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# Impact of external magnetic and Aharanov-Bohm (AB) Fields on the energy spectra of the Varshni-type potential in arbitrary dimensions

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## Abstract

In this article, the Varshni potential is analyzed taking into consideration the effects of magnetic and AB flux fields within the non-relativistic regime using the Nikiforov-Uvarov-Functional Analysis method (NUFA) method. The energy equation and wave function of the system are obtained in close form. We find that the entirety of effects of these fields removes degeneracy as a consequence and raises the bound state energy of the system. More so, it could be deduced that to regulate the energy spectra of this system, the AB-flux and magnetic field will do so greatly. The results from this study can be applied in condensed matter physics, atomic and molecular physics.

**Keywords:** NUFA method; Magnetic and AB fields; Varshni potential; D-dimensions

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## 1. Introduction

Researchers have paid great attention to studies concerned with the confinement of a non-relativistic particle by a potential model in the past 20 years. Solving this non-relativistic equation (.e. Schrodinger equation (SE)) with diverse potentials have been a subject of great attention. The reason for this unrivalled attention is due to the fact that its solutions gives the very important information required to understand the behavior a quantum system [1-10]. These solutions to this equation different potentials have been applied to study several physical systems, example; quantum dots [11], quarks [12], diatomic molecules [13], etc. Amongst the numerous potentials proposed and adopted for such studies over the years is the Varshni-type potential proposed by Varshni-type [14]. This model is given as;

$$V(r) = a - \frac{abe^{-\eta r}}{r} \quad (1)$$

Where $a$ are they potential parameters, $\eta$ is the screening parameter and $r$ is the inter-nuclear distance [15]. A number of authors have adopted this model to carry out some interesting studies. For instance, Oluwadare and Oyewumi [16] noted that the Varshni-type potential is very similar to the Hellmann potential.

Another very interesting area of research within quantum mechanics is the effects of the perturbation of external fields on a quantum system. This line of research of has been a subject of interest since the early days of quantum mechanics. A number of authors have carried out research regarding this concept recently [17-20]. Ikot et al. [21] studied effects of

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magnetic and AB fields on the energy spectra and thermo-magnetic properties of the screened Kratzer potential (SKP). Edet and Ikot [22] studied the effects of magnetic and AB fields on the energy spectra and thermal properties of some diatomic molecules using the Hulthen-Kratzer potential (HKP) model. Abu-shady et al. [23] investigated the dissociation of quarkonia in a thermal QCD medium in the background of AB and strong magnetic fields. Edet and Ikot [4] studied the effects of magnetic and AB fields on the energy spectra and thermomagnetic properties of CO diatomic molecule using screened modified Kratzer. Ikot et al. [25] studied the effects of external magnetic and AB fields on the thermodynamic variable of some diatomic molecules via superstatistics.

In view of this, we are interested in providing answers to the following questions; what happens to the energy spectra of the Varshni-type potential in the presence of the all-inclusive effect of magnetic and Aharanov- Bohm (AB) fields? What happens when there is a solitary effect? This questions have motivated us to carry out this study.

In the present work, our goal is to solve the SE with the Varshni-type potential in the presence of magnetic and AB flux fields using the Nikiforov-Uvarov Functional Analysis (NUFA) method. We will discuss the effects of the fields on the energy spectra of the system.

The paper is organized as follows. In section 2, we solve of the 2D Schrödinger equation with the Varshni-type potential under the combined effects of magnetic and AB flux fields. In section 3, we discuss the effects of the fields on the behavior of the energy spectra of the Varshni-type potential. Finally, a brief concluding remark is given in section 4.

## 2. Nu-functional analysis (nufa) method

Ikot et al. [26] proposed the Nikiforov-Uvarov-Functional Analysis method (NUFA) as a simple and elegant method for solving a second order differential equation of the hypergeometric form. The Nikiforov Uvarov (NU) method[27], the parametric NU method [28], and the functional analysis method[29, 30, 31] were used. This method, like the parametric NU method, is simple and straightforward. The NU is well-known for solving a second-order differential equation with the form [28].

$$\psi_n''(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\psi_n'(s)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi_n(s)=0 \quad , \tag{2}$$

Where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at most of second degree, and $\tilde{\tau}(s)$ is a first-degree polynomial. Tezcan and Sever [28] latter introduced the parametric form of NU method in the form

$$\psi''+\frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\psi'+\frac{1}{s^2(1-\alpha_3 s)^2}\left[-\xi_1 s^2+\xi_2 s-\xi_3\right]\psi(s)=0 \tag{3}$$

where $\alpha_i$ and $\xi_i (i=1,2,3)$ are all parameters. It can be observed in equation (3) that the differential equation has two singularities at $s\to 0$ and $s\to 1$, thus we take the wave function in the form,

$$\psi(s)=s^{\lambda}(1-s)^{\nu} f(s) \tag{4}$$

Substituting equation (4) into equation (3) leads to the following equation,

$$\begin{aligned}
&s(1-\alpha\ s) f''(s)+\left[\alpha_1+2\lambda-(2\lambda\alpha_3+2\nu\alpha_3+\alpha_2)s\right] f'(s)\\
&-\alpha_3\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3}-1+\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3}}\right)\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3^2}-1+\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3^2}}\right)\\
&+\left[\frac{\lambda(\lambda-1)+\alpha_1\lambda-\xi_3}{s}+\frac{\alpha_2\nu-\alpha_1\alpha_3\nu+\nu(\nu-1)\alpha_3-\frac{\xi_1}{\alpha_3}+\xi_2-\xi_3\alpha_3}{(1-\alpha_3 s)}\right] f(s)=0
\end{aligned} \tag{5}$$

Equation (5) can be reduced to a Gauss hypergeometric equation if and only if the following functions vanished,

$$\lambda(\lambda-1)+\alpha_1\lambda-\xi=0_3 \quad (6)$$

$$\alpha_2\nu-\alpha_1\alpha_3\nu+\nu(\nu-1)\alpha_3-\frac{\xi_1}{\alpha_3}+\xi_2-\xi_3\alpha=0 \quad (7)$$

Thus, equation (5) becomes

$$s(1-\alpha_1 s)f''(s)+\left[\alpha_1+2\lambda-(2\lambda\alpha_3+2\nu\alpha_3+\alpha_2)s\right]f'(s)$$
$$-\alpha_3\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3}-1+\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3}}\right)\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3^2}-1+\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3^2}}\right)f(s)=0 \quad (8)$$

Solving equations (6) and (7) completely give,

$$\lambda=\frac{(1-\alpha_1)\pm\sqrt{(1-\alpha_1)^2+4\xi_3}}{2} \quad (9)$$

$$\nu=\frac{(\alpha_3+\alpha_1\alpha_3-\alpha_2)\pm\sqrt{(\alpha_3+\alpha_1\alpha_3-\alpha_2)^2+4\left(\frac{\xi_1}{\alpha_3}+\alpha_3\xi_3-\xi_2\right)}}{2} \quad (10)$$

Equation (8) is the hypergeometric equation type of the form,

$$x(1-x)f''(x)+\left[c+(a+b+1)x\right]f'(x)-abf(x)=0 \quad (11)$$

Using equations (4), (8) and (11), we obtain the energy equation and the corresponding wave equation respectively for the NUFA method as follows:

$$\lambda^2+2\lambda\left(\nu+\frac{\alpha_2}{\alpha_3}-1+\frac{n}{\sqrt{\alpha_3}}\right)+\left(\nu+\frac{\alpha_2}{\alpha_3}-1+\frac{n}{\sqrt{\alpha_3}}\right)^2-\left(\frac{\alpha_2}{\alpha_3}-1\right)^2-\frac{\xi_1}{\alpha_3^2}=0 \quad (12)$$

$$\psi(s)=Ns^{\frac{(1-\alpha_1)+\sqrt{(1-\alpha_1)^2+4\xi_3}}{2}}(1-\alpha_3 s)^{\frac{(\alpha_3+\alpha_1\alpha_3-\alpha_2)+\sqrt{(\alpha_3+\alpha_1\alpha_3-\alpha_2)^2+4\left(\frac{\xi_1}{\alpha_3^2}+\alpha_3\xi_3-\xi_2\right)}}{2}}\,{}_2F_1(a,b,c;s) \quad (13)$$

Where $a,b,c$ are given as follows,

$$a=\sqrt{\alpha_3}\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3}-1+\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3}}\right) \quad (14)$$

$$b=\sqrt{\alpha_3}\left(\lambda+\nu+\frac{\alpha_2}{\alpha_3}-1-\sqrt{\left(\frac{\alpha_2}{\alpha_3}-1\right)^2+\frac{\xi_1}{\alpha_3}}\right) \quad (15)$$

$$c=\alpha_1+2\lambda \quad (16)$$

## 3. Schrödinger equation with Varshni-type potential (VP) with AB flux and magnetic fields

For a charged particle in an electromagnetic field bearing in mind the effects of the magnetic and AB fields, Edet [33] have established a robust framework that can treat basically all class of exponential-type models. Following the work of Edet [33], we obtain the differential equation of the form;

$$R''_{nm}(r)+\frac{2\mu}{\hbar^2}\left[\begin{array}{l}E_{nm}-\left(a-\frac{abe^{-\eta r}}{r}\right)-\hbar\omega_c(m+\xi)\frac{e^{-\eta r}}{\left(1-e^{-\eta r}\right)r}-\left(\frac{\mu\omega_c^2}{2}\right)\frac{e^{-2\eta r}}{\left(1-e^{-\eta r}\right)^2}\\-\frac{\hbar^2}{2\mu}\frac{\gamma_m}{r^2}\end{array}\right]R_{nm}(r)=0 \quad (17)$$

For our consideration, where $m'=(m+\xi)^2-\frac{1}{4}$, $\xi=\frac{\phi_{AB}}{\phi_0}$ is an integer with the flux quantum $\phi_0=\frac{hc}{e}$ and $\omega_c=\frac{e\vec{B}}{\mu c}$ denotes the cyclotron frequency.

Eq. (17) is not exactly solvable due to the presence of centrifugal term. Therefore, we employ the Greene and Aldrich approximation scheme [34, 35] to overcome the centrifugal term. This approximation is given;;

$$\frac{1}{r^2}=\frac{\eta^2}{\left(1-e^{-\eta r}\right)^2} \quad (18)$$

We point out here that this approximation is only valid for small values of the screening parameter $\eta$ .

Inserting Eqs. (7) into Eq. (6) and introducing a new variable $s=e^{-\eta r}$ allows us to obtain

$$\frac{d^2R_{nm}(s)}{ds^2}+\frac{1}{s}\frac{dR_{nm}(s)}{ds}+\frac{1}{s^2(1-s)^2}\left[-(\varepsilon_{nm}+t_0+t_2)s^2+(2\varepsilon_{nm}+t_0-t_1)s-(\varepsilon_{nm}+\gamma)\right]R_{nm}(s)=0 \quad (19)$$

For Mathematical simplicity, let's introduce the following dimensionless notations;

$$,-\varepsilon_{nm}=\frac{2\mu(E_{nm}-a)}{\hbar^2\eta^2},t_0=\frac{2\mu ab}{\hbar^2\eta},t_1=\frac{2\mu\omega_c}{\hbar^2\eta}(m+\xi),t_2=\frac{\mu^2\omega_c^2}{\hbar^2\eta^2}\ \ and\ \ \gamma=(m+\xi)^2-\frac{1}{4} \quad (20)$$

Comparing (19) and (3), we obtain the following;

$$\alpha_1=\alpha_2=\alpha_3=1,\ \nu=\sqrt{\varepsilon_{nm}+\gamma}\ \text{ and }\ \sigma=\frac{1}{2}+\sqrt{\frac{1}{4}+t_2+t_1+\gamma} \quad (21)$$

The energy is then obtained from (12) as follows;

$$(\nu+\sigma)-\left(\sqrt{\varepsilon_{nm}+t_0+t_2}\right)+n=0 \quad (22)$$

By carrying out some simple algebraic manipulations, we obtain the expression below;

$$\varepsilon_{nm}=-\gamma+\frac{1}{4}\left[\frac{t_0+t_2-\gamma-\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+t_2+t_1+\gamma}\right)^2}{\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+t_2+t_1+\gamma}\right)}\right]^2 \quad (23)$$

Substituting the expressions in eq. (20), we obtain the energy of the Varshni-type potential under influence of magnetic and AB fields with topological defects as follows;

$$E_{nm}=\frac{\hbar^2\eta^2\gamma}{2\mu}+a-\frac{\hbar^2\eta^2}{8\mu}\left[\frac{\frac{2\mu ab}{\hbar^2\eta}+\frac{\mu^2\omega_c^2}{\hbar^2\eta^2}-\gamma-\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{\mu^2\omega_c^2}{\hbar^2\eta^2}+\frac{2\mu\omega_c}{\hbar^2\eta}(m+\xi)+\gamma}\right)^2}{\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{\mu^2\omega_c^2}{\hbar^2\eta^2}+\frac{2\mu\omega_c}{\hbar^2\eta}(m+\xi)+\gamma}\right)}\right]^2 \tag{24}$$

For completeness sake, we move to find the wave function of the system. Let us now calculate the wave function of this system. The wave function of the system is given as;

$$R_{nm}(s)=(-1)^n N_{nm}\frac{\Gamma(2\nu+1+n)}{\Gamma(2\nu+1)}s^{\nu}(1-s)^{\sigma}\,{}_2F_1(-n,2(\nu+\sigma)+n;2\nu+1;s) \tag{25}$$

The full wave function is written as follows;

$$\psi(r,\varphi)=\frac{1}{\sqrt{2\pi r}}e^{im\varphi}(-1)^n N_{nm}\frac{\Gamma(2\nu+1+n)}{\Gamma(2\nu+1)}\left(e^{-\eta r}\right)^{\nu}\left(1-e^{-\eta r}\right)^{\sigma}\,{}_2F_1\left(-n,2(\nu+\sigma)+n;2\nu+1;e^{-\eta r}\right) \tag{26}$$

The three-dimensional non-relativistic energy solutions are obtained by setting $m=\ell+\frac{1}{2}$, in Eq. (24) to obtain;

$$E_{n\ell}=\frac{\hbar^2\eta^2\ell(\ell+1)}{2\mu}+a-\frac{\hbar^2\eta^2}{8\mu}\left[\frac{\frac{2\mu ab}{\hbar^2\eta}-\ell(\ell+1)-\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\ell(\ell+1)}\right)^2}{\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\ell(\ell+1)}\right)}\right]^2 \tag{27}$$

Where $\ell$ is the rotational quantum number?

The arbitrary dimensional non-relativistic energy solutions are obtained by setting $\ell(\ell+1)\to\frac{(D+2\ell-1)(D+2\ell-3)}{4}$, in Eq. (27) to obtain;

$$E_{n\ell}=\frac{\hbar^2\eta^2(D+2\ell-1)(D+2\ell-3)}{8\mu}+a-\frac{\hbar^2\eta^2}{8\mu}\left[\frac{\frac{2\mu ab}{\hbar^2\eta}-\frac{(D+2\ell-1)(D+2\ell-3)}{4}-\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{(D+2\ell-1)(D+2\ell-3)}{4}}\right)^2}{\left(n+\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{(D+2\ell-1)(D+2\ell-3)}{4}}\right)}\right]^2 \tag{28}$$

## 4. Results and discussion

**Table 1** Energy values for the Varshni-type potential under the influence of AB flux and external magnetic fields with various values of magnetic quantum numbers. We have used the following fitting parameters; $a = 0.15, b = 0.15, \hbar = c = e = \mu = 1$. and $\eta = 0.005$.

| $m$ | $n$ | $\vec{B}=0,\xi=0$ | $\vec{B}=5,\xi=0$ | $\vec{B}=0,\xi=5$ | $\vec{B}=5,\xi=5$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.148984 | 0.149994 | 0.150091 | 0.149932 |
| | 1 | 0.149929 | 0.149969 | 0.150061 | 0.149783 |
| | 2 | 0.149992 | 0.149919 | 0.150021 | 0.14961 |
| | 3 | 0.149995 | 0.149845 | 0.149974 | 0.149412 |
| -1 | 0 | 0.149959 | 0.150006 | 0.150084 | 0.149945 |
| | 1 | 0.150007 | 0.150006 | 0.150055 | 0.14982 |
| | 2 | 0.150005 | 0.149981 | 0.150019 | 0.149672 |
| | 3 | 0.149987 | 0.149932 | 0.149976 | 0.149498 |
| 1 | 0 | 0.149959 | 0.149981 | 0.150095 | 0.14992 |
| | 1 | 0.150007 | 0.149932 | 0.150064 | 0.149746 |
| | 2 | 0.150005 | 0.149857 | 0.150023 | 0.149549 |
| | 3 | 0.149987 | 0.149758 | 0.149973 | 0.149326 |

**Table 2** Energy values for the Varshni-type potential under the influence of AB flux and external magnetic fields with various values of magnetic quantum numbers. We have used the following fitting parameters; $a = 0.15, b = 0.15, \hbar = c = e = \mu = 1$. and $\eta = 0.01$

| $m$ | $n$ | $\vec{B}=0,\xi=0$ | $\vec{B}=5,\xi=0$ | $\vec{B}=0,\xi=5$ | $\vec{B}=5,\xi=5$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.148975 | 0.149975 | 0.150184 | 0.14973 |
| | 1 | 0.149953 | 0.149876 | 0.150082 | 0.149139 |
| | 2 | 0.149983 | 0.149678 | 0.149937 | 0.148452 |
| | 3 | 0.14993 | 0.149381 | 0.149757 | 0.147668 |
| -1 | 0 | 0.150025 | 0.150025 | 0.150175 | 0.149779 |
| | 1 | 0.150025 | 0.150025 | 0.150076 | 0.149285 |
| | 2 | 0.149964 | 0.149926 | 0.149941 | 0.148695 |
| | 3 | 0.149869 | 0.149727 | 0.149775 | 0.148008 |
| 1 | 0 | 0.150025 | 0.149926 | 0.150189 | 0.149682 |
| | 1 | 0.150025 | 0.149727 | 0.150086 | 0.148993 |
| | 2 | 0.149964 | 0.149431 | 0.149934 | 0.148209 |
| | 3 | 0.149869 | 0.149036 | 0.149743 | 0.14733 |

Table 1 shows the numerical energy values for the VP under the influence of AB flux and magnetic fields with various values of magnetic quantum numbers. We observe that when both fields are absent, there exist degeneracy in the energy spectra. By introducing only magnetic field to the system, the energy eigenvalues is increased and takes away the degeneracy as well. Nevertheless, as the strength of the magnetic field is raised so is the energy. This suggests that the energy values of the VP can be altered or regulated to a highest level by the application a strong magnetic field. The application of the AB field only, raises the energy values and degeneracies are eliminated. The energy spectra become more negative and the system becomes strongly attractive as the quantum number n increases for fixed m. The combined effect of both fields is robust and therefore, there is an upward shift in the bound state energy of the system. The combined effect completely eliminates the degeneracy. The complete effects shows that the system is strongly attractive while the localizations of quantum levels change and the eigenvalues increase. Also, the combined effect of the fields is strong and consequently, there is a significant upward shift in the bound state energy of the system. The effect of the magnetic field is seen to be stronger than the combined effect. The same behavior is observed in Table 2 but the energy is dropped in this case due to an increased value of the screening parameter.

In Table 3, we present the numerical results for Varshni-type potential in natural units with potential strength, $a = 2$ and$b = 1$. In Table 2, we calculate the energy eigenvalue for the Varshni-type potential for varying values of the potential strength;$a = 2, b = 1, a = 1, b = 1$,$a = 1, b = 2$, and $a = 2, b = -1$. More so, in Table 3, we observed that at each energy level, the energy increases with increasing $l$ except at the fourth energy level where the energy values remained constant. This also happened in the five dimensions computed for as it is evident in Table 3. A closer look at Table 4 shows a quasi-monotonicity in the behavior of the energy eigenvalues.

**Table 3** The bound state energy levels (in units of fm−1) of the Varshni-type potential for various values of $n$, $l$ and for $\hbar = \mu = b = 1$, $a = 2$ and $\eta = 0.2$

| $D$ | $l$ | $E_0$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.19500000 | 1.68000000 | 1.93277778 | 1.99500000 | 1.99500000 | 1.96444444 |
| | 1 | 0.19500000 | 1.68000000 | 1.93277778 | 1.99500000 | 1.99500000 | 1.96444444 |
| | 2 | 1.79500000 | 1.99500000 | 2.03875000 | 2.03020000 | 1.99500000 | 1.94193878 |
| | 3 | 2.10611111 | 2.11875000 | 2.09580000 | 2.05277778 | 1.99500000 | 1.92468750 |
| 2 | 0 | -6.00500000 | 1.27500000 | 1.83820000 | 1.96887755 | 1.99500000 | 1.97847107 |
| | 1 | 1.37277778 | 1.87980000 | 1.99500000 | 2.01475309 | 1.99500000 | 1.95239645 |
| | 2 | 1.99500000 | 2.06846939 | 2.07104938 | 2.04260331 | 1.99500000 | 1.93277778 |
| | 3 | 2.17459184 | 2.15500000 | 2.11533058 | 2.06127219 | 1.99500000 | 1.91749135 |
| 3 | 0 | 0.19500000 | 1.68000000 | 1.93277778 | 1.99500000 | 1.99500000 | 1.96444444 |
| | 1 | 1.79500000 | 1.99500000 | 2.03875000 | 2.03020000 | 1.99500000 | 1.94193878 |
| | 2 | 2.10611111 | 2.11875000 | 2.09580000 | 2.05277778 | 1.99500000 | 1.92468750 |
| | 3 | 2.22000000 | 2.18220000 | 2.13111111 | 2.06846939 | 1.99500000 | 1.91104938 |
| 4 | 0 | 1.37277778 | 1.87980000 | 1.99500000 | 2.01475309 | 1.99500000 | 1.95239645 |
| | 1 | 1.99500000 | 2.06846939 | 2.07104938 | 2.04260331 | 1.99500000 | 1.93277778 |
| | 2 | 2.17459184 | 2.15500000 | 2.11533058 | 2.06127219 | 1.99500000 | 1.91749135 |
| | 3 | 2.25179012 | 2.20326446 | 2.14411243 | 2.07464444 | 1.99500000 | 1.90524931 |
| 5 | 0 | 1.79500000 | 1.99500000 | 2.03875000 | 2.03020000 | 1.99500000 | 1.94193878 |
| | 1 | 2.10611111 | 2.11875000 | 2.09580000 | 2.05277778 | 1.99500000 | 1.92468750 |
| | 2 | 2.22000000 | 2.18220000 | 2.13111111 | 2.06846939 | 1.99500000 | 1.91104938 |
| | 3 | 2.27500000 | 2.22000000 | 2.15500000 | 2.08000000 | 1.99500000 | 1.90000000 |

**Table 4** The bound state energy levels of the Varshni-type potential for various values of $n$, $l$ and for $\hbar = \mu = 1$ and $\eta = 0.5$ in 3D

| $n$ | $l$ | $E_{nl}$ $a = 2, b = 1$ | $E_{nl}$ $a = 1, b = 1$ | $E_{nl}$ $a = 1, b = 2$ | $E_{nl}$ $a = 2, b = -1$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.19500000000 | 0.5950000000 | -0.805000000 | 1.28000000 |
| 1 | 0 | 1.68000000000 | 0.9550000000 | 0.680000000 | 1.28000000 |
| | 1 | 1.99500000000 | 1.0394444440 | 0.995000000 | 1.50611111 |
| 2 | 0 | 1.93277777800 | 0.9994444444 | 0.932777778 | 1.53277778 |
| | 1 | 2.03875000000 | 1.0200000000 | 1.038750000 | 1.58875000 |
| | 2 | 2.09580000000 | 1.0318000000 | 1.095800000 | 1.59980000 |
| 3 | 0 | 1.99500000000 | 0.9887500000 | 0.995000000 | 1.59500000 |
| | 1 | 2.03020000000 | 0.9822000000 | 1.030200000 | 1.59820000 |
| | 2 | 2.05277777800 | 0.9777777778 | 1.052777778 | 1.58611111 |
| | 3 | 2.06846938800 | 0.9745918368 | 1.068469388 | 1.57051020 |
| 4 | 0 | 1.99500000000 | 0.9550000000 | 0.995000000 | 1.59500000 |
| | 1 | 1.99500000000 | 0.9311111111 | 0.995000000 | 1.57277778 |
| | 2 | 1.99500000000 | 0.9133673470 | 0.995000000 | 1.54602041 |
| | 3 | 1.99500000000 | 0.8996875000 | 0.995000000 | 1.52000000 |
| | 4 | 1.99500000000 | 0.8888271605 | 0.995000000 | 1.49623457 |

## 5. Conclusion

In this research article, the Varshni-type potential is analyzed with the influence of magnetic and AB flux fields. To this end, the Hamiltonian operator containing the external fields and the potential model is transformed into a second-order differential equation. We solve this differential equation using the NUFA method to get the energy equation and wave function of the system. The effect of the fields on the energy spectra of the system is closely examined. The D-dimensional analysis of this system is also carried out in the absence of external fields. It was found out that the magnetic and AB fields remove degeneracy. The results of this study will find possible applications in condensed matter physics, atomic and molecular physics.

## Compliance with ethical standards

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### *Disclosure of conflict of interest*

The authors declare no conflicts of interest.

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