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On class (n, mBQ) Operators

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Abstract

In this paper, we introduce the class of (n, mBQ) operators acting on a complex Hilbert space H. An operator if $T \in B(H)$ is said to belong to class (n, mBQ) if $T^{*2m}T^{2n}$ commutes with $(T^{*m}T^{n})^2$ equivalently $[T^{*2m}T^{2n}, (T^{*m}T^{n})^2] = 0$, for a positive integers n and m. We investigate algebraic properties that this class enjoys. Have. We analyze the relation of this class to (n,m)-power class (Q) operators.

Keywords: (n,m)-power Class (Q); Normal; Binormal operators; N-power class (Q); (BQ) operators; (n,mBQ) operators

1. Introduction

H denotes Hilbert space over the complex field throughout this paper while B(H) the Banach algebra of all bounded linear algebra on an infinite dimensional separable Hilbert space H . A bounded linear operator T is said to be in class (Q) if $T^{*2}T^{2} = (T^{*}T)^{2}$ (2), (n,m)-power class (Q) if $T^{*2m}T^{2n} = (T^{*m}T^{n})^2$ for positive integers n and m (1).

The class of (Q) operators was expanded to many classes such as the following classes, almost class (Q) (4), n-power class (Q) (2), (α, β)-class (Q) (3), K* Quasi-n- Class (Q) Operators (6) and quasi M class (Q). An operator $T \in B(H)$ is said to belong to class (BQ) if $T^{*2}T^{2}(T^{*}T)^{2} = (T^{*}T)^{2}T^{*2}T^{2}$ (5), $T \in B(H)$ is said to belong to class (n, mBQ) if $T^{*2m}T^{2n}(T^{*m}T^{n})^{2} = (T^{*m}T^{n})^{2}T^{*2m}T^{2n}$. A conjugation on a Hilbert space H is an anti-linear operator C from Hilbert space H onto itself that satisfies. Cξ, Cζi = hζ, ξi for every $\xi, \zeta \in H$ and $C^2 = I$. An operator T is said to be complex symmetric if $T = CT^{*}C$.

2. Main results

2.1. Theorem 1

Let $T \in B(H)$ be such that $T \in (n, mBQ)$, then the following holds for (N, mBQ);

(i). λT for any real λ

(ii). Any $S \in B(H)$ that is unitarily equivalent to T.

(iii). the restriction T /M to any closed subspace M of H.

Proof. (i). the proof is straight forward.

(ii). Let $S \in B(H)$ be unitarily equivalent to T, then there exists a unitary operator U

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∈ B (H) with

$S^n = U^* T^n U$ and $S^{*m} = U^* T^{*m} U$ for non-negative integers n and m. Since T ∈ (n, mBQ), we have;

$T^{*2m} T^{2n} (T^{*m} T^n)^2 = (T^{*m} T^n)^2 T^{*2m} T^{2n}$, hence

$S^{*2m} S^{2n} (S^{*m} S^n)^2 = UT^{*2m} U^* UT^{2n} U^* (UT^{*m} U^* UT^n U^*)^2$

$= UT^{*2m} U^* U^* T^{2n} U^* UT^{*m} U^* UT^{*m} U^* UT^n U^* UT^n U^*$

$= UT^{*2m} T^{2n} (T^{*m} T^n)^2 U^*$

$= U (T^{*m} T^n)^2 T^{*2m} T^{2n} U^*$

And

$(S^{*m} S^n)^2 S^{*2m} S^{2n} = (UT^{*m} U^* UT^n U^*)^2 UT^{*2m} U^* UT^{2n} U^*$

$= UT^{*m} U^* UT^n U^* UT^{*m} U^* UT^n U^* UT^{*2m} U^* UT^{2n} U^*$

$= UT^{*m} T^n T^{*m} T^n T^{*2m} T^{*2m} U^*$

$= U (T^{*m} T^n)^2 T^{*2m} T^{2n} U^*$

Thus S is unitarily equivalent to T.

(iii) . If T is in class (n, mBQ), then;

$T^{*2m} T^{2n} (T^{*m} T^n)^2 = (T^{*m} T^n)^2 T^{*2m} T^{2n}$.

Hence;

$(T/M)^{*2m} (T/M)^{2n} \{(T/M)^{*m} (T/M)^n\}^2$

$= (T/M)^{*2m} (T/M)^{2n} \{(T/M)^{*m} (T/M) n\}^2$

$= (T^{*2m}/M) (T^{2n}/M) \{(T^{*m}/M) (T^n/M)\} \{(T^{*m}/M) (T^n/M)\}$

$= \{(T^{*m} T^n)^2/M\} \{T^{*2m} T^{2n}/M\}$

$= \{(T^{*m}/M) (T^n/M)\}^2 (T/M)^{*2m} (T/M)^{2n}$

Thus T/M ∈ (n, mBQ).

2.2. Theorem 2

If T ∈ B (H) is in (n,m)-power Class (Q), then T ∈ (n,mBQ).

Proof. If T ∈ (Q), then

$T^{*2m} T^{2n} = (T^{*m} T^n)^2$

Post multiplying both sides by $T^{*2m} T^{2n}$;

$T^{*2m} T^{2n} T^{*2m} T^{2n} = (T^{*m} T^n)^2 T^{*2m} T^{2n}$

$T^{*2m} T^{2n} T^{*m} T^n T^{*m} T^n = (T^{*m} T^n)^2 T^{*2m} T^{2n}$

$T^{*2m} T^{2n} (T^{*m} T^n)^2 = (T^{*m} T^n)^2 T^{*2m} T^{2n}$.

2.3. Theorem 3

Let $S \in (n, mBQ)$ and $T \in (n, mBQ)$. If both S and T are doubly commuting, then

ST is in (n, mBQ).

Proof.

$(ST)^{*2m} (ST)^{2n} ((ST)^{*m} (ST)^{n})^{2}$

$=S^{*2m} T^{*2m} S^{2n} T^{2n} ((ST)^{*m} (ST)^{n}) ((ST)^{*m} (ST)^{n})$

$=S^{*2m} T^{*2m} S^{2n} T^{2n} ((S^{*m} T^{*m}) (S^{n}T^{n})) ((S^{*m}T^{*m}) (S^{n}T^{n}))$

$=S^{*2m} T^{*2m} S^{2n} T^{2n} S^{*m} T^{*m} S^{n}T^{n}S^{*m}T^{*m} S^{n}T^{n} S^{*m} T^{*m} S^{n}T^{n}$

$= S^{*2} T^{*2} S^{2} T^{2} S^{*} ST^{*}TS^{*} ST^{*} T$

$=T^{*2m} T^{2n} S^{*2m} S^{2n} S^{*m} S^{n}S^{*m} S^{n}T^{*m}T^{n}T^{*m}T^{n}$

$=T^{*2m} T^{2n} S^{*2m} S^{2n} (S^{*m}S^{n})^{2} T^{*m}T^{n}T^{*m}T^{n}$

$=T^{*2m} T^{2n} (S^{*m} S^{n})^{2} S^{*2m} S^{2n} T^{*m} T^{n}T^{*m}T^{n}$ (Since $S \in (n, mBQ)$).

$= (S^{*m}S^{n})^{2} T^{*2m} T^{2n} T^{*m} T^{n}T^{*m}T^{n}S^{*2m} S^{2n}$

$= (S^{*m} S^{n})^{2} T^{*2m} T^{2n} (T^{*m}T^{n})^{2} S^{*2m} S^{2n}$

$= (S^{*m} S^{n})^{2} (T^{*m}T^{n})^{2} T^{*2m} T^{2n} S^{*2m} S^{2n}$ (Since $T \in (n, mBQ)$).

$= ((S^{*m}S^{n}) (T^{*m}T^{n}))^{2} T^{*2m} S^{*2m} T^{2n} S^{2m}$

$= ((S^{*m}T^{*m}) (S^{n}T^{n}))^{2} S^{*2m} T^{*2m} S^{2n} T^{2n}$

$= ((ST)^{*m} (ST)^{n})^{2} (ST)^{*2m} (ST)^{2n}$

Thus $ST \in (n. mBQ)$.

2.4. Theorem 4

Let $T \in B(H)$ be a class (n, mBQ) operator such that $T = CT^{*}C$ for positive integers n and m with C being a conjugation on H. If C is such that it commutes with $T^{*2m} T^{2n}$ and $(T^{*m}T^{n})^{2}$, then T is an

(n,m)-power class (Q) operator.

Proof. Let $T \in (n, mBQ)$ and complex symmetric, then we have; $T^{*2m} T^{2n} (T^{*m}T^{n})^{2} = (T^{*m}T^{n})^{2} T^{*2m} T^{2n}$

And $T = CT^{*}C$.

Hence;

$T^{*2m} T^{2n} (T^{*m}T^{n})^{2} = (T^{*m}T^{n})^{2} T^{*2m} T^{2n}$

$T^{*2m}T^{2n} CT^{n}CCT^{*m} CCT^{n}CCT^{*m} C = (T^{*m}T^{n})^{2} CT^{n}CCT^{*m} CCT^{n}CCT^{*m} C.$

$T^{*2m} T^{2n} CT^{n}T^{*m} T^{n}T^{*m} C = (T^{*m}T^{n})^{2} CT^{n}T^{*m}T^{n}T^{*m} C$

$T^{*2m} T^{2n} CT^{2n} T^{*2m} C = (T^{*m}T^{n})^{2} CT^{*m}T^{n}T^{*m}T^{n}C$

$T^{*2m} T^{2n} C T^{*2m} T^{2n} C = (T^{*m} T^n)^2 C (T^{*m} T^n)^2 C$.

C commutes with $T^{*2m} T^{2n}$ and $(T^{*m} T^n)^2$ hence we obtain;

$T^{*2m} T^{2n} T^{*2m} T^{2n} = (T^{*m} T^n)^2 (T^{*m} T^n)^2$.

Which implies;

$T^{*2m} T^{2n} = (T^{*m} T^n)^2$ and thus $T \in$ (n,m)-power class (Q).

2.5. Theorem 5

Let $T \in B(H)$ be (n-1, m)-class (Q) operator, if T is a complex symmetric

Operator such that C commutes with $(T^{*m} T)^2$ for a positive ineteger m, then T is an (n, m)-power class (Q) operator.

Proof. With T being complex symmetric and (n-1, m)-class (Q), we have;

$T = CT^*C$ and $T^{*2m} T^{2n-2} = (T^{*m} T^{n-1})^2$.

We obtain;

$T^{*2m} T^{2n-2} T^2 = (T^{*m} T^{n-1})^2 T^2$.

Hence;

$T^{*2m} T^{2n} = (T^{*m} T^{n-1})^2 T^2$.

$T^{*2m} T^{2n} = T^{*2m} T^{2n-2} T^2 = T^{2n-2} T^{*2m} T^2$

$T^{*2m} T^{2n} = T^{2n-2} T^{*m} T^{*m} TT = T^{2n-2} CTCCTCCT^{*m} CCT^{*m} C = T^{2n-2} CTTT^{*m} T^{*m} C$.

$= T^{*2m} T^{2n} = T^{2n-2} CT^2 T^{*2m} C = T^{2n-2} C (T^{*m} T)^2 C$

Since C commutes with $(T^{*m} T)^2$ we obtain;

$T^{*2m} T^{2n} = T^{2n-2} (T^{*m} T)^2 CC = T^{2n-2} T^{*2m} T^2 CC = T^{2n-2} T^2 T^{*2m} CC = T^{*2m} T^{2n} = (T^{*m} T^n)^2$

Hence T is n-power class (Q).

3. Conclusion

The study of class (n,mBD) operators will help in the enhancement of study of properties of various classes such as class (Q) operators, normal operators and binormal operators.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declared no conflict of interest.

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