World Journal of Advanced Research and Reviews
eISSN: 2581-9615 CODEN (USA): WJARAI
Cross Ref DOI: 10.30574/wjarr
Journal homepage: https://wjarr.com/

(Research Article)

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Simulation of unsteady MHD flow of incompressible fluid between two parallel plates using Laplace-Adomian decomposition method

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World Journal of Advanced Research and Reviews, 2022, 14(03), 136–145

Publication history: Received on 15 April 2022; revised on 04 June 2022; accepted on 06 June 2022

Article DOI: https://doi.org/10.30574/wjarr.2022.14.3.0456

Abstract

The aim of this study is to numerically obtain the solution to an unsteady incompressible fluid confined between two parallel plates with constant velocity. The nonlinear partial differential equation governing the flow is first converted to a nonlinear ordinary differential equation using similarity transformation and solved using the combination of Laplace transform and Adomian decomposition method (LADM). The model parameters affecting the flow geometry is analysed and presented in tables and graphs. The method produces an accurate, elegant, computable, and approximately convergent solution which gives great insight but with less mathematical sophistication.

Keywords: MHD; Unsteady; Incompressible fluid; Laplace Adomian decomposition method (LADM); Similarity transformation

1. Introduction

Unsteady MHD flow of an incompressible fluid between two infinitely long parallel plates has been extensively studied since the fall of the twentieth century. Technically called Couette flow is one which has a fixed lower plate and an upper plate which is moving with uniform velocity in the x direction. The pioneering study of this phenomena was first presented by Sancheti and Bhatt [1]. Since its conception, several researchers have simplified the Navier-Stokes equation with underlying assumptions to solve for the exact solution. Several methods have been applied to solve this problem for the exact solution. The unsteady MHD Couette flow of heat transfer of dusty fluid with variable viscosity of varying properties have been examined by Attia [2]. He extended the above study to include two parallel plates with variable physical properties [3]

[4] have investigated the heat and mass transfer of MHD free convection from a vertical surface incorporating viscous dissipation and ohmic heating. The viscous fluid between two oscillating plates of an unsteady hydromagnetic flows of a dusty fluid was studied by [5]. [6] have analysed the heat transfer between two parallel plates of a stretching surface. The study reveals that, the velocity component has an underlying influence on the velocity profiles as well as the concentration profile. Similarly, unsteady flow of a dusty flow of a dusty incompressible between two parallel plates under the influence of impulsive pressure gradient has been examined by [7]. Nagarajan [8] extended the above study to including viscosity and a conducting liquid between the plates. The analysis of pulsatile flow of unsteady dusty fluid through rectangular channel have been considered by [9]. [10] have studied the unsteady flow of dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. The result obtained agreed with literature. [11] have examined an extended study of MHD flow of dusty viscous conducting liquid between parallel plates.

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The fusion of Laplace transformation and Adomian decomposition (LADM) as a hybrid semi-analytical method have also been extensively applied to solve many practical problems in science and engineering. This method has an advantage over other semi-analytical methods in that it does not require perturbation, discretization, linearization, or unnecessary assumptions that affect the physical nature of the equations. This practice of academics combining methods for analytical solution though relatively new have equally attracted attention from scholars in the nonlinear science to solve several problems ranging from linear and nonlinear differential as well as functional equations, Jeffery-Hamel flow problems namely, linear, nonlinear, and coupled partial differential equation. Ebiwareme et al. [12-16] have used the LADM to embark on analytical study of the Hepatitis E Virus model, Dynamics of atmospheric carbon (iv) oxide incorporating Pade approximation, modified Adomian decomposition method for the numerical approximation of the crime deterrence model, numerical solution of the food chain ecoepidemic model and analytical treatment for the SIR infectious disease model respectively. LADM have also been applied to investigate the following problems: Numerical solution to the logistic equation, convection diffusion-dissipation equations, nonlinear coupled partial differential equations, two-dimensional viscous fluid with shrinking sheet, nth order integro-differential equations, approximate solution of nonlinear fractional differential equations, Volterra integro-differential equations, linear and nonlinear Volterra integral equations with weak kernel, system of ordinary differential equations, Newell-Whitehead-Segel equation, HIV infection model of CD4+T cells, Falker-Skan equation, Duffing Equation [17-29].

In this study, we propose the Laplace Adomian decomposition method (LADM) to solve the resulting nonlinear ordinary differential equation from our governing equations upon using similarity transformation. Our main objective in this study therefore is to obtain the velocity and temperature profiles subject to the prescribed boundary conditions and present the results in tables and graph. The study is organized as follows: In section two, the formulation of the problem subject to the given boundary conditions is given. Similarity transformation to convert the governing equations to nonlinear ordinary differential equation is carried out in section 3. The fundamentals of the Laplace Adomian decomposition method are presented in section 4. Section 5 gives the mathematical analysis of the problem using the proposed method. The presentation and discussion of the results in graphical and tabular form is presented in section 6.

2. Formulation of the problem

Let us consider the flow of a viscous incompressible fluid confined between two parallel plates separated by a distance h. Assuming the plates extend to infinity in the x and z directions, and the $y-$axis is perpendicular to the plane containing them. The fluid in the confined space between the parallel plates is assumed to be at rest, and the lower plate at $y = 0$ at $t = 0$ is suddenly set in motion with constant velocity U in the $x-$direction. This sudden motion generates a two-dimensional motion of the fluid between the space of the fluid. The only nonzero component of the velocity is u, and it's a function of y and t alone, where $\vec{q} = (u, v, w)$ and $u = u(y, t)$. Suppose the pressure is constant, then the Navier-Stokes's equation in the absence of body forces is reduced to the form. The governing continuity and Navier-Stokes equations of two-dimensional incompressible viscous fluid is given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Following the assumption stated above

$$v = 0, \Longrightarrow \frac{\partial u}{\partial x} = 0, \frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = o$$

Eq. (2) now reduced to the equivalent form

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} \tag{4}$$

Subject to the initial and boundary condition

$$u = 0 \text{ when } t = 0 \text{ for all } y \tag{5}$$

$$\left.\begin{array}{l} u = U \;\; at\; y = 0 \\ u = 0 \; at\; y = \infty \end{array}\right] \text{when } t > 0 \tag{6}$$

3. Similarity Transformation

To solve the partial differential equation in (4) subject to Eqs. (5) − (6), we first convert it to an ordinary differential equation using similarity transformation of the form

$$\eta = \frac{y}{2\sqrt{\nu t}} \tag{7}$$

$$f(\eta) = \frac{u}{U} \tag{8}$$

Manipulating the above, we get the following of derivates as follows

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial t} = U\frac{\partial f}{\partial \eta}\left(\frac{-y}{4\sqrt{\nu}t^{3/2}}\right)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = U\frac{\partial f}{\partial \eta}\left(\frac{1}{2\sqrt{\nu t}}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial y}\right)\frac{\partial \eta}{\partial y} = U\frac{\partial^2 f}{\partial \eta^2}\left(\frac{1}{4\nu t}\right)$$

Plugging the above into Eq. (1), the equation reduced to the form in terms of the similarity variable

$$\frac{\partial f}{\partial \eta}(-2\eta) = \frac{\partial^2 f}{\partial \eta^2}$$

That is, $\frac{\partial^2 f}{\partial \eta^2} + 2\eta\frac{\partial f}{\partial \eta} = 0$ (9)

And the corresponding boundary conditions are

$$f(\eta) = 1, \text{at } \eta = 0$$

$$f(\eta) = 0, \text{at } \eta = \infty \tag{10}$$

To solve Eq. (9) using (10), we apply the Laplace Adomian decomposition method. The basics of this hybrid method is discussed as follows.

4. Fundamentals of the Laplace Adomian Decomposition Method (LADM)

In this subsection, we discuss the basics of the hybrid Laplace Adomian decomposition algorithm for the nonlinear first order differential equations governing the problem. For convenience, we consider a first order nonhomogeneous functional differential equation subject to initial condition of the form

$$L[u(x)] + R[u(x)] + N[u(x)] = g(x) \tag{11}$$

$$u(0) = f(x) \tag{12}$$

$$L[u(x)] = g(x) - R[u(x)] - N[u(x)] \tag{13}$$

Applying Laplace transform to both sides of Eq. (11), and using the differentiation property, we get

$$s\mathcal{L}\{u(x)\} - f(x) = \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$s\mathcal{L}\{u(x)\} = f(x) + \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$\mathcal{L}\{u(x)\} = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}\{g(x)\} - \frac{1}{s}\mathcal{L}\{Ru(x)\} - \frac{1}{s}\mathcal{L}\{Nu(x)\} \quad (14)$$

Applying the inverse Laplace transform to both sides of Eq. (14), we obtain

$$u(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ru(x)\} - \frac{1}{s}\mathcal{L}\{Nu(x)\}\right] \quad (15)$$

Where $\phi(x)$ is the term arising from the first three terms on the right hand side of Eq. (15).

Next, we assume the solution of the problem as a decomposing series in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \quad (16)$$

Similarly, the nonlinear terms are written in terms of the Adomian polynomials as

$$Nu(x) = \sum_{n=0}^{\infty} A_n \quad (17)$$

Where the $A_n'^s$ represents the Adomian polynomials defined in the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty} \lambda^i y_i\right)\right]_{i=0}, n = 0,1,2,3 \quad (18)$$

Plugging Eqs. (16) and (17) into Eq. (18), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{R\sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s}\mathcal{L}\{N\sum_{n=0}^{\infty} A_n\}\right] \quad (19)$$

Matching both sides of Eq. (19), we obtain an iterative algorithm in the form

$$u_0(x) = \phi(x)$$

$$u_1(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_0(x)\right\} - \frac{1}{s}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_0\right\}\right]$$

$$u_3(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{R\sum_{n=0}^{\infty} u_2(x)\} - \frac{1}{s}\mathcal{L}\{N\sum_{n=0}^{\infty} A_2\}\right] \quad (20)$$

$$u_{n+1}(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x)\right\} - \frac{1}{s}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_n\right\}\right]$$

Then the solution of the differential equation is obtained as the sum of decomposed series in the form

$$u(x) \approx u_0(x) + u_1(x) + u_2(x) + \cdots \quad (21)$$

5. Mathematical Analysis using LADM

To solve Eq. (9) subject to (10), we proceed as follows

We take the Laplace transform of Eq. (9), we have the expression

$$\mathcal{L}\{f''\} + 2\eta\mathcal{L}\{f'\} = 0 \quad (22)$$

Where $\mathcal{L}\{F\}$ denotes the Laplace transformation of the function, F

Using the property of the Laplace transform, we have

$$s^2\mathcal{L}\{f\} - sf(0) - f'(0) + 2\eta[s\mathcal{L}\{f\} - f(0)] = 0 \tag{23}$$

Using the initial condition in Eq. (10), we obtain the following

$$s^2\mathcal{L}\{f\} - s - \alpha + 2\eta[s\mathcal{L}\{f\} - 1] = 0$$

Rearranging the above gives

$$\mathcal{L}\{f\} = \frac{s+\alpha+2\eta}{s^2+2\eta s} \tag{24}$$

Applying the principle of partial fraction, the right-hand side takes the form

$$\mathcal{L}\{f\} = \frac{1+\alpha/2\eta}{s} + \frac{\alpha}{s(s+2\eta)} \tag{25}$$

Following Adomian decomposition method, we express the function, f in the form of an infinite series

$$f = \sum_{n=0}^{\infty} f_n \tag{26}$$

Plugging Eq. (26) into Eq. (25), we have the form

$$\mathcal{L}\{\sum_{n=0}^{\infty} f_n\} = \frac{1+\alpha/2\eta}{s} + \frac{\alpha}{s(s+2\eta)} \tag{27}$$

Matching both sides of Eq. (27) yield the iterative algorithm

$$\mathcal{L}\{f_0\} = \frac{1+\alpha/2\eta}{s} + \frac{\alpha}{s(s+2\eta)}$$

Taking the inverse Laplace transform of the above expression give

$$f_0 = 1 + \frac{\alpha}{2\eta} - \frac{(-1+e^{-2t\eta})\alpha}{2\eta} \tag{28}$$

The solution of the temperature profile is given by

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) \tag{29}$$

Using the boundary condition, $f(\infty) = 0$, we obtain the value of the constant as

$$\alpha = -\frac{1}{\int_0^{\infty} e^{-\eta^2} d\eta} = -\frac{2}{\sqrt{\pi}} \tag{30}$$

Putting Eq. (30) into the concentration profile in Eq. (29), we have the expression

$$f(\eta) = 1 - \left[\frac{\int_0^{\eta} e^{-\eta^2} d\eta}{\int_0^{\eta_0} e^{-\eta^2} d\eta}\right] \tag{31}$$

Therefore, the velocity distribution is given by

$$f(\eta) = \frac{u}{U_0} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta \tag{32}$$

$$\Rightarrow \frac{u}{U_0} = f(\eta) = 1 - Erf(\eta) \tag{33}$$

Where $Erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$ is called the error function or probability distribution.

6. Results and discussion

In this section, we present the results obtained from solving the nonlinear ordinary differential equation in tables and graph. The effects of the governing parameters on the concentration and velocity profiles are shown in figures.

Table 1 Concentration & Temperature profiles of the viscous flow

η	$Erf(\eta)$	$\frac{u}{U_0}$
0	0.000000	1.00000
0.2	0.222703	0.777297
0.4	0.428392	0.571608
0.6	0.603856	0.396144
0.8	0.742101	0.257899
1.0	0.842701	0.157299
1.2	0.910324	0.089686
1.4	0.952285	0.0477149
1.6	0.976348	0.0236516
1.8	0.989091	0.0109095
2.0	0.995322	0.0046777

Table 2 Computed of values of velocity profile for different values of height,y and constant values of $v = 2, x = 5$

η	$y = 0.3$	$y = 0.5$	$y = 0.7$	$y = 0.9$	$y = 1.1$
0	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	1.33333	0.80000	0.571429	0.44444	0.363636
1.0	5.33333	3.20000	2.285710	1.777778	1.454555
1.5	12.0000	7.20000	5.142860	4.00000	3.272727
2.0	21.33333	12.80000	9.142860	7.111111	5.181818
2.5	33.33333	20.00000	14.28570	11.11111	9.090910
3.0	48.00000	28.80000	20.57140	16.00000	13.09090
3.5	65.33333	39.20000	28.00000	21.77778	17.81820
4.0	85.33333	51.20000	36.57140	28.44444	23.27270
4.5	108.0000	64.80000	46.28570	36.00000	29.45450
5.0	133.3333	80.00000	57.14290	44.44444	36.36360

Table 3 Computed values for velocity profile with variation in the width, x and constant values of $y = 1, v = 5$ on the channel

$\boldsymbol{\eta}$	$\boldsymbol{x = 0.5}$	$\boldsymbol{x = 0.7}$	$\boldsymbol{x = 0.9}$	$\boldsymbol{x = 1.1}$	$\boldsymbol{x = 1.3}$
0	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	4.00000	2.85714	2.22222	1.18181	1.53846
1.0	16.0000	11.4286	8.88889	7.27273	6.15385
1.5	36.0000	25.7143	20.0000	16.3636	13.8462
2.0	64.0000	45.7143	35.5556	29.0909	24.6154
2.5	100.000	71.4286	55.5556	45.4545	38.4615
3.0	144.000	102.857	80.0000	65.4545	55.3846
3.5	196.000	140.000	108.889	89.0909	75.3846
4.0	256.000	182.857	142.222	116.364	98.4615
4.5	324.000	231.429	180.000	147.273	124.615
5.0	400.000	285.714	222.222	181.818	153.846

Table 4 Computed values of velocity profile for variation in the viscosity, v and constant values of $y = 1, x = 2$

$\boldsymbol{\eta}$	$\mathbf{v = 0.2}$	$\mathbf{v = 0.3}$	$\mathbf{v = 0.4}$	$\mathbf{v = 0.5}$	$\mathbf{v = 0.6}$
0	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.04000	0.06000	0.08000	0.10000	0.12000
1.0	0.16000	0.24000	0.32000	0.40000	0.48000
1.5	0.36000	0.54000	0.72000	0.90000	1.00000
2.0	0.64000	0.96000	1.28000	1.60000	1.92000
2.5	1.00000	1.50000	2.00000	2.50000	3.00000
3.0	1.44000	2.16000	2.88000	3.60000	4.32000
3.5	1.96000	2.94000	3.92000	4.90000	5.88000
4.0	2.56000	3.84000	5.12000	6.40000	7.68000
4.5	3.24000	4.86000	6.48000	8.10000	9.72000
5.0	4.00000	6.00000	8.00000	10.0000	12.0000

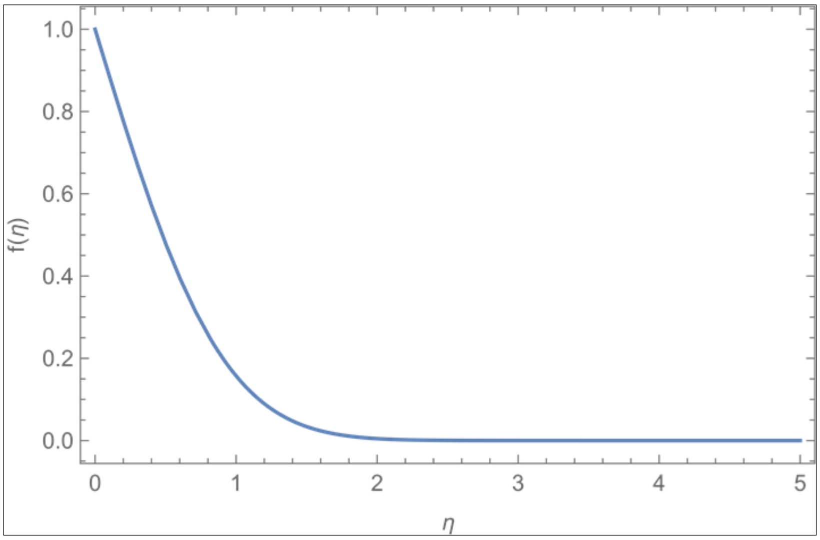

Figure 1 Effect of concentration profile on the unsteady flow with constant viscosity

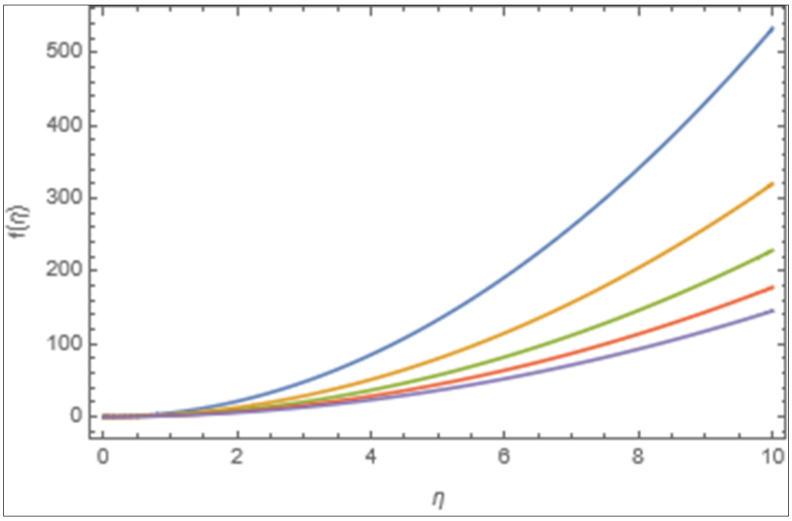

Figure 2 Variation of the concentration profile for different values of the height

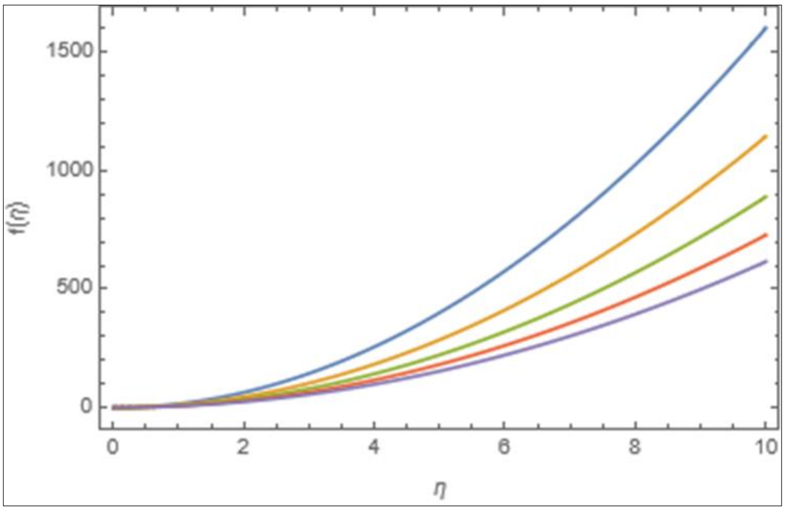

Figure 3 Variation of the concentration profile for different values of the width

7. Conclusion

In this study, the analytical investigation of an unsteady MHD incompressible fluid flow sandwiched between two infinite parallel plates is carried out using the Laplace Adomian decomposition method. The findings of the study are summarized as follows.

- The velocity decreases continuously and tends to a limiting value zero as $\eta \longrightarrow \infty$
- The limiting value is reached at exactly $\eta = 2$. This agrees with practical results
- Distance and kinematic viscosity vary proportionally.
- Decrease in the thickness of the layer leads to a corresponding decrease in the product of kinematic viscosity and time, hence the flow eventually become a boundary layer flow.

Compliance with ethical standards

Acknowledgments

The first author is grateful to his wife Mrs. Diepreye Ebiwareme for her excellent typesetting of the manuscript and to all reviewers whose comments improve the quality of this article.

Disclosure of conflict of interest

The authors have declared that there is no conflict of interest

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