

Scheduling in hospital administration, OPD, surgeries and emergencies

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Abstract

This paper attempts to provide the solution in Hospital Administration, to get various jobs completed well in time, with limited Resources, viz., Medical Equipments (MRI, Cat Scanning, Ultrasound, X – Ray Machines), Medical Laboratories, Manpower, Paramedical Staff, Nurses, Doctors, Surgeons, by using Optimization Criteria, for Flowshop Sequencing Problems, with Optimizer, as Waiting Time. The problem is solved with the objective of obtaining a schedule of the given jobs which minimizes a certain performance measure, as Total Flow time & Waiting Time.

Keywords: Optimization; Sequencing; Flowtime; Total Elapsed Time; Makespan; Waiting Time; Bicriteria; Hospital Administration; OPD; Surgeries and Emergencies

1. Introduction

With the increase in population and because hospitals are open 24/7, the demand for better healthcare and hospital administration also increases.

Due to the evolving nature of the medical industry, vast complexes involving different departments, ensuring the proper workflow in such an environment is not an easy task.

Sequencing Problems involve placing items in a certain sequence or order for service. The well – known sequencing problem is to determine the sequence in which two or more jobs should be processed on one or more machines in order to optimize some measure of effectiveness. Sequencing simply refers to the determination of order in which the jobs are to be processed on various machines while Scheduling refers to the time table that includes the start time and the completion time of the jobs on machines.

In the Flowshop problem, the jobs flow between the machines in the same order. That is, a flowshop is one, in which all the jobs / patients, follow essentially the same path from one machine to another. In flow shop problems, a job may not ‘ pass ’ another job while waiting for processing on a machine.

2. Scheduling

In typical scheduling problems, a set of jobs I has to be scheduled on a set of given resources O such that some (often time-oriented) objective is optimized.

For treatment of patients in the Hospital and to keep the hospital running smoothly, the waiting time of the patients have to be minimized.

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3. Scheduling in Hospital Administration

Hospital administration manages all departments to ensure they operate as a whole. In order to manage, schedule, coordinate, direct, and track the effects of medical and health services, And keeping a large organization healthy requires a robust and multidimensional skillset. This is where the role of hospital administration comes in & to coordinate all this, Hospital Administrator is employed. Hospital administrators can be called on at all hours to resolve disputes or address a crisis. They must be calm under pressure and long work hours as this job can be very demanding.

The role of the Hospital Administrator, with his administration team is to create and ensure a proper work order for the institution, to ensure maximum productivity, that is to coordinate the resources of a healthcare institute for maximum efficiency, done by the subordination of work and by being proactive for emergencies, with skills of competent in analysis, communication, and maintaining collaborative relationships.

Since the role of a Hospital Administrator is multidimensional, to keep a large organization healthy, it requires a robust and multidimensional skillset duties including:

- Organizing the activities and personnel involved in the healthcare system.
- Serving as a linkage between the facility staff, doctors, management and patients.
- Maintaining interdepartmental communication.
- Overseeing the collection, storage and use of both patient and facility data.
- Developing new policies and procedures for improved patient care.
- Human resource management.
- Record management.
- Planning
- Cooperation
- Directing and designing the organization's strategies.
- Ensuring compliance with governmental policies.
- Law and order policing.
- Coordination
- Innovating
- Managing the hiring, training, and evaluation of human resources
- Budgeting
- Finance and business management.
- Designing budgets and establishing prices for services provided.
- Managing the hiring, training, and evaluation of human resources.
- This assures fluidic functioning and ease for patients availing services, Synchronising departments such as OPD, radiology, medical laboratory, surgery etc.

4. Scheduling in OPD

In many planning environments, not all jobs or their characteristics are known at the time of a scheduling decision, so that there is considerable uncertainty about future demand for resources and thus about ideal resource utilization. Sometimes, there is Queue lengths at waiting rooms of OPD. And sometimes, there is problem on patients late arrivals

5. Minimizing the Expected Waiting Time of Emergency Jobs

We consider a scheduling problem where a set of known jobs needs to be assigned to a set of given parallel resources such that the expected waiting time for a set of uncertain emergency jobs is kept as small as possible. On the basis of structural insights from queuing theory, we develop deterministic scheduling policies that reserve resource capacity in order to increase the likelihood of resource availability whenever an emergency job arrives.

In many planning environments, not all jobs or their characteristics are known at the time of a scheduling decision, so that there is considerable uncertainty about future demand for resources and thus about ideal resource utilization. At the heart of many scheduling problems therefore lies a form of risk management that needs to trade off the consequences of scheduling a known job on some resource against future demand induced by uncertain jobs. The importance of the scheduling decision is exacerbated if jobs cannot be interrupted once started and in particular

whenever the uncertain jobs foreseeably have an emergency character, so that the success of the total system crucially depends on whether emergency jobs can be served in a timely fashion.

6. Minimizing the Expected Waiting Time in Surgeries

In the field of surgical operations scheduling in hospitals, where high-priority but uncertain emergencies compete for scarce operating room capacity with elective surgeries of lower priority. We had compared our approaches with other policies from the literature in a comprehensive simulation study of a surgical operations unit.

7. Scheduling Applied for Optimal Results in Hospital Management Efficiency

7.1. Notations & Definitions

7.1.1. Johnsons Sequence

Johnson [8] in 1954, has given the solution for the Optimal Sequence, with respect to Total Elapsed Time, for the problems:

$n / 2 / F / F_{\max}$ and $n / 3 / F / F_{\max}$ problems.

- For $n / 2 / F / F_{\max}$ problem:

Job i precedes job j in an optimal sequence w.r.to minimum Total Elapsed Time, if

$$\text{Min} (p_{i1}, p_{j2}) \leq \text{Min} (p_{i2}, p_{j1}).$$

- For $n / 3 / F / F_{\max}$ problem:

$$\text{Min} p_{i1} \leq \text{Max} p_{i2}$$

$$\text{Min} p_{i3} \leq \text{Min} p_{i2}$$

- The Total Waiting Time of Job i : [W_i]:

$$W_i = \sum_{k=1}^m W_{ik}$$

- Completion Time of Job i [C_i]:

Completion Time of job i is defined as the time at which processing of the last operation of the job is completed.

$$C_i = r_i + W_{i1} + p_{i1} + W_{i2} + p_{i2} + W_{i3} + p_{i3} + \dots + W_{im} + p_{im}$$

$$= r_i + \sum_{k=1}^m (W_{ik} + p_{ik})$$

- Flowtime of the Job i [F_i]:

Flowtime of the job i is defined as the total time that the job spends in the shop / office / hospital.

$$F_i = W_{i1} + p_{i1} + W_{i2} + p_{i2} + W_{i3} + p_{i3} + \dots + W_{im} + p_{im}$$

$$= \sum_{k=1}^m (W_{ik} + p_{ik})$$

$$= C_i - r_i$$

$$F_i = C_i - r_i$$

- Makespan or Total Elapsed Time:

Total Elapsed Time, i.e., minimization of the time taken to complete the processing of all the jobs.

- Idle Time, I_k , on Machine k :

$$I_k = C_{\max} - \sum_{i=1}^n p_{ik}$$

where C_{\max} is the maximum completion times, over all the jobs.

8. Minimizing bicriteria

Only one single criteria for optimization, that is, either minimization of makespan or minimization of total flowtime has been often taken in Scheduling. But A survey of scheduling literature has revealed the desirability of an optimal schedule being evaluated by more than one performance measure or criterion and is developed by various authors. Gupta and Dudek

[7] has conducted an experimental study of comprehensive performance measure in the flowshop scheduling problem and the study has revealed and recommended ***the use of a combination of these two criterias ; viz., makespan and total Flowtime.*** Dileepan and Sen [6] have extensively surveyed the bicriterion static scheduling research for a single machine. Chandrasekharan [5] has given a technique based on Branch and Bound method and satisfaction of certain conditions to obtain a sequence which minimizes total flowtime subject to minimum makespan, in a two-stage flowshop problem. In this chapter, Chandrasekharan's method is firstly modified for the

$n \times 2$ flowshop problem, for the same bicriteria and is applied further, for the specially - structured problems in the general $n \times m$ flowshop problem.

Minimizing Bicriteria is divided into to sections:

- Section 1: Two - Stage General Case Problem
- Section 2: M - Stage Specially - Structured Problems.

8.1. Section 1: Two-Stage General Case Problem

The present section provides a simple and direct procedure for obtaining a sequence which minimizes total flowtime subject to minimum Makespan in a two - stage Flowshop scheduling problem. The procedure of this section, which is based only on the Branch and Bound technique [2] for the above bicriteria is better than the one given by Chandrasekharan's [5], which is based on the Branch and Bound method and satisfaction of certain conditions. The same example, as taken by Chandrasekharan, has been discussed here, of 5×2 flowshop problem and the computational results have also been provided for various jobs - sized problems.

Notations

S = any sequence of n -jobs

i = jobs to be performed, i.e., $i = 1, 2, \dots, n$.

S_1 = the sequence obtained by applying Johnson's procedure.

M = minimal makespan

J_r = any partial schedule of r - jobs.

σ = too from the set of jobs other than in J_r

π = of jobs other than in σ and J_r , arranged according to Johnson's sequence S_1 .

$Z(i, X)$ = completion time of the i the job on machine X
 ($X = A, B$).

The problems call be mathematically formulated as:

To obtain a sequence S^* which satisfies the bicriteria:

Minimize $\sum_{i \in S^*} Z(i, B)$, for the sequence S^* .

The lower bound on total flowtime $LBTF [J_r \sigma]$, of the partial schedule $J_r \sigma$ of $(r + 1)$ - jobs can be computed by Branch and Bound technique, similar to that as, given by Ignall and Schrage's [9].

$$LBTF [J_r \sigma] = \sum_{i \in J_r \sigma} Z(i, B) + \text{Max} [g_1, g_2]$$

$$\text{where } g_1 = \sum_{p=1}^{n-(r+1)} [Z(J_r \sigma, A) + \sum_{q=1}^p A'_q + B'_p]$$

$$\text{such that } A'_1 \leq A'_2 \leq \dots \leq A'_{n-(r+1)}$$

$$\text{where } g_2 = \sum_{p=1}^{n-(r+1)} [Z(J_r \sigma, B) + \sum_{q=1}^p B'_q]$$

$$\text{such that } B'_1 \leq B'_2 \leq \dots \leq B'_{n-(r+1)}$$

..... Formula (1)

With additional notations:

A'_p = Processing time of the p th job (of the set of jobs other than in J_r and σ) on machine A.

B'_p = Processing time of the p th job (of the set of jobs other than in J_r and σ) on machine B.

$Z(J_r \sigma, X)$ = completion time of the last job (i.e., σ) of the partial schedule $J_r \sigma$ of $(r + 1)$ - jobs on machine X . ($X = A, B$)

8.2. The following algorithm provides the optimal solution which minimizes the Bicriteria, of $n \times 2$ flowshop problem :

8.2.1. Algorithm 1

- Step 1: Obtain $LBMS (J_r \sigma)$ = lower bound on makespan for schedule $J_r \sigma \pi$, except for those $J_r \sigma$, which follows Johnson's sequence ; by direct enumeration.
- Step 2: Obtain $LBTF (J_r \sigma)$ = lower bound on total Flowtime for the node $J_r \sigma$, by Formula (1), which have $LBMS (J_r \sigma) = M$.
- Step 3: Choose the minimum $LBTF$ among all the unbranched nodes, replace $J_r \sigma$ by J_r and repeat Step 2, for those $J_r \sigma$ which have $LBMS (J_r \sigma) = M$.
- Step 4: The Branch and Bound technique is applied till the complete sequence is obtained with minimum $LBTF$ and $LBMS = M$.

Example 1:

Consider the 5 – jobs, 2 - machines flowshop problem with processing times as in Table 1.

Machines →	A	B
Jobs ↓		
1	15	19
2	5	10
3	16	12
4	5	20
5	7	12

The Johnson's sequences obtained from the Table 1 are

$$S_1 : 2 - 4 - 5 - 1 - 3 \text{ and } 4 - 2 - 5 - 1 - 3.$$

M = Makespan for the sequence $S_1 = 78$ units.

If initially, $J_r = \{ \emptyset \}$, then σ can be either 1, 2, 3, 4 or 5 ; so that, $J_r \sigma$ becomes 1, 2, 3, 4 or 5.

Obtaining LBMS ($J_r \sigma$) ; for $J_r \sigma = 1, 3$ and 5 (Step 1), by direct enumeration.

The completion times In - Out table for computing LBMS (1), is shown as in Table 2, as follows:

Machines →	A	B
Jobs ↓	In - Out	In - Out
1	0 - 15	15 - 34
2	15 - 20	34 - 44
4	20 - 25	44 - 64
5	25 - 32	64 - 76
3	32 - 48	76 - 88

where $\pi = 2 - 4 - 5 - 3$

Thus LBMS (1) = 88, where $\pi = 2 - 4 - 5 - 3$ or $4 - 2 - 5 - 3$.

Similarly, LBMS (3) = 89, where $\pi = 2 - 4 - 5 - 1$ or $4 - 2 - 5 - 1$

And LBMS (5) = 80 ; where $\pi = 2 - 4 - 1 - 3$ or $4 - 2 - 1 - 3$.

[LBMS ($J_r \sigma$) ; $J_r \sigma = 2, 4$ are not computed in Step 1, since they follow order of Johnson's sequence and will thus lead the value of LBMS as 78].

Since for none of LBMS ($J_r \sigma$) ; $J_r \sigma = 1, 3, 5$; is 78, thus, LBTF will not be computed for any one of them.

Computing LBTF ($J_r \sigma$) ; $J_r \sigma = 2, 4$, by Formula (1) (Step 2) ;

$$LBTF (2) = Z (2, B) + \text{Max} [g_1, g_2] = 15 + \text{Max} [g_1, g_2]$$

where $g_1 = 165$; $g_2 = 202$

Thus, $LBTF (2) = 15 + \text{Max} [165, 202] = 15 + 202 = 217$

Similarly, $LBTF (4) = 244$

and the corresponding scheduling tree for LBTF is as in Figure 1, below:

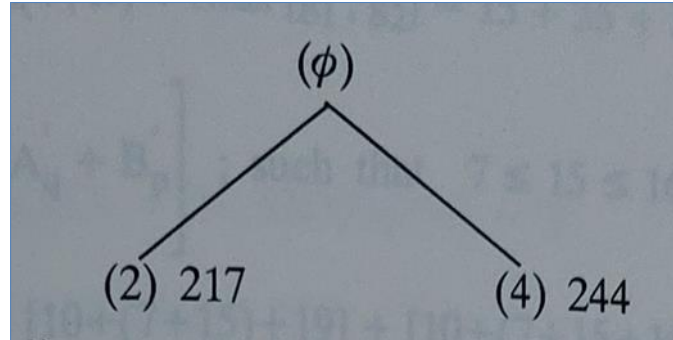


Figure 1 Scheduling tree showing generation of the first level of nodes

By Step 3, since the minimum $LBTF = 217$; for $J_r \sigma = 2$, among all the unbranched nodes (see Figure 1), call this partial schedule $J_r \sigma (= 2)$ by J_r . Again, for $J_r = 2$, σ can be either 1, 3, 4 or 5, so that, $J_r \sigma$ becomes either 21, 23, 24 or 25.

The completion times In - Out table for computing $LBMS (21)$, can be obtained, as shown in Table 3 below:

Machines →	A	B
Jobs ↓	In - Out	In - Out
2	0 - 5	5 - 15
1	5 - 20	20 - 39
4	20 - 25	39 - 59
5	25 - 32	59 - 71
3	32 - 48	71 - 83

where $\pi = 4 - 5 - 3$

Since $LBMS (21) = 83$, $LBMS (23) = 84$, $LBMS (25) = 78$

(Step 1), thus, computing $LBTF (J_r \sigma)$; for $J_r \sigma = 24$ and 25

(Step 2), by Formula (1).

$LBTF (24) = 15 + 35 + \text{Max} (140, 184) = 50 + 184 = 234$

Similarly, $LBTF (25) = 217$.

And the corresponding scheduling tree is as given in Figure 2 below:

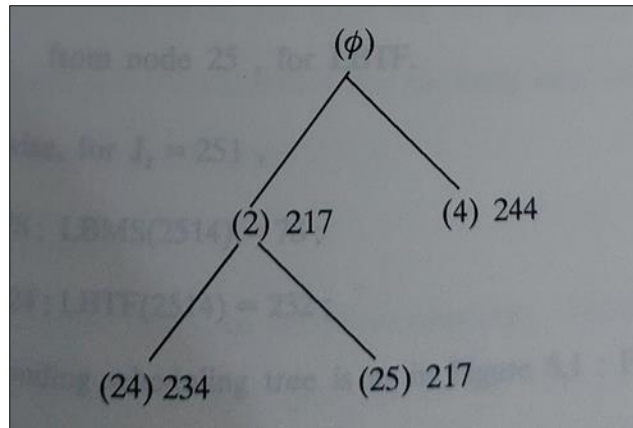


Figure 2 Scheduling tree showing generation of the second level of nodes, from node 2, for LBTF

Since the minimum LBTF among all the unbranched nodes is 217 corresponding to $J_r \sigma = 25$ (see Figure 2), therefore, call $J_r \sigma = 25$,

Again, for $J_r = 25$,

$LBMS (251) = 78$; $LBMS (253) = 79$; $LBMS (254) = 78$

and $LBTF (251) = 224$; $LBTF (254) = 226$;

and the corresponding scheduling tree is as in Figure 3 below:

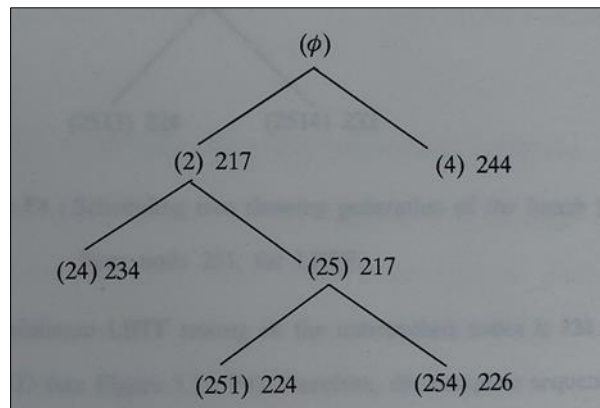


Figure 3 Scheduling tree showing generation of the third level of nodes, from node 25, for LBTF

Proceeding likewise, for $J_r = 251$,

$LBMS (2513) = 78$; $LBMS (2514) = 78$;

$LBTF (2513) = 224$; $LBTF (2514) = 232$;

and the corresponding scheduling tree is as in Figure 4, below:

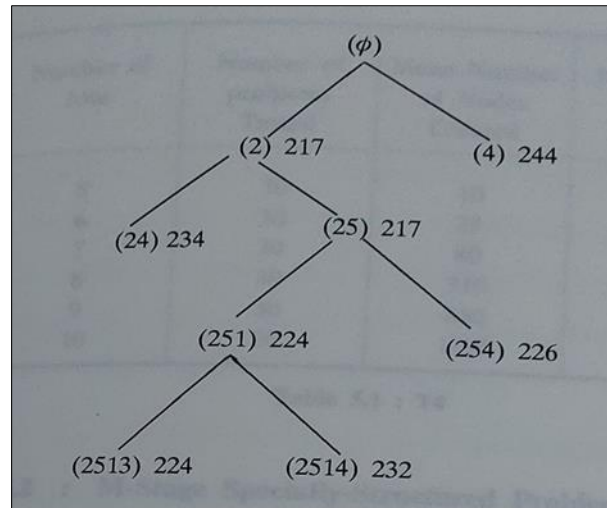


Figure 4 Scheduling tree showing generation of the fourth level of nodes, from node 251, for LBTF

Since the minimum LBTF among all the unbranched nodes is 224 units, corresponding to $J_r \sigma = 2513$ (see Figure 4), therefore, the complete sequence is 2 - 5 - 1 - 3 - 4, with minimum total flowtime as 224 units and the makespan as 78 units.

Hence, the sequence 2 - 5 - 1 - 3 - 4 satisfies the bicriteria with total flowtime as 224 units and the makespan as 78 units.

8.2.2. Computational Results

The proposed technique has been tested on computer. Thirty problems each of job - size varying from 5 to 10 have been considered. The processing times have been taken at random, and the average number of nodes and C.P.U time, has been recorded.

8.3. Section 2: M - Stage Specially - Structured Problems :

In the present section, the same bicriteria of optimization of Section 1, viz., minimizing the total flowtime subject to obtaining the minimum makespan, applied for the generalized $n \times m$ flowshop problem and for the special cases on its processing times. The algorithm for the generalized $n \times m$ flowshop problem, is based on the Branch and Bound method. Some special cases are further considered, where the processing times follow certain conditions and it is being observed that instead of applying the algorithm for the general case, the simplified algorithms can be applied in each special case (3 cases).

The basic strategy adopted in the algorithms presented in this section reduces (intractable) $n \times m$ flowshop optimization problems, to polynomial time $n \times 2$ flowshop optimization problems, when certain conditions on the processing times considered here, hold. The algorithms presented for the special cases, are very efficient algorithms for minimizing the bicriteria, viz., minimizing total flowtime, subject to a schedule with minimum makespan, as these are reduced to two - machines algorithm. The algorithms presented here are polynomial time and are based on the Branch and Bound technique. The schedules generated guaranteed to be optimal. Thus, in practice, all those situations of $n \times m$ flowshop problems, where the processing times follow the above criterion, the optimal solution can be obtained by simple algorithms of two - machines.

Notations

S = any sequence of n - jobs.

i = jobs to be performed i.e., $i = 1, 2, \dots, n$.

$Y (i, k)$ = the processing time of job i on machine k .

$Z (i, k)$ = the time when the i th job completes the processing on machine k .

where $i = 1, 2, \dots, n ; k = 1, 2, \dots, m$

S_i = the sequences obtained by applying Johnson's procedure ;

$i = 1, 2, \dots$

$M =$ minimal makespan

$J_r =$ any partial schedule of r - jobs.

$\sigma =$ any job from the set of jobs other than in J_r .

$\pi =$ a set of jobs other than in J_r and σ , arranged according to that Johnson's sequence for which M is the makespan.

An $n \times m$ flowshop problem can be mathematically formulated as:

To obtain a sequence S^* which satisfies the bicriteria

$$\text{Minimize } \sum_{i \in S^*} Z(i, m)$$

subject to $Z(n, m) = M$, for the sequence S^* .

The following algorithm provides the optimal solution for the required bicriteria in general case:

8.3.1. Algorithm 2

- Step 1: Apply the usual Branch and Bound technique to minimize the makespan.
- The procedure terminates only when the makespan of sequence(s) is strictly less than the value attached to all the unbranched vertices.
- Step 2: Enumerate the total flowtime, for all the sequences, obtained in Step 1 and comparing their total flowtime, obtain the sequence(s), which gives the minimum total flowtime.
- Step 3: The sequence(s), obtained in Step 2 is (are) the optimal sequence(s) satisfying the bicriteria with makespan value, as obtained in Step 1 and the total flowtime value, as obtained in Step 2.

Example 2 :

Consider the 4 - jobs, 5 - machines sequencing problem with processing times as in Table 4 below:

Machines →	A	B	C	D	E
Jobs ↓					
1	2	7	8	6	2
2	3	7	9	7	1
3	6	5	7	4	5
4	1	7	8	5	4

Applying the Branch and Bound technique to minimize the makespan for 4 x 5 sequencing problem (Step 1), the scheduling tree is obtained as in Figure 5 below:

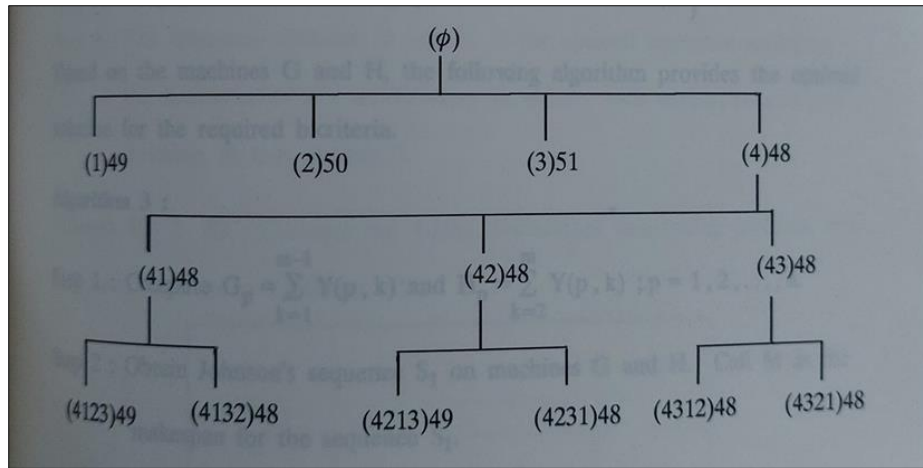


Figure 5 Sequences

From Figure 5, the sequences which minimize the makespan are 4 - 1 - 3 - 2 ; 4 - 2 - 3 - 1 ; 4 - 3 - 1 - 2 and 4 - 3 - 2 - 1 with the makespan value as 48 units.

Enumerating, the total flowtime for 4 - 1 - 3 - 2 ; 4 - 2 - 3 - 1 ; 4 - 3 - 1 - 2 and 4 - 3 - 2 - 1 are 145, 147, 144 and 145 units.

Comparing their total flowtime, the sequence 4 - 3 - 1 - 2 has the minimum value of total flowtime, i.e, 144 units (Step 2).

Hence, the sequence 4 - 3 - 1 - 2 is the optimal sequence satisfying the bicriteria with makespan value as 48 units and the total flowtime as 144 units (Step 3).

8.4. Special Cases of n x m Flowshop Problem

This section deals with the special cases of n x m flowshop problem with the objective for minimizing the bicriteria, viz., minimizing total flowtime, subject to a schedule with minimum makespan or total elapsed time. Various Special cases with restrictions in processing times conditions have also been considered.

The three special cases taken in this section are:

- Case 1:

$$Y(i, k) \geq Y(j, k + 1) ; \forall i \& j ; i \neq j ; k = 1, 2, \dots, m - 2$$
- Case 2:

$$Y(i, k) \leq Y(j, k + 1) ; \forall i \& j ; i \neq j ; k = 1, 2, \dots, m - 2$$
- Case 3:

$$Y(i, k) \geq Y(j, k + 1) ; \forall i \& j ; i \neq j ; 2 \leq k \leq m - 1$$

Algorithms are considered below, for each case :

Case 1:

$$Y(i, k) \geq Y(j, k + 1) ; \forall i \& j ; i \neq j ; k = 1, 2, \dots, m - 2$$

Since this case

$$\text{Min} [G_j, H_{j+1}] \leq \text{Min} [G_{j+1}, H_j]$$

$$m - 1$$

where $G_p = \sum Y(p, k)$ and

$$k = 1$$

$$H_p = \sum_{k=2}^m Y(p, k); p = 1, 2, \dots, n$$

..... Formula (2)

Based on the machines G and H, the following algorithm provides the optimal solution for the required bicriteria:

8.4.1. Algorithm 3

- Step 1: Compute

$$\text{where } G_p = \sum_{k=1}^{m-1} Y(p, k) \text{ and}$$

$$H_p = \sum_{k=2}^m Y(p, k); p = 1, 2, \dots, n$$

- Step 2: Obtain Johnson's sequence S_1 on machines G and H. Call M as the makespan for the sequence S_1 .
- Step 3: Obtain LBMS ($J_r \sigma$) = lower bound on makespan for schedule $J_r \sigma \pi$, of $n \times 2$ flowshop problem (on machines G and H), by direct enumeration except for those $J_r \sigma$, which follows Johnson's sequence.
- Step 4: Obtain LBTF ($J_r \sigma$) = Lower bound on total flowtime for the node $J_r \sigma$, of $n \times 2$ flowshop problem, by Formula (1), on machines G and H, which have LBMS ($J_r \sigma$) = M
- Step 5: Choose the minimum LBTF among all the unbranched nodes, replace $J_r \sigma$ by J_r and repeat Step 4, for those, which have LBMS ($J_r \sigma$) = M.
- Step 6: The Branch and Bound technique is applied till the complete sequence is obtained with minimum LBTF and LBMS = M, of $n \times 2$ flowshop problem.
- Step 7: Enumerate the makespan and the total flowtime for the complete sequence, obtained in Step 6, of the $n \times m$ flowshop problem.
- Step 8: The sequence obtained in Step 7, is the optimal sequence satisfying the bicriteria, of $n \times m$ flowshop problem, with makespan and total flowtime, as that of Step 7.

Case 2:

$$Y(i, k) \leq Y(j, k + 1); \forall i \& j; i \neq j; k = 1, 2, \dots, m - 2$$

8.4.2. Algorithm 4

Step 1: Obtain all the Johnson's sequences on the last two machines. Call these sequences as $S_i; i = 1, 2, \dots$. Obtain $Z_i(n, m);$ for each $S_i; i = 1, 2, \dots;$ by the formula:

$$Z(n, m) = \sum_{k=1}^{m-2} Y(1, k) + \text{makespan value of schedule on last two machines.}$$

Call $M = \text{Min } Z_i(n, m).$

Step 2: Obtain LBMS ($J_r \sigma$) = lower bound on Makespan for schedule $J_r \sigma \pi$, of $n \times m$ flowshop problem by the formula:

$$Z(n, m) = \sum_{k=1}^{m-2} Y(1, k) + \text{makespan value of schedule on last two machines}$$

by direct enumeration.

LBMS ($J_r \sigma$) are obtained except for those $J_r \sigma$, which follows any of the Johnson's sequences $S_i; i = 1, 2, \dots$

Step 3: Obtain LBTF ($J_r \sigma$) = the lower bound on total flowtime for the node $J_r \sigma$, of $n \times m$ flowshop problem, by Formula (2), which have $LBMS(J_r \sigma) \leq M$.

Step 4: Choose the minimum LBTF among all the unbranched nodes, replace $J_r \sigma$ by J_r and repeat Step 3, for those $J_r \sigma$, which have $LBMS(J_r \sigma) \leq M$.

Step 5: The Branch and Bound technique is applied till the complete sequence is obtained with minimum LBTF and $LBMS \leq M$ of $n \times m$ flowshop problem.

Step 6: The sequence, so obtained in Step 5, is the optimal sequence satisfying the bicriteria with its corresponding Makespan value and the total flowtime.

Case 3:

$$Y(i, k) \geq Y(j, k+1); \forall i \& j; i \neq j; 2 \leq k \leq m-1$$

8.4.3. Algorithm 5

Step 1: Obtain all the Johnson's sequences on the first two machines. Call these sequences as $S_i; i = 1, 2, \dots$

Obtain $Z_i(n, m)$, for each $S_i; i = 1, 2, \dots$, by the formula:

$$Z(n, m) = Z(n, 2) + \sum_{k=3}^m Y(n, k)$$

Let $M = \text{Min } Z_i(n, m)$.

Step 2: Obtain LBMS ($J_r \sigma$) = lower bound on Makespan for schedule $J_r \sigma \pi$, of $n \times m$ flowshop problem by the formula:

$$Z(n, m) = Z(n, 2) + \sum_{k=3}^m Y(n, k)$$

where $Z(n, 2)$ is to be computed by direct enumeration.

LBMS ($J_r \sigma$) are obtained except for those $J_r \sigma$, which follows any of the Johnson's sequences $S_i; i = 1, 2, \dots$

Step 3: Obtain LBTF ($J_r \sigma$) = the lower bound on total flowtime for the node $J_r \sigma$, of $n \times m$ flowshop problem, by Formula (2), which have $LBMS(J_r \sigma) \leq M$.

Step 4: Choose the minimum LBTF among all the unbranched nodes, replace $J_r \sigma$ by J_r and repeat Step 3, for those $J_r \sigma$, which have $LBMS(J_r \sigma) \leq M$.

Step 5: The Branch and Bound technique is applied till the complete sequence is obtained with minimum LBTF and $LBMS \leq M$, of $n \times m$ flowshop problem.

Step 6: The sequence, so obtained in Step 5, is the optimal sequence satisfying the bicriteria with its corresponding Makespan value and the Total Flowtime.

9. Conclusion

This paper attempts to present the heuristic approach with the help of Mathematical Models, focussing on optimal scheduling / ordering of jobs / patients, Sequencing with Realistic Applications applied for Hospital affairs, dealing with Medical Laboratories, Radiology, OPD's, Surgeries, Emergencies etc., minimizing waiting time of the patients, giving them final clearance, with the objective, to optimize the use of available facilities to effectively process the items or the jobs / patients.

The optimal sequences obtained above in three different cases, along with the total elapsed time to complete all jobs well in time, with minimum time taken could be very useful, in completing Hospital work, Patients rush, completing in an optimal manner, yielding Maximum Results & if adopted, could be fruitful, giving maximum benefits, with limited resources / machines / manpower, which is at par, excellence in Hospital Administration & Management.

This assures fluidic functioning and ease for patients availing services, Synchronising departments such as OPD, radiology, medical laboratory, surgery etc.

Compliance with ethical standards

Disclosure of conflict of interest

The authors have no conflicts of interest to declare. Both Coauthors have seen and agree with the manuscript and there is no financial interest to report.

References

- [1] Bagga and Ambika (1997), Sequencing with Restrictions in Processing Times, *Opsearch*, 34, 2, 116 – 127.
- [2] Bagga and Ambika Bhambani (1997), Bicriteria in Flowshop Scheduling Problem, *Journal of Combinatorics, Information and System Sciences, USA*, 22, 1, 63 – 83.
- [3] Bagga and Ambika (2002), Bicriteria in Special Flow shop Problems, Vol 33, No. 4, 435 – 441, *Indian Journal of Pure and Applied Mathematics*, April 2002, INSA.
- [4] Bagga and Ambika (2000), Bicriteria in a Two – Stage Flow shop Scheduling Problem, Vol 31, No. 11, 1475- 1481, *Journal of Pure and Applied Mathematics, INSA*, Nov 2000
- [5] Chandrasekharan Rajendran (1992), Two Stage Flowshop Scheduling Problem with Bicriteria, *Operational Research Society, U.K*, 43, 9, 871 – 884.
- [6] Dileepan and Sen, T (1988), Bicriterion State Scheduling Research for a Single Machine, *Omega*, 16, 53 – 59.
- [7] Gupta, J.N.D and Dudek, R.A (1971), Optimality Criteria for Flowshop Schedule, *AIIE Transactions*, 3, 3, 199 – 205.
- [8] Johnson, S.M (1954), Optimal Two – and Three – Stage Production Schedules with Set – Up – Times Included, *Naval Research Logistics*, 1, 61 – 78.
- [9] Ignall, E and Schrage, L (1965), Application of the Branch and Bound Technique to Some Flowshop Scheduling Problems, *Operations Research*, 13, 3, 400 – 412.
- [10] Lomnicki, Z.A (1965), A Branch and Bound Algorithm for the Exact Solution of the Three – Machine Scheduling, *Operational Research*, 16, 89 – 100.
- [11] Szwarc, W (1974), Mathematical Aspects of the $3 \times n$ Job – shop Sequencing Problem, *Naval Research Logistics*, 21, 145 – 153