engineering and, in particular, medicine (here, one can consider Reuters' report-www.reuters.com-on a South Korean scientist described as rock-star like in status who was found guilty of fabrication in his team's stem cell research; by the way, where were the reviewers and pre-review editorial staff). As Table (1) shows, substitution of charge radii into Eq. (12) gives after calculation values that are smaller than the time-tested Rydberg constant. *The validity or quality of any result, quantitative or mathematical, cannot be assessed on the basis of nationality, economic status, or the hosting journal and publisher listed by Thomson Reuters or Scopus. Enough is enough!*

A measurement using muonic hydrogen atoms, in which a measurement of lamp-shift was taken, found a substantial discrepancy in the length of the so-called charge radius of the proton compared with previous results [14]. The shorter length of the proton radius led to what has been termed the "proton-radius-puzzle" [14]. But a greater puzzle ought to be expected if, despite the freely available literature materials, the scholars did not notice the values of proton radius shorter than 0.831 fm. [11]. The value of proton charge radius, regarded as a shorter radius and probably a more accurate value for which the scientists were puzzled, is even longer than 0.8 fm., a value which was never cited by those researchers, even though the literature materials were freely available as published by highly reputable publishers. Whose fault? Could it be the fault of the publisher, the reviewers, the authors, the editor, *etc.*? The only speculation left for now is that some scholars wish to be the first to discover, be recognized, and possibly be awarded the highest price known to man till date, and the publisher may be interested in hosting such a paper and being rated very high. As useful as this is, it demonstrates double standards if there is evidence of discrimination by the author(s) and the reviewers/editors fail to log on to the website to find similar papers or articles and advise that similar articles be cited. Any price awarded under such circumstances is null and void. The entire scenario is no better than a predatory behaviour.

The citation of an article does not necessarily imply that such information, quantitative or qualitative, is correct or incorrect, but researchers must categorically conform to the highest international standards in reportage (this means reporting exactly what was done, observed, and recorded, not just statistical issues, but conditions as well, and means or methods) and publishing of research findings. No contemporary events of disease (e.g., COVID-19) should overshadow other positive occurrences, as if in an attempt to divert attention and derecognize other achievements to the advantage of the usual favorites in the character of conspiracy. Perhaps it is as a result of the adherence to the usual favorites, that there seems to be a reoccurring misinterpretation of Heisenberg Uncertainty Principle (HUP), despite the view that the HUP is the mathematical expression for the statistical error in the variables of the wave function, such as those assigned to the position and momentum of the electron [19].

It should be noted that any discussion of Bohr's equation and its result not only for hydrogen but other larger atoms, cannot be conducted in isolation of any criticism leveled at it; thus, the constant references to HUP and the Schrödinger equation (SE), which served as replacement models are pertinent. However, failure to see Bohr's equation as a deterministic mathematical model will always flaw HUB and SE. It does not need to explain every physical phenomenon observed in hydrogenic or nonhydrogenic atoms. "It is not a broad-spectrum antibiotic that cures many diseases." If the Rydberg constant can be applied in the solution to the problem of the mass radius of any nucleon, it means that given a theoretical and experimental method that can determine the mass radius, the Rydberg constant can also be calculated using the derived equation in this research. For instance, an article by Burchell [25] shows that the measured radius of the proton is 1.11 fm. Based on the liquid drop model, Utama *et al.* [26] calculated the constant parameter, r_0 (= ($3m_N/4\rho$ r_0) where ρ is nuclear density) in the equation of nuclear radius to be 1.15 fm. By adopting the mass defect of neutron Sha [27], the calculated mass radius value is similar to the Burchell [25] result.

As written earlier, the charge radius of the proton as observed in the literature and the negative radius of the neutron cannot be explored for the calculation of Rydberg constant according to the derived equation. Then the question is what is valid? The answer may not easily be found according to the new models, HUP and SE, considering the view that "despite their advances, quantum mechanics and the inherent Heisenberg Uncertainty Principle remain primarily mathematical, have yet to produce a unified theory, and have yet to be proven to be based on reality" [19]. Yet results of recent investigations continues to show different variants of HUP ("not all about variant of COVID-19") and in one notable instance purport to disproof HUP [28] without forgoing the notion of uncertainty (or error in measurement) in position and momentum despite post-and pre-2005 results and views by eminent experimental and theoretical physicist to the effect that [19]: "The Schrödinger equation predictions are called into question by numerous new experiments, reanalyzes of previous data, and electrons in superfluid helium, among other things. Many eminent physicists contest quantum mechanics. In this regard, Einstein insisted that a more detailed, wholly deterministic theory must underlie the vagaries of quantum mechanics". [19] The Einstein view is therefore, supportive of the defensive view that the two different approaches, the deterministic approach of Bohr and the stochastic approach that characterizes the Heisenberg principle and Schrödinger-Dirac mathematical formalism, are "strange bed fellows" [4].

Table 1 Calculated Rydberg constant as a function of mass radii of proton and selected charge radii

Mass radii of proton/fm.	Rydberg constant (R_∞)/exp. (+7) m	Calculated value of R_∞ as percentage (%) of 2016 CODATA value.
1.101682527*	1.094540411 (1.096049145)	≈ 99.74 (99.88)
1.11 [25]	1.104324087	≈ 100.63
1.15 [26]	1.14411955	≈ 104.26
0.831[17]	0.8267507356	≈ 75.34
0.8 [24]	0.7959092521	≈ 72.53

The asterisk (*) indicates the value calculated by substituting all CODATA [7] fundamental constants into Eq. (14). The value of the Rydberg constant in parenthesis is obtained using the reduced mass of the electron. The value of the Rydberg constant calculated using Eq. (15b) is 1.097373156 exp. (+7) /m. Another value from the literature is 1.0967700 exp. (+7)/m [29].

According to Edwin Cartridge (www.physicsworld.com/electronic scattering experiment), the proton radius can also be described as the spatial extent of the proton's electric charge, which therefore implies that the mass radius must be shorter than the spatial extent of the proton's charge; a simple analogy is the case of any shape of magnet; though with a preference for spherical magnetic objects, one can say that the mass radius of the object is shorter than the spatial extent of the magnetic field. The values of the Rydberg constant calculated using the value of the mass radius of the proton calculated in this research in particular (Table 1) and those obtained from literature [25, 26] are closer to the literature value than the values calculated using the charged radii from electron-proton scattering experiments. This leaves one with the conclusion that the mass radii calculated in this research and in the literature [10] are valid answers to the question of the mass radii of the elementary particles. This does not require extraordinary (or rather "super") intelligence, just common sense!

5. Conclusion

The equation, which shows that the Rydberg constant is one of the fundamental physical constants that can be used to calculate nuclear matter parameters such as the mass radius of the nucleon, was derived. The calculation of the Rydberg constant using the derived equation after substituting the calculated mass radius of the proton in this research and other literature values and the well-known physical constants reproduced the Rydberg constant to between 99.88 to 104.26 % of the 2016 CODATA value. Calculations based on charge radii from electron-proton scattering experiments produced results ranging from 72.53 to 75.34 % of the CODATA value. This summarily validated the procedure in this research and the equations derived in this research and in the literature. Significant cases of ethical breaches involving failures in citing relevant papers by other authors and failure to identify flaws or errors in published equations were identified; stakeholders were accordingly advised to end unethical behavior. In the light of current findings in this research, the derivation of Rydberg constant dependent equations for the calculation of periodic atomic parameters for multi-electron atoms is recommended for research in the future.

Compliance with ethical standards

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Disclosure of conflict of interest

The author has no competing interests of any kind.

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Author's short biography

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My interest covers mainly subjects such as biochemistry, biophysics (and related field) and very limited extra-curricular subject such as atomic and nuclear physics. My general interest includes field of science amenable to basic mathematics.

Any biological field in which a challenging problem or observation is made and poses a challenge to my imagination and curiosity is also of interest to me. I earned a Ph.D in Biochemistry from Ambrose Alli University, Ekpoma, Edo State, Nigeria.