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(Research Article)

Flow of an incompressible non-Newtonian fluid in a lipid concentrated cylindrical channel under the influence of magnetic field and metabolic heat

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Abstract

In this research, we used mathematical models in the investigation of incompressible non-Newtonian fluid flow through a lipid-concentrated cylindrical channel with the effect of metabolic heat and magnetic treatment. The nonlinear partial differential equations were scaled into linear partial differential equations and then solved using the Laplace method to obtain the exact solutions. The numerical simulation was carried out using Wolfram Mathematica, version 12, where graphical results were obtained showing the variation of various pertinent parameters and their effects on the velocity profile of the fluid. The graphs revealed the effects of the parameters such as the metabolic heat, Grashof parameter, magnetic field; Prandtl number, pressure gradient, and porosity parameter have on the flow profile. The significance of this study is in the therapeutic investigation of hyperthermia, especially on the regulation of blood flow and lipids in the blood.

Keywords: Non-Newtonian; Lipid Concentration; Metabolic Heat; Magnetic Treatment; Cylindrical Channel; Atherosclerosis; Laplace method

1. Introduction

Atherosclerosis is the leading cause of death in western societies. It is characterized by an accumulation of excessive cholesterol and inflammatory cells and lipids in the intima and media, leading to their thickening and hence to a constriction of the arterial lumen. It is now well accepted that a significant first step in the initiation of the early atherosclerotic process is a dysfunction of the endothelium, allowing the penetration of low-density lipoproteins (LDL) through the monolayer of endothelial cells into the vessel wall. Blood is a fluid, primarily made up of red blood cells (RBCs), white blood cells (WBCs), and platelets suspended in plasma. Oxygenated blood flows away from the heart to different organs through systemic circulation, according to Bunonyo *et al.* [1]. One major health risk is poor blood circulation in the body as a result of occlusion/blockage in arteries. Arteries allow oxygenated blood and other nutrients from the heart to reach the tissues of the body in the circulatory system of the human body. Blood is a viscous fluid circulating in the artery or vein. Atherosclerosis is the hardening of a blood vessel from a buildup of plaque as a result of excessive cholesterol (lipoprotein) intake. Plaque is made of fatty deposits, cholesterol, and calcium. Plaque buildup causes the artery to narrow and harden, which is a serious risk to human life. It can inhibit and even stop blood circulation, Bunonyo and Eli [2]. This means the tissue supplied by the artery is cut off from its blood supply, and as such, humans must watch the quantity of cholesterol they consume. Several years ago, there were theoretical and experimental studies on blood circulation in humans, and here is some literature in that regard: Shit and Roy [3] studied the effect of an induced magnetic field on blood flow through a constricted channel, and demonstrated that increasing the strength of the magnetic field reduces the velocity of the blood flow at the center. Rahbari *et al.* [4] carried out a study on blood flow containing nano-particles through porous blood vessels in the presence of a magnetic field using

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the Homotopy Perturbation Method (HPM). The heat and mass transfer of blood flow considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating, and a finite heat source was discussed by Sharma et al. [5]. According to Chakravarty and Sannigrahi [6], the investigation of basic BFD flow problems attracts interest due to the numerous proposed applications in bioengineering and medical sciences. Bio-fluids in the presence of a magnetic field with dissipation find their applications in various upcoming fields like innovative drug targeting and surgical operations. Singh and Mathew [7] studied the effects that injection and suction have on oscillating hydrodynamic magnetic flow in a horizontal channel that is rotating. Attia and Kotb [8] examined magneto-hydrodynamic flow between parallel plates having heat transfer. In their work, the upper plate is given a constant velocity while the lower plate is kept stationary. The viscosity of the fluid is assumed to vary with temperature, and the governing equations were solved numerical. In a similar vein, Swapna *et al*. [9] studied the mass transfer in mixed convective periodic flow through a porous medium in an inclined channel, and a closed form solution of the problem was obtained. The various flow parameters on the velocity, temperature and concentration fields were discussed with the aid of graphs. Bunonyo *et al.* [10] investigated blood flow in a stenosed artery with heat in the presence of a magnetic field. In their investigation, it was observed that magnetic field increases inhibit blood flow as a result of the Lorentz force. In this present research, we study incompressible non-Newtonian fluid flow through a lipid concentrated cylindrical channel with the effect of metabolic heat and magnetic treatment with mathematical models and solve the system using the Laplace method where various flow profiles shall be obtained and simulation shall be carried out using Wolfram Mathematica, a computational software, version 12.

2. Mathematical Formulation

We consider blood circulation in humans by considering the assumption that the blood vessel is a cylindrical channel and blood as an incompressible non-Newtonian fluid, viscous and electrically conducting in nature. The viscosity of blood is a result of the percentage of erythrocytes (RBC), the main carriers of hemoglobin and lipid. The lipoprotein is the protein in the blood that is the major constituent that increases the concentration. The oxidation of metabolic fuels such as carbohydrates and fatty acids in the mitochondria of the fibres produces adenosine triphosphate. Through the hydrolysis of adenosine triphosphate, energy is released to support muscle contraction. However, hydrolysis of adenosine also releases heat. The pumping action of the heart causes the blood to circulate. In addition, we also take into consideration the velocity of the fluid, the lipid concentration, and the temperature of the fluid, respectively. The equations governing the flow of the fluid through a cylindrical channel following Bunonyo and Ebiwareme (2022)[11], are stated as coupled differential equations as stated below.

2.1. Geometry of the lipid concentrated region

$$R = \begin{cases} R_0 - \dfrac{\delta^*}{2}\left(1 + cos\dfrac{2\pi x^*}{\lambda}\right) & \text{at } d_0 \le x^* \le \lambda^* \\ R_0 & \text{at } 0 \le x^* \le d_0 \end{cases} \quad \text{.......(1)}$$

where

$$x^* = \left(d_0 + \frac{\lambda}{2}\right) \quad \text{.......(2)}$$

2.2. Momentum equation

$$\rho_b \frac{\partial w^*}{\partial t^*} = -\frac{\partial P^*}{\partial x^*} + \mu_b\left(\frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial w^*}{\partial r^*}\right) - \frac{\mu_b \varphi}{k^*}w^* - \sigma B_0^2 w^* + \rho_b g \beta_C\left(T^* - T_\infty\right) \quad \text{.......(3)}$$

2.3. Energy equation

$$\rho_b c_b \frac{\partial T^*}{\partial t^*} = k_{Tb}\left(\frac{\partial^2 T^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T^*}{\partial r^*}\right) - Q_0\left(T^* - T_\infty\right) + Q_1\left(C^* - C_\infty\right) \quad \text{.......(4)}$$

2.4. Concentration equation

$$\frac{\partial C^*}{\partial t^*} = D_m\left(\frac{\partial^2 C^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial C^*}{\partial r^*}\right) - k_0\left(C^* - C_\infty\right) \quad (5)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &w^* = 0, T^* = T_w, C^* = C_w \ \text{ at } \ r^* = R \\ &w^* \neq 0, T^* \neq T_\infty, C^* \neq C_\infty \ \text{ at } \ r^* = 0 \end{aligned}\right\} \quad (6)$$

2.5. Dimensionless Parameters

$$\left.\begin{aligned} &x = \frac{x^*}{\lambda}, r = \frac{r^*}{R_0}, t = \frac{t^*\upsilon_b}{R_0^2}, w = \frac{w^* R_0}{\upsilon_b}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, Rd_1 = \frac{Q_0 R_0^2}{\mu_b c_b}, Rd_3 = \frac{k_0 R_0^2}{\upsilon_b} \\ &Gc = \frac{g\beta_C\left(C_w - C_\infty\right)R_0^3}{\upsilon_b^2}, Gr = \frac{g\beta_T\left(T_w - T_\infty\right)R_0^3}{\upsilon_b^2}, M = B_0 R_0\sqrt{\frac{\sigma}{\mu_b}}, \frac{1}{k} = \frac{\varphi R_0^2}{k^*}, \ r = h \\ &Sc = \frac{\upsilon_b}{D_m}, \delta^* = \frac{\delta R_0 e^{at}}{R_T}, Pr = \frac{\mu_b c_b}{k_{Tb}}, P = \frac{R_0^3 P^*}{\lambda\mu_b\upsilon_b}, Rd_2 = \frac{Q_1 R_0^2\left(C_w - C_\infty\right)}{k_{Tb}\left(T_w - T_\infty\right)} \end{aligned}\right\} \quad (7)$$

With the help of equation (7), the governing equations (1)-(6) are reduced to:

$$\frac{R}{R_0} = \begin{cases} 1 - \dfrac{\delta}{2R_T} e^{at}\left(1 + cos2\pi x\right) & \text{at } \ d_0 \le x^* \le \lambda^* \\ 1 & \text{at } \ 0 \le x^* \le d_0 \end{cases} \quad (8)$$

where

$$x = \frac{1}{\lambda}\left(d_0 + \frac{\lambda}{2}\right) \quad (9)$$

$$\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial x} + \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \frac{1}{k}w - M^2 w + Gr\theta \quad (10)$$

$$Pr\frac{\partial \theta}{\partial t} = \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) - Rd_1 Pr\theta + Rd_2\phi \quad (11)$$

$$Sc\frac{\partial \phi}{\partial t} = \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r}\right) - Rd_3 Sc\phi \quad (12)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &w \neq 0, \theta \neq 0, \phi \neq 0 \ \text{ at } \ r = 0 \\ &w = 0, \theta = 1, \phi = 1 \ \text{ at } \ r = \frac{R}{R_0} \end{aligned}\right\} \quad (13)$$

Since the pressure is generated by the ventricular action and in the axial direction, in order to obtain the various profiles according to Kubugha and Amos [12], let us consider the equations (10)-(13) to be in the following form:

$$\left.\begin{aligned} w(r,t) &= w_0(r)e^{i\omega t}, \theta(r,t) = \theta_0(r)e^{i\omega t} \\ \phi(r,t) &= \phi_0(r)e^{i\omega t}, -\frac{\partial P}{\partial x} = P_0 e^{i\omega t} \end{aligned}\right\} \quad (14)$$

In order to reduce equations (10)-(13) to an ordinary differential equation, we apply equation (14), which reduces them to:

$$\frac{d^2 w_0}{dr^2} + \frac{1}{r}\frac{dw_0}{dr} - \beta_1^2 w_0 = P_0 - Gr\theta_0 \quad (15)$$

$$\frac{d^2 \theta_0}{dr^2} + \frac{1}{r}\frac{d\theta_0}{dr} - \beta_2^2 \theta_0 = -Rd_2\phi_0 \quad (16)$$

$$\frac{d^2 \phi_0}{dr^2} + \frac{1}{r}\frac{d\phi_0}{dr} - \beta_4^2 \phi_0 = 0 \quad (17)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &w_0 \neq 0, \theta_0 \neq 0, \phi_0 \neq 0 && \text{at } r = 0 \\ &w_0 = 0, \theta_0 = e^{-i\omega t}, \phi_0 = e^{-i\omega t} && \text{at } r = \frac{R}{R_0} \end{aligned}\right\} \quad (18)$$

2.6. Method of Solution

To solve equations (15)-(17) with the boundary conditions in equation (18), let us apply the Laplace method. The method can be stated as:

$$L\{w_0(r)\} = w_0(s) = \int_0^\infty w_0(r)e^{-rs}dr \quad (19)$$

$$L\{\theta_0(r)\} = \theta_0(s) = \int_0^\infty \theta_0(r)e^{-rs}dr \quad (20)$$

$$L\{\phi_0(r)\} = \phi_0(s) = \int_0^\infty \phi_0(r)e^{-rs}dr \quad (21)$$

To solve equation (17), we shall apply equation (21), which is

$$L\left\{r\frac{d^2\phi_0}{dr^2}\right\} + L\left\{\frac{d\phi_0}{dr}\right\} + \beta_{41}^2 L\{r\phi_0\} = 0 \quad (22)$$

Where

$$\beta_{41} = i\beta_4$$

Simplifying equation (22), we have

$$L\left\{r\frac{d^2\phi_0}{dr^2}\right\}+L\left\{\frac{d\phi_0}{dr}\right\}+\beta_{41}^2L\{r\phi_0\}=0=-\frac{d}{ds}\left(s^2\phi_0(s)-s\phi_0(0)-\dot{\phi}_0(0)\right)+s\phi_0(s)-\phi_0(0)-\beta_{41}^2\frac{d\phi_0}{ds} \quad \text{.................. (23)}$$

Simplifying equation (23), we have

$$\frac{d\phi_0}{ds}+\frac{s}{\left(s^2+\beta_{41}^2\right)}\phi_0(s)=0 \quad \text{.................................. (24)}$$

Solving equation (24), we have

$$\phi_0(r)=\left(\frac{e^{-i\omega t}}{I_0(\beta_4h)}\right)I_0(\beta_4r) \quad \text{.................................. (25)}$$

where

$$J_0(i\beta_4r)=I_0(\beta_4r)$$

The lipid concentration profile is obtained after substituting equation (25) into equation (14), which is

$$\phi(r,t)=\left(\left(\frac{e^{-i\omega t}}{I_0(\beta_4h)}\right)I_0(\beta_4r)\right)e^{i\omega t} \quad \text{.................................. (26)}$$

To investigate the effect of lipid concentration on the fluid temperature, let us differentiate equation (25) twice and substitute the resulting solution into equation (17), which is:

$$\frac{d^2\theta_0}{dr^2}+\frac{1}{r}\frac{d\theta_0}{dr}-\beta_2^2\theta_0=-\left(\frac{Rd_2e^{-i\omega t}}{I_0(\beta_4h)}\right)\left(1+\frac{\beta_4^2r^2}{4}+\frac{\beta_4^4r^4}{64}+\frac{\beta_4^6r^6}{2304}+\dots\right) \quad \text{.................................. (27)}$$

Applying the Laplace method in solving equation (27), the homogenous solution is:

$$\theta_{0h}(r)=A_0I_0(\beta_2r) \quad \text{.................................. (28)}$$

The particular solution to equation (27) is:

$$\theta_{0p}(r)=A_1+A_2r^2+A_3r^4 \quad \text{.................................. (29)}$$

So that the general solution of equation (27), which is

$$\theta_0(r)=A_0I_0(\beta_2r)+A_1+A_2r^2+A_3r^4 \quad \text{.................................. (30)}$$

$$A_1=\frac{Rd_2e^{-i\omega t}}{\beta_2^2I_0(\beta_4h)}\left(1+\left(\frac{\beta_4^2}{\beta_2^2}+\frac{\beta_4^4}{\beta_2^4}\right)\right),\ A_2=\left(\frac{Rd_2e^{-i\omega t}}{I_0(\beta_4h)}\right)\left(\frac{\beta_4^2}{4\beta_2^2}+\frac{\beta_4^4}{4\beta_2^4}\right),\ A_3=\frac{\beta_4^4}{64\beta_2^2}\left(\frac{Rd_2e^{-i\omega t}}{I_0(\beta_4h)}\right)$$

See appendix for all the other coefficients.

The temperature profile is obtained after substituting equation (30) into equation (14), which is:

$$\theta(r,t)=\left(A_0 I_0\left(\beta_2 r\right)+A_1+A_2 r^2+A_3 r^4\right)e^{i\omega t} \qquad (31)$$

where

$$\beta_{21}=i\beta_2$$

To investigate the effect of temperature change on blood momentum, we shall substitute equation (30) into equation (15), which is

$$\frac{d^2 w_0}{dr^2}+\frac{1}{r}\frac{dw_0}{dr}-\beta_1^2 w_0=P_0-GrA_1-Gr\left(A_0 I_0\left(\beta_2 r\right)+A_2 r^2+A_3 r^4\right) \qquad (32)$$

Applying equation (19) on the homogenous part of equation (32), this is:

$$L\left\{r\frac{d^2 w_0}{dr^2}\right\}+L\left\{\frac{dw_0}{dr}\right\}+\beta_{11}^2 L\left\{rw_0\right\}=0 \qquad (33)$$

Simplifying equation (33), we have

$$L\left\{r\frac{d^2 w_0}{dr^2}\right\}+L\left\{\frac{dw_0}{dr}\right\}+\beta_{11}^2 L\left\{rw_0\right\}=0=-\frac{d}{ds}\left(s^2 w_0(s)-sw_0(0)-\dot{\theta}_0(0)\right)+sw_0(s)-w_0(0)-\beta_{11}^2\frac{dw_0}{ds} \qquad (34)$$

Simplifying equation (34), we obtained the homogenous solution as:

$$w_{0h}(r)=L^{-1}\left\{\frac{B_3}{\sqrt{\left(s^2+\beta_{11}^2\right)}}\right\}=B_3 L^{-1}\left\{\frac{1}{\sqrt{\left(s^2+\beta_{11}^2\right)}}\right\}=B_3 J_0\left(\beta_{11} r\right)=B_3 J_0\left(i\beta_1 r\right) \qquad (35)$$

Whereas the particular solution of equation (32) is as follows:

$$w_{0p}(r)=A_4+A_6 I_0\left(\beta_2 r\right)+A_7 r^2+A_8 r^4$$

$$A_4=-\frac{P_0}{\beta_1^2}+\frac{GrA_1}{\beta_1^2}+4\left(\frac{GrA_2}{\beta_1^4}+\frac{16GrA_3}{\beta_1^6}\right),A_6=\frac{GrA_0}{\beta_1^2},A_7=\frac{GrA_2}{\beta_1^2}+\frac{16GrA_3}{\beta_1^4},A_8=\frac{GrA_3}{\beta_1^2}$$

The solution for equation (32) is as follows:

$$w_0(r)=B_3 I_0\left(\beta_1 r\right)+A_4+A_6 I_0\left(\beta_2 r\right)+A_7 r^2+A_8 r^4 \qquad (37)$$

$$B_3=-\frac{A_4}{I_0\left(\beta_1 h\right)}-\frac{A_6 I_0\left(\beta_2 h\right)}{I_0\left(\beta_1 h\right)}-\frac{A_7 h^2}{I_0\left(\beta_1 h\right)}-\frac{A_8 h^4}{I_0\left(\beta_1 h\right)} \qquad (37)$$

Having simplified the concentration effect on blood velocity, the solution for equation (32) is:

$$w(r,t)=\left(B_3 I_0\left(\beta_1 r\right)+A_4+A_6 I_0\left(\beta_2 r\right)+A_7 r^2+A_8 r^4\right)e^{i\omega t} \qquad (38)$$

The volumetric flow rate can be calculated using the mathematical formula

$$Q=2\pi\int_{r=0}^{r=h} w(r,t)\,rdr \qquad (39)$$

Using equation (39) to calculate the flow rate, we have:

$$Q = 2\pi e^{i\omega t} \int_{r=0}^{r=h} \left(B_3 r I_0(\beta_1 r) + A_4 r + A_6 r I_0(\beta_4 r) + A_7 r^3 + A_8 r^5 \right) dr \quad \text{..................................... (40)}$$

Simplifying equation (40), we have:

$$Q = 2\pi e^{i\omega t} \left(B_3 h I_1(\beta_1 h) + A_6 h I_1(\beta_4 h) + \frac{A_4 h^2}{2} + \frac{A_7 h^4}{4} + \frac{A_8 h^6}{6} \right) \quad \text{..................................... (41)}$$

3. Results

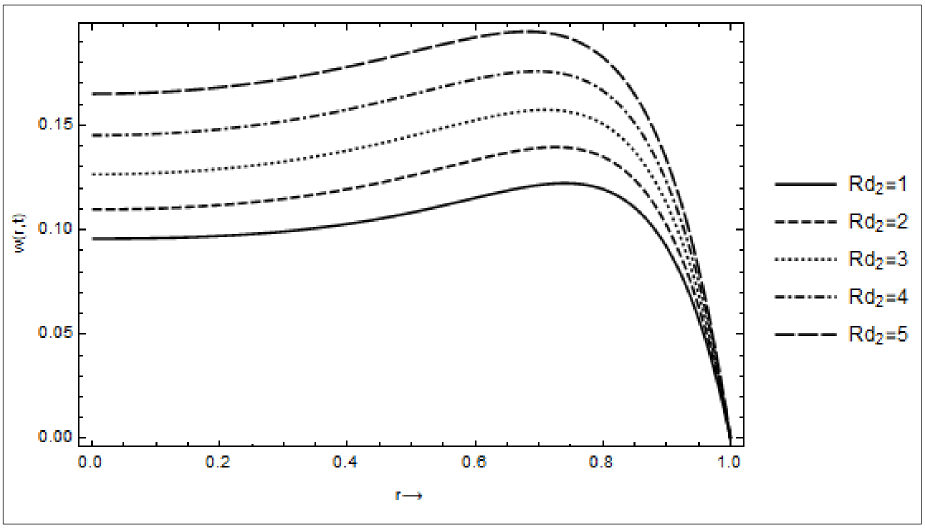

Figure 1 Effect of metabolic heat parameter on velocity with other values as

$$Gr = 15, Pr = 2.1, Sc = 2, Rd_3 = 2, Rd_1 = 2, \omega = 0.3, k = 0.05, M = 1.5, x = 0.5$$

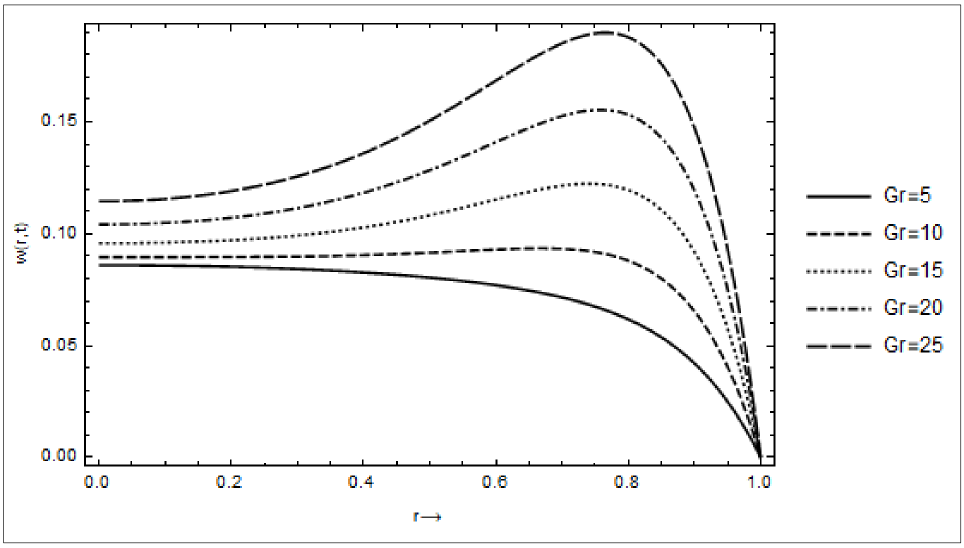

Figure 2 Effect of Grashof number on velocity with other values as

$$Rd_2 = 1, Pr = 2.1, Sc = 2, Rd_3 = 2, Rd_1 = 2, \omega = 0.3, k = 0.05, M = 1.5, x = 0.5$$

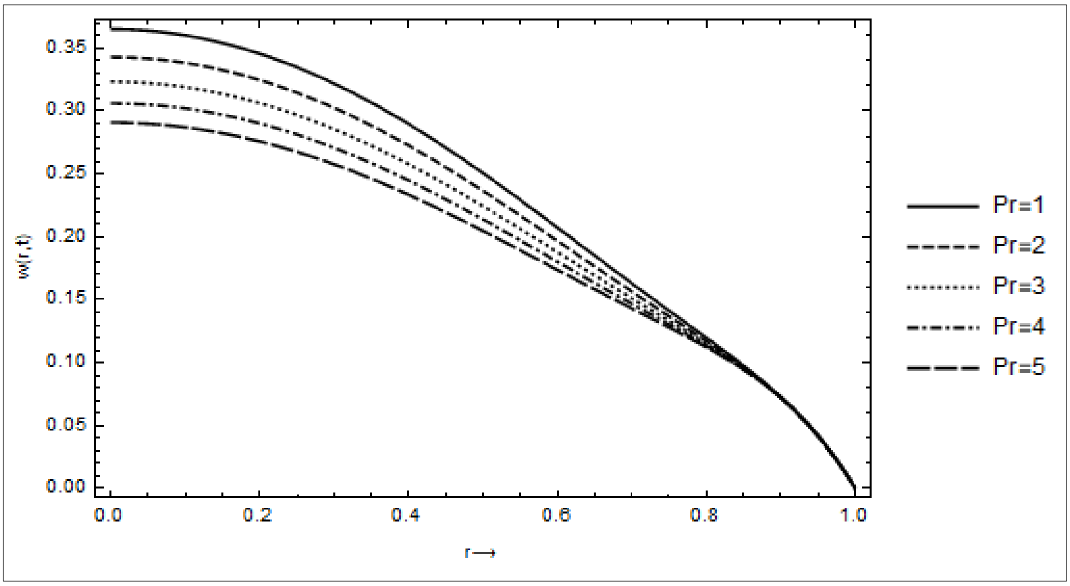

Figure 3 Effect of Prandtl number on velocity with other values as

$$Gr = 15, Rd_2 = 1, Sc = 2, Rd_3 = 2, Rd_1 = 2, \omega = 0.3, k = 0.05, M = 1.5, x = 0.5$$

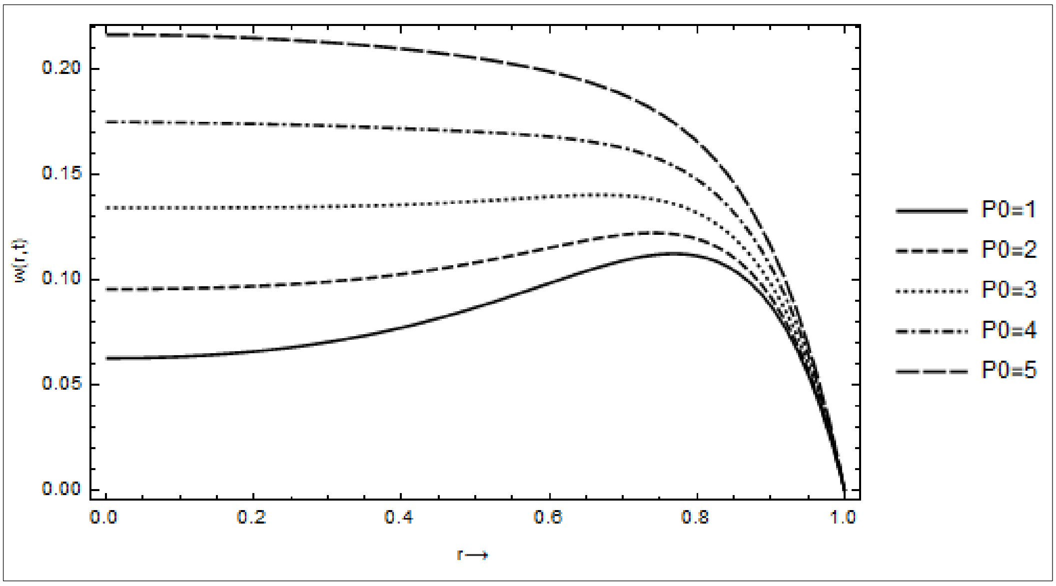

Figure 4 Effect of Pressure change on velocity with other values as

$$Gc = 15, Pr = 2.1, Sc = 2, Rd_3 = 2, Rd_2 = 1, Rd_1 = 2, \omega = 0.3, k = 0.05, M = 1.5, x = 0.5$$

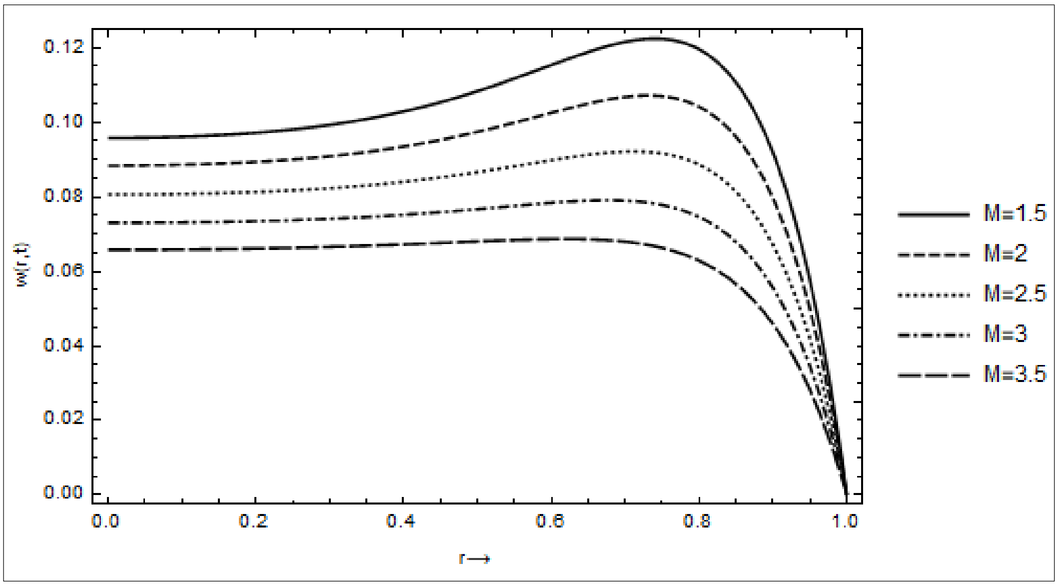

Figure 5 Effect of Magnetic field parameter on velocity with other values as

$$Gr = 15, Pr = 2.1, Sc = 2, Rd_3 = 2, Rd_1 = 2, \omega = 0.3, k = 0.05, Rd_2 = 1, x = 0.5$$

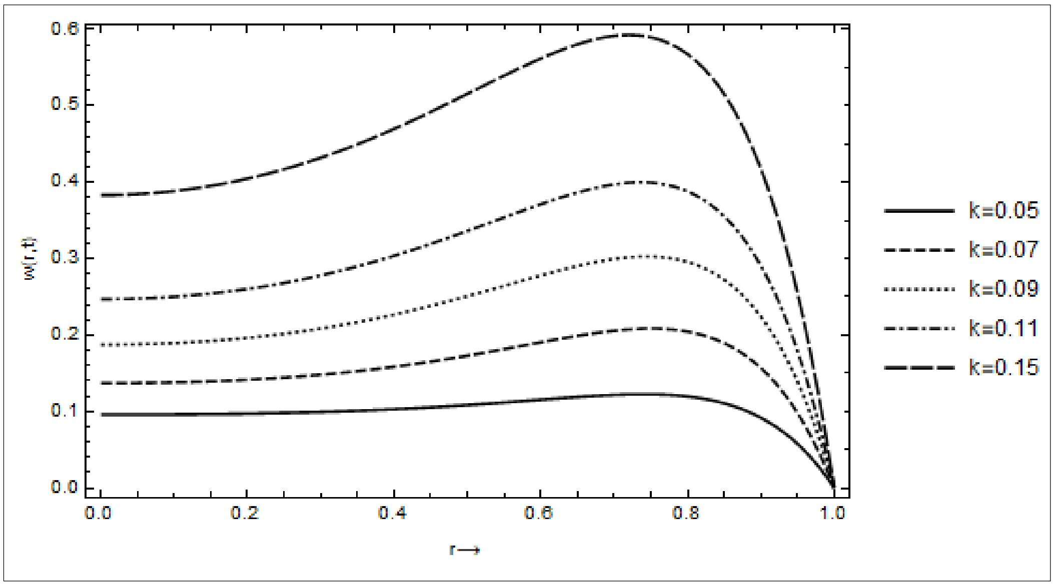

Figure 6 Effect of porosity parameter on velocity with other values as

$$Gr = 15, Pr = 2.1, Sc = 2, Rd_3 = 2, Rd_2 = 1, Rd_1 = 2, \omega = 0.3, M = 1.5, x = 0.5$$

4. Discussion

Figure 1 illustrates the impact of the metabolic heat on the velocity profile of the fluid. It was observed that the fluid velocity increased from 0.095724, 0.109743, 0.126569, 0,145229, and 0.165103 for an increase in metabolic heat from 1 to 5 units, respectively. The investigation was carried out at the arterial segment of $x = 0.5$. The effect of thermal Grashof number was investigated and the result is presented in Figure 2. The figure shows that the fluid velocity increases for different values of the Grashof number, while every other parameter value as shown in the figure remains the same. The result shows a decrease in fluid velocity at the segment, with the velocity decreasing as 0.365028, 0.342867, 0.323376, 0.306164, and 0.290907 and the Prandtl number increasing from 1 to 5 units, as shown in the figure. In this research, we also investigated the impact of the change in systolic pressure gradient due to the pumping rate in the vessel as against the velocity profile as depicted in Figure 4. The result illustrates an increase in velocity of 0.0630003, 0.095724, 0.134253, 0.17479, and 0.216209 for different units of pressure gradient. This result is a clear indication that faster flow can be achieved if we induce the pressure, which is a very useful study in resuscitating cardiac arrest patients. Figure 5 shows the effect of magnetic therapy treatment on fluid flow through a sclerotic segment. The result indicates that the fluid velocity decreases with an increase in magnetic field intensity. This reduction is as a result of the Lorentz force, which opposes the flow of electrically conducting fluids such as blood. The velocity decrease is seen as 0.095724, 0.08832, 0.0805624, 0.0729485, and 0.065759. The level of porosity of the channel and fluid velocity were investigated as shown in Figure 6. The result indicates that the fluid velocity increases by 0.095724, 0.136806, 0.187319, 0.246793, and 0.383187 for an increase in the porosity of the channel. The results obtained in this research are highly correlated with previous results by Bunonyo and Eli [13], Bunonyo and Ebiwareme [14], Hanvey and Bunonyo [15] and Misra and Adhikary [16], respectively.

Nomenclature

- x^* : Dimensional coordinate along the channel
- r^* : Dimensional coordinate perpendicular to the channel
- R : Radius of an abnormal channel
- R_0 : Radius of normal channel
- P_0 : Systolic pressure
- Rd_2 : Metabolic heat parameter
- Rd_3 : Chemical reaction parameter
- k_{Tb} : Blood thermal conductivity
- w^* : Dimensional velocity profile
- w : Dimensionless velocity profile
- w_0 : Perturbed velocity profile
- C^* : Dimensional lipid particle concentration
- C_∞ : Far field cholesterol particle concentration
- c_{bp} : The specific heat capacity of the fluid
- t^* : Dimensionless time
- T : Temperature of the fluid
- T_∞^* : Far field temperature
- T_w^* : Temperature at the wall
- B_0 : Magnetic field
- M : Magnetic field parameter
- a : Growth rate of LDL-cholesterol

Greek Symbols

- υ_b : Kinematic viscosity of the fluid
- μ_b : Dynamic viscosity of the fluid
- Pr : Prandtl number of the fluid
- g : Acceleration due to gravity
- δ^* : Dimensional height of stenosis
- σ_e : Electrical conductivity

- λ^* : Length of stenosis
- ω : Oscillatory frequency
- θ : Dimensionless temperature of the fluid
- ϕ : Dimensionless concentration of lipid in the fluid
- θ_0 : Perturbed temperature of the fluid
- ϕ_0 : Perturbed lipid concentration
- ρ_b : Density of the fluid

Subscripts

- w : Wall
- b : Blood
- e : Electrical
- T : Thermal
- ∞ : Far field
- **MMDARG**: Mathematical Modeling and Data Analytics Research Group

Appendix

where

$$\beta_1^2 = \left(\frac{1}{k} + M^2 + i\omega\right), \beta_2^2 = \left(Rd_1 + i\omega\right)Pr, \beta_4^2 = \left(Rd_3 + i\omega\right)Sc$$

$$\left.\begin{aligned} A_0 &= \frac{e^{-i\omega t}}{I_0\left(\beta_2 h\right)} - \frac{A_1}{I_0\left(\beta_2 h\right)} - \frac{A_2 h^2}{I_0\left(\beta_2 h\right)} - \frac{A_3 h^4}{I_0\left(\beta_2 h\right)}, A_1 = \frac{Rd_2 e^{-i\omega t}}{\beta_2^2 I_0\left(\beta_4 h\right)}\left(1 + \left(\frac{\beta_4^2}{\beta_2^2} + \frac{\beta_4^4}{\beta_2^4}\right)\right), \\ A_2 &= \frac{Rd_2 e^{-i\omega t}}{4 I_0\left(\beta_4 h\right)}\left(\frac{\beta_4^2}{\beta_2^2} + \frac{\beta_4^4}{\beta_2^4}\right), A_3 = \frac{\beta_4^4}{64\beta_2^2}\frac{Rd_2 e^{-i\omega t}}{I_0\left(\beta_4 h\right)}, A_4 = -\frac{P_1}{\beta_1^2} + 4\left(\frac{GrA_2}{\beta_1^4} + \frac{16GrA_3}{\beta_1^6}\right), \end{aligned}\right\}$$

$$A_6 = \frac{GrA_0}{\beta_1^2}, A_7 = \frac{GrA_2}{\beta_1^2} + \frac{16GrA_3}{\beta_1^4}, A_8 = \frac{GrA_3}{\beta_1^2}, P_1 = P_0 - GrA_1$$

$$B_3 = -\frac{A_4}{I_0\left(\beta_1 h\right)} - \frac{A_6 I_0\left(\beta_2 h\right)}{I_0\left(\beta_1 h\right)} - \frac{A_7 h^2}{I_0\left(\beta_1 h\right)} - \frac{A_8 h^4}{I_0\left(\beta_1 h\right)}$$

5. Conclusion

In this paper, we have studied incompressible non-Newtonian fluid flow through a lipid-concentrated cylindrical channel with the effect of metabolic heat and magnetic treatment. The governing equations were solved using the Laplace method. From the study, we conclude as follows:

- An increase in metabolic heat causes an increase in the velocity of the fluid.
- An increase in the Grashof number resulted in an increase in the velocity of the fluid.
- The velocity of the fluid is seen to have decreased as a result of the increase in Prandtl number.
- For different unit values of pressure, there is a corresponding increase in the velocity of the fluid.
- The effect of the Lorentz number caused a decrease in the velocity of the fluid for different values of the magnetic field intensity.
- The velocity of the fluid was observed to increase with an increase in the porosity.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors have not reported any conflicts of interest.

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