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Analysis of fixation exercises for a High School class on quantized Space-time via Loop Quantum Gravity

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Abstract

The work addresses a part of the application with a class of 3rd year of high school, being a master's research project in Physics Teaching of the Federal University of the State of Rio de Janeiro, Brazil, within the program of the Brazilian Society of Physics, through the National Professional Master's degree in Physics Teaching. We treat in an introductory and high school level about the aspect of the quantization of space-time in the approach of Quantum Gravity in Loop and apply a fixation exercise for analysis of the results of the student-researchers.

Keywords: Space-Time; Spin network; Student-researchers; Quantization

1. Introduction

The study is part of one of the classroom applications of a master's research in Physics Teaching at the Federal University of the State of Rio de Janeiro. It was applied to a class in Saint Cristopher, Rio de Janeiro, Brazil, with 23 students from the 3rd year of high school. This work sought an innovative proposal using the classroom research method, making students have a critical constructivist character in the educational process and researcher posture, being called student-researchers.

The challenge also occurred in the introduction of a theme still under development in the field of research for the quantization of gravity, in which we currently have two most promising theories being superstrings and quantum gravity in loop. The student-researchers took previous lessons on Planck's constant, quantum numbers and spin projections to later arrive at the proposed theme of space-time quantization. In this article will be presented some topics covered as the concept of space-time, space-time as quantum object, spin network as graph-shaped structure of its quantization, application, and discussion of the results.

2. The concept of space-time

We started the work and survey of questions through the initial concept of space-time. We present the ideas of René Descartes (1569-1650), where he considered everything related to the movement or rest of a body, with dependence on a reference for the analysis of such body, considering that we live in three dimensions.

For Isaac Newton (1642-1727) in a thought opposite to Descartes, he believed in the eternity and immobility of space, as he considered it to be absolute and time following the constant regular course, regardless of anything. For Albert Einstein (1879-1955), he believed that differentiating past, present and future would be just an illusion.

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Throughout this classroom discussion, the students realized that there was an idea of their own for the time of space and time. The idea of the junction of space and time came with Minkowski (1864-1909), where he used the space-time diagram, because he believed that the union would preserve independence. He also considered time as a dimension, being three spatial coordinates and one in time, where we would live in four dimensions according to this idea [1] .

Several studies are being discussed on the space-time issue throughout this time for the scientific community and according to Hendrik Lorentz (1853-1928), he sought to explain the propagation of light in space-time, making the substitutions of Galileo's transformations, calling Lorentz transformations. These transformations have shown that the space-time coordinates of a given physical event depend on an inertial reference from which they are being observed [2].

We impose D *the* interval in space-time, associated with the coordinates, being invariant in relation to Lorentz transformations, and for the ordered pair (t, x), we have the form

$$D^2 = c^2(\Delta t)^2 - (\Delta x)^2 \quad (1)$$

Where Δt the temporal distance and Δx the spatial distance between events. And to study and better understand this discussion, we present the space-time diagram, to map the events, according to the graphic 1.

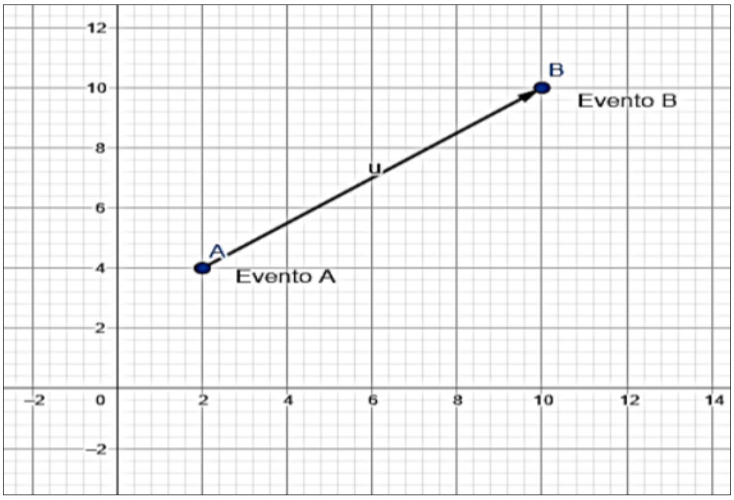


Figure 1 Example of a space-time diagram. Horizontal axis representing space and vertical axis time. Offset from event A to B

The diagram is also known as the Minkowski diagram. The trajectory from A to B is called the world line, where it indicates the variation of the position as a function of time. For the case of four dimensions, we will have in the form (t, x, y, z). We will see between two events (x_1, y_1, z_1) and (x_2, y_2, z_2) and for the relativistic interval, we must

$$D^2 = c^2(t_2 - t_1)^2 - [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^2 \quad (2)$$

Which are the Lorentz transformations and depending on the interval we $D^2 > 0$will have, being of the time $D^2 < 0$type, being of the space type and $D^2 = 0$, the interval of the light type [3].

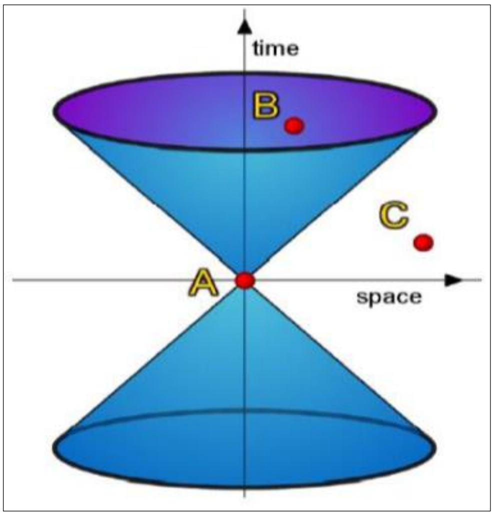


Figure 2 Representation of events in the light cone. A light type, B type time and C type space [**3**]

In the studies with the student-researchers, we *considered c = 1*, to facilitate the calculations.

2.1. Space-time as quantum object

Through the need to quantize gravity, because the other elementary forces of nature were quantized namely: Weak, strong, and electromagnetic, we have the gravitational, but we still find some conflicts for its quantization. General Relativity describes the geometry of space-time, considering the effects of gravity, describing the Universe, and celestial objects, with their interactions, giving a quantitative treatment with enormous values. When we unify with quantum mechanics, conflicts appear, because it's like we're going to measure tiny particles with a very large ruler [4].

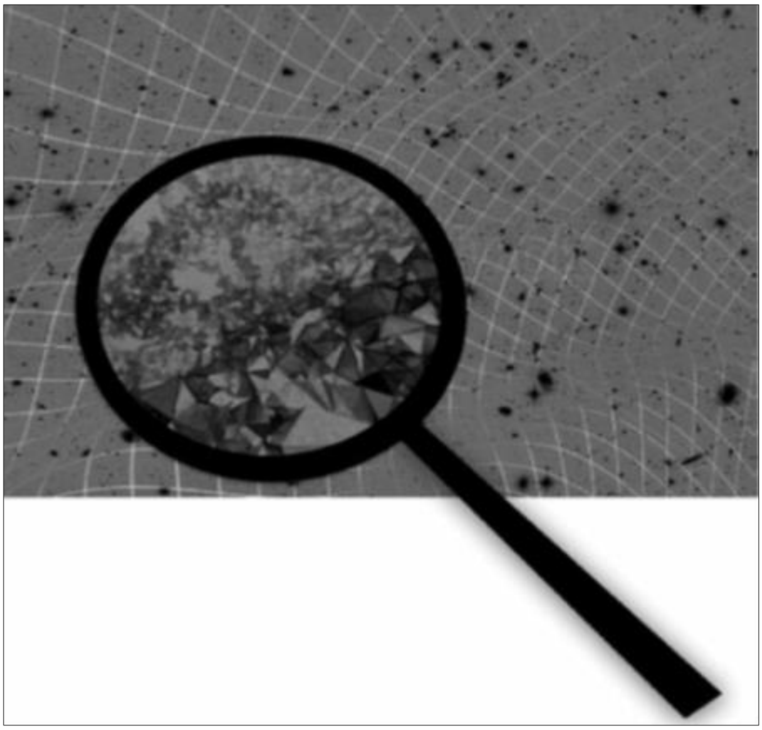


Figure 3 Ultra powerful lens showing the granular structure of space [**5**]

In the proposal of work with students, we developed an introductory discussion about the geometric structure of space-time in the approach of Loop Quantum Gravity (LQG). From the point of view of LQG, space-time is like a shirt that we see from afar as a piece of cloth, where it gives the idea of continuous and when we approach our view and put a magnifying glass, we observe that this shirt, have separate wires. If the quantizing process involves packet analysis discretely, then the LQG theory observes space-time in a discrete way as well[4].

As general relativity considers the geometry of space-time, then LQG suggests this type of proposal for its quantization. The Loops that appear at the end of the theory are the atoms of space intertwined in similar ones, forming a network of relationships that weave space. This network is the space itself[5].

2.2. Spin network

The region of the space is granular and when moving from one grain to another until closing a circuit, returning to the starting grain, we make a ring or loop[6] . In the quantization process, considering the geometry of space time, the area and volume operators of platonic solids will now be treated. In the case of spin is where it is observed in elementary particles are intrinsic in their structure. In the case of fermions that are particles of matter, their spins are semi-integers while the bosons that are mediating particles possess the entire spin.

In the LQG theory, in the discussion of the spin network, we will work on the geometric structure using the area and volume operated of a platonic solid, as well as the junction of the spin treatment. Initially we will discuss its geometric structure. For the quantization process it is natural to use the rule. Areas and volumes are restricted with specific quantities with certain combinations of numbers for the lines and nodes of the network. It's like putting together geometric figures respecting your regular space. For example, we see in the figure the pyramid on top of the cube, figure (c), then we have two polyhedral, then we will have two volumes, as a result, two nodes. For each face, we will have the lines connected to the nodes that connect the polyhedral, where we will have their respective spin network, figure 3 (d)[4].

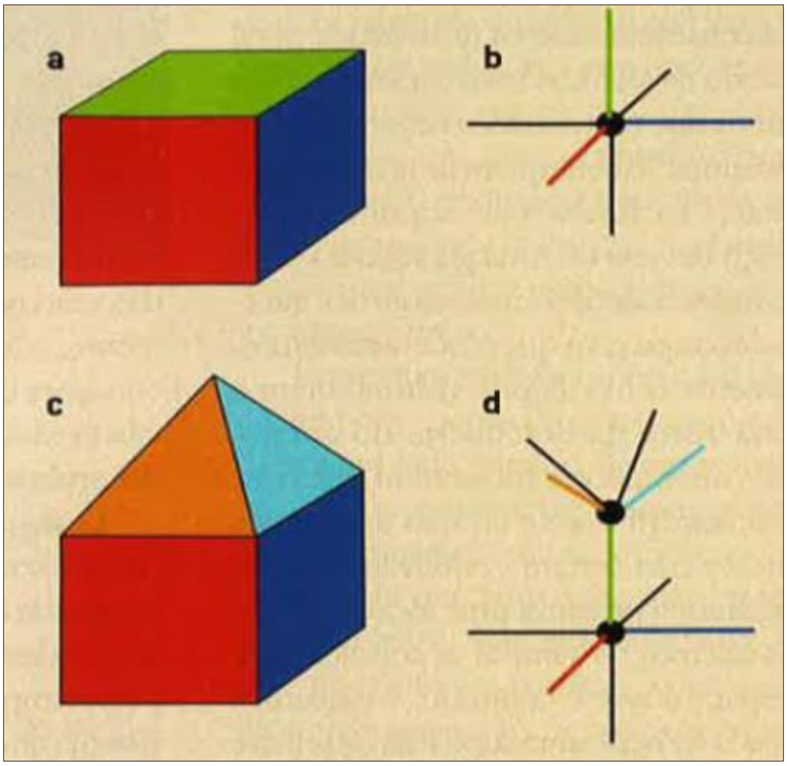


Figure 4 a) cube. b) hub net. c) pyramid on top of the cube. d) pyramid-cube joint network [**4**]

Because the volume spectrum is discrete, in the representation of the spin network, representing a graph, with nodes and lines, in which each node represents the volume and line the area. [6].

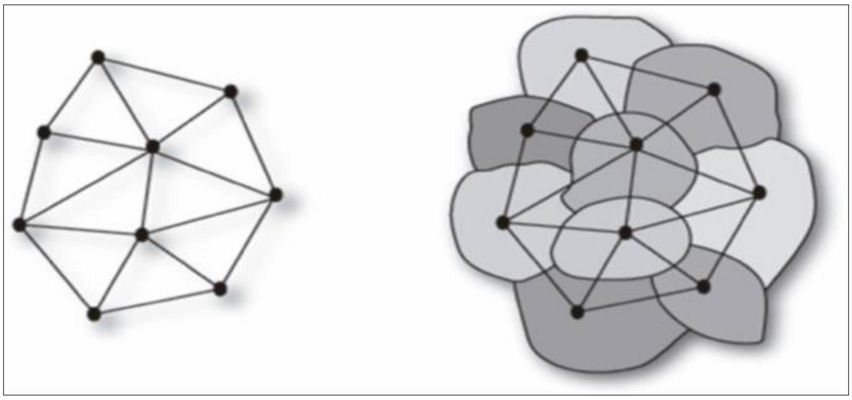


Figure 5 On the left, a graph formed by nodes and lines. On the right, the grains of space that the graph represents. The lines represent the adjacent grains, separated by surfaces [**6**]

About the quantization of the geometry of space-time it is necessary to analyze the geometric object that will be analyzed. As the regime is quantum it is natural the emergence of Planck's constant in the description of the operators. Before we treat the area and volume operators, we have the length, being quantized in the form

$$\hat{L} = 8\pi\hbar G\sqrt{j(j+1)} \qquad (3)$$

Where the spins j accompanies the quantization process, so spin network. Spins on a triangular basis of the network obey the same principle as triangular inequalities. For example, a triangle with sides L_1, L_2 *and* L_3, we have for inequalities

$$|L_1 - L_2| \leq L_3 \leq L_1 + L_2 \qquad (4)$$

In this work with the student-researchers, we used the geometric figure of the tetrahedron for an initial didactic exploration of the subject, but we also developed other geometric figures. When using the tetrahedron, its geometry is characterized by the length of its sides, the area of the faces, its volume, the dihedral angles at its edges and the angles at the vertices of their faces. Because geometry is the same thing as the gravitational field, these geometric amounts are related to each other. We will not consider the dihedral angles and angles at the vertices in our study.

The tetrahedron has 4 faces, which are 4 triangles, $\overrightarrow{L_a}$ so the vector area passing through the center of each triangle are the normal vectors that represent the lines that depart from a node. Its norm is the area of the face [7].

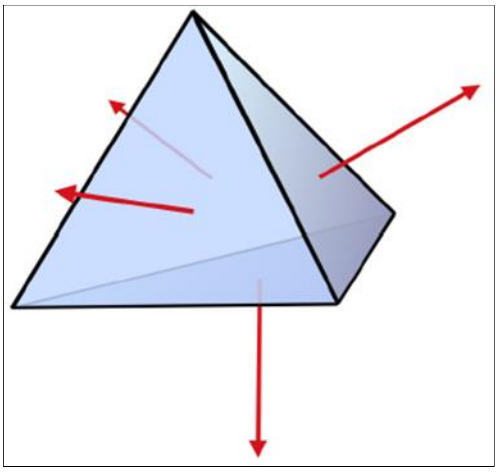


Figure 6 The four normal vectors of tetrahedron faces [**7**]

Area operators are responsible for the quantization process of the space-time area. For example, if a spin network crosses an *S surface*, cross-sectionally, then this surface has an area defined in this state, given as the sum over the j spins *entering* the edges, staying in the shape

$$\hat{A} = 8\pi\gamma\sqrt{j(j+1)}, \text{with } \ell_P = 1 \text{ e } j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \ldots \tag{5}$$

For Planck length equal to 1. The area operator has a discrete spectrum. If we consider the length of Planck, we will have for the area operator

$$\hat{A} = 8\pi\gamma{\ell_P}^2\sqrt{j(j+1)} \ldots \tag{6}$$

Or the way

$$\hat{A} = 8\pi\gamma\hbar G\sqrt{j(j+1)}, with\ c = 1 \ldots \tag{7}$$

In the case of the tetrahedron, we have 6 edges, 4 areas (faces) and 4 vertices. What is the quantum state of the tetrahedron? As only the spectrum of the area and volume are considered, we have the 4 areas and only 1 volume, so the quantum state of the tetrahedron is 5. For example, what is the quantum state of the cube? We have 6 areas and 1 volume, so its quantum state is 7.

There are two main characteristics of quantum geometry which is the area and volume and for didactic understanding, considering the constants that precede the relationship of the spin, being $8\pi\gamma{\ell_P}^2 = 1$, then we will have for the area the form $\hat{A} = \sqrt{j(j+1)}$. Considering *the* side of a tetrahedron, then the area of one of the faces of this tetrahedron is of the form

$$A_F = \frac{a^2\sqrt{3}}{4} \ldots \tag{8}$$

$$a^2 = \frac{4A_F}{\sqrt{3}}, with\ a = \left(\frac{4A_F}{\sqrt{3}}\right)^{\frac{1}{2}} \ldots \tag{9}$$

Since we are studying the quantization of geometry, then we can relate $\hat{A} = A_F$. In the case of tetrahedron volume, we have [8].

$$V = \frac{a^3\sqrt{2}}{12} \ldots \tag{10}$$

When making the relationship of Equations (8) and (9) with Equation (10), isolating a, we have for the volume of the tetrahedron in the form

$$V = \hat{A}^{\frac{3}{2}}.2^{\frac{3}{2}}.3^{-\frac{7}{4}} \ldots \tag{11}$$

If $\hat{A} = \sqrt{j(j+1)}$, then the volume of the tetrahedron is in the

$$\hat{V} = 2^{\frac{3}{2}}.3^{-\frac{7}{4}}\left(\sqrt{j(j+1)}\right)^{\frac{3}{2}} \ldots \tag{12}$$

Let's take another example with the cube. If the cube side is a, the area of one of the faces is of the shape

$$A_F = a^2 \ldots \tag{13}$$

In the case of the volume of the cube, we have in the

$$V = a^3 \ldots \tag{14}$$

Also, to the previous example, making the ratio of Equations (13) and (14), we have for the quantized volume of the cube, in the form

$$\hat{V} = \left(\sqrt{j(j+1)}\right)^{\frac{3}{2}} \quad (15)$$

We could mention several polyhedral making the area and volume relationships quantized for didactic purposes, but the objective is to understand the real meaning that the theory addresses, showing us the importance of the geometric structure of space-time [8].

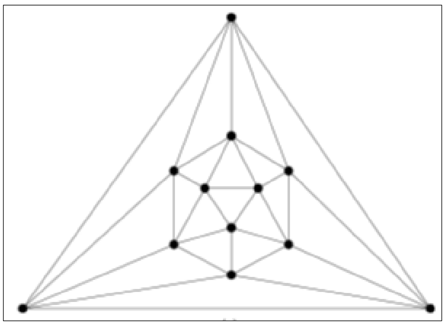
3. Application and discussion of results

We applied the fixation exercises for a class with 23 students, containing 5 questions on the proposed topic in the classroom. The following is the questionnaire applied:

- Consider the coordinates of space-time in the shape (t, x). Mark the alternative that represents the relativistic interval of the time type.

  (A) P(3,6) e Q(1, 4) (B) M(8, 2) e N(2, 6) (C) E(2, 3) e F(5, 12) (D) R(7, 5) e S(4, 8)

- In space-time in four dimensions of the shape (t, x, y, z), we have an event from A (8, 6, 6, 7) to B (t, 12, 8, 4). What t-values of event B will we have for the relativistic interval to be light type?
- Consider the spin network in the figure below. What is the quantum state of this network?



- Is it possible to build a triangular-based spin network with spins j = 1/2, 3/2 and 1?
- Consider the structure of space-time in the form of an icosahedron. The face area is $A_F = \frac{a^2\sqrt{3}}{4}$ and its volume is $V = \frac{5a^3(3+\sqrt{5})}{12}$. What is the representation of the quantized volume in this structure with spin j = 1/2, knowing that $\hat{A} = A_F = \sqrt{j(j+1)}$?

The correct option is the letter B because it shows, according to the classroom work on the treatment of the light cone and about 91% of the student-researchers settled the question, demonstrating the ability to formalize the contents through mathematical methods, when well worked, because they initially had difficulties. The importance of exercises is precisely to condition students in which, in the case of physics, they will come across, especially in mathematical issues. This percentage if there was no interaction with the teacher prior to the delivery of the exercise sheets, it would certainly be very reduced, because they could make confusion of the space-time positioning, as did the 9% of the students who could not get it right and one of the factors for the effectiveness in solving the problems is attention, even in seemingly easy questions to students.

The second question is discursive, and the student-researchers should develop in their calculations and find the correct value and about 74% of the student-researchers effectively were able to answer the statement. The question already involves more variables and comes to a 2nd degree equation. Attention is doubled to the student in the resolution of the calculations. Misconceptions are perceived in the second term of the equation that corresponds to that of spatial coordinates. At the site of the sum, they placed subtraction and others at the subtraction site the sum. Surely the result will be different.

In the third question about 87% of the students settled the question. A relatively easy question, because it is necessary only to count the links, with the sum of the nodes.

The fourth question involves the knowledge of the calculation to form a triangle with certain values for its sides and about 96% of the students got it right, because the calculations of triangular inequality in the classroom were emphasized.

The fifth and last question, considered particularly the most difficult, places the values of the area and volume in the classical regime and the objective is to know what the value of the volume would be quantized with spin also given by the utterance. It really is an accessible but laborious issue because the student-researcher will put the spin in the face area and implement in the icosahedron volume equation. About 61% settled the issue, as it involves accuracy in the processes of replacing both the spin, both after the implementation of the volume.

After the exercises, the student-researchers discussed with the teacher in a fruitful way, removing doubts and placing their positions in relation to the fixation exercises, ensuring consistency in learning.

4. Conclusion

The proposal of the work in general was to introduce an innovative theme of Contemporary Physics in high school, as well as classroom research, making students have a more active posture, seeking answers by themselves, building their own path in the educational process. Despite some difficulties in the activities that required mathematical treatment, they realized the importance of its use in the description of Physics as Science and the role of researchers to discover new paths for humanity.

Compliance with ethical standards

Acknowledgments

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