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(RESEARCH ARTICLE)



Establishment of local geometric geoid model for Busoga, Uganda

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Abstract

The importance of the local geoid model for the computation of accurate geoid heights, as well as orthometric heights used for engineering constructions, necessitated its establishment in areas, regions or countries. Consequently, this study establishes the local geometric geoid model of Busoga, Uganda, using the geometric method. A total of 26 points were used in the study, 20 points for the development of the model and 6 test points. GNSS observations were acquired with Trimble GNSS dual-frequency receivers and processed with Bernese (V5.2) and Spectra Precision Survey Office (v4.1) software to obtain the coordinates and ellipsoidal heights of the points. Differences between the existing orthometric and ellipsoidal heights were computed to obtain the geoid heights. The Least squares adjustment technique was applied to determine the fit, as well as the Bicubic and Multiquadratic models' parameters. The Root Mean Squares Error (RMSE) index was used to compute the accuracy of the models. The geoid models were compared with their RMSE, as well as accuracy to determine which of them is more suitable for application in the study area. A Microsoft Excel program was developed for the application of the model in the study area.

Keywords: Geoid; Orthometric; Undulation; Ellipsoidal; Height; Local; Model; Geometric

1. Introduction

The geoid is a surface adopted as a reference for practical, as well as orthometric heights computation. According to Jalal *et al.* [1], the geoid is a level surface, which is defined as a closed equigeopotential surface of the Earth's gravity nearest to the Mean Sea Level (MSL). The determination of the local geoid models of areas and regions has become crucial as the global geopotential models such as EGM 08, EGM 96, EGM 84 and EIGEN 6C4 do not best fit areas and regions. Also, the Global Navigation Satellite System (GNSS) ellipsoidal, as well as theoretical heights are not suitable for engineering construction as are computed on a specified ellipsoid which is not a level surface. By applying the ellipsoidal heights in an area, water cannot flow from one point to the other. But with orthometric heights, water can flow from one point to another.

The GNSS has the capability of determining the 3-D coordinates of points on the earth surface. The height component is theoretical. It is not a practical height obtained relative to the geoid. Orthometric height is obtained relative to the geoid using a level by spirit levelling. The process is laborious, costly and time-consuming. By applying it in a large area may result in significant errors in the observations; thereby reduce the resolution of the obtained heights. The ellipsoidal heights of points are converted to orthometric heights if their geoid heights are known. According to Mårtensson [2], obtaining orthometric heights this way, could in certain circumstances, depending on the required accuracy, replace conventional spirit levelling and thus make the levelling procedure cheaper and faster. Geoid heights are vertical

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distances between the ellipsoid and the geoid. It is called geoid undulation, as well as geoid-ellipsoid separation. Figure 1 shows the relationship among the geoid, ellipsoidal and orthometric heights.



Figure 1 Relationship among the Geoid, Ellipsoidal and Orthometric Heights.

Source: Hasan and Ismat [3]

From Figure 1, orthometric heights can be computed if the ellipsoidal and the geoid heights are known using [4]

H = h - N

Where,

H = Orthometric height

h = Ellipsoidal height

N = Geoid height

The geometric method has been applied by various researchers in various parts of the world with different accuracy achieved. Jürgenson *et al.* [5] fitted a gravimetric geoid surface to existing geometric geoid heights in Estonian with an accuracy of 1.3cm; El-Hallaq [6] developed a local GPS-Levelling (geometric) geoid model for Gaza Strip Area and obtained an accuracy of 4.28cm. Tata and Ono [4] also applied the geometric method in Akure environs, Nigeria and achieved RMSE of 0.511m. Ismail *et al.* [7] established a new fitted geoid model in Universiti Teknologi Malaysia with an accuracy of 8mm. Das *et al.* [8] determined the local geoid model of Papua New Guinea by comparing three polynomial surfaces (second, third and fourth degree polynomial surfaces) and recommended the third degree polynomial geoid models (Model 1 and Model 2) for orthometric heights interpolation in FCT, Abuja, Nigeria and recommends Model 1 with an accuracy of 11cm for application in the study area. Kyamulesire *et al.* [10] also determined the local geoid model of Sampala, Uganda by comparing three plane geometric geoid surfaces (Model 1, Model 2 and Model 3) and recommended the three models for application in the study area with more reliability, as well as greater confidence in model 2.

The European Improved Gravity model of the Earth by New techniques, EIGEN-6C4 [11, 10] is a global geopotential model adopted for the conversion of ellipsoidal heights to orthometric heights in Uganda. It is well known that global geoid models best the entire globe but not areas, regions or countries. Their application in areas, regions or countries, yields very low accuracy. A truncated model at a lower spherical harmonic degree was applied in Jinja, Uganda, an accuracy of 1.390m achieved is considered very low. Considering the low resolution of the global geoid model in the country, the local geoid model of Kampala which is the capital city of Uganda was determined using the geometric method [10]. Busoga is a sub-region of Uganda. It is a developing area that requires accurately obtained orthometric heights for the fast execution of infrastructural and civil engineering development programs such as highways, water flow, and in general, topographical base or working plans preparation. Consequently, this study establishes the local geoid model of Busoga, Uganda, using the geometric method.

1)

1.1. The Study Areas

The study area is Busoga, Uganda. According to Isiko [12], Busoga sub-region is located in the eastern part of Uganda, curved by Lake Kyoga in the north. Victoria Nile in the west separates Busoga from Buganda. In the south, Lake Victoria acts as the boundary that separates Busoga from the islands that belong to the Basamia and Buganda. Mpologoma River in the east separates Busoga from the districts that make up the former Bukedi region. The area covered by Busoga is about 8920 square kilometres, it is relatively flat. Busoga region lies between the equator and 2° 00" N and longitudes 33° 00" E and 34° 00" E, the altitude above sea level varies between 1036.32 and 1219.20 metres [13]. Figure 2 shows the map of Uganda sub-regions.



Figure 2 Map of Uganda Sub-Regions Showing Busoga

1.2. Geometric Geoid Surfaces

Geometric geoid surfaces are mathematical interpolation surfaces fitted to geoid heights to enable geoid heights of new points to be determined using variable such as geographic or rectangular coordinates. These surfaces include plane surface, bi-linear surface, second degree surface, third degree polynomial and fifth degree polynomial. The surface to be adopted, as well as the degree and order of the polynomial, depends on the size of the study area and variation of the geoid heights. For a small area; the plane surface is used, for a relatively large area; the second and third-order polynomial surfaces are used [14, 15].

The study area (Busoga) is relatively large; as a result, the Bicubic and Multiquadratic model surfaces were applied in the study. The Bicubic model is a third degree polynomial surface used for geometric geoid modelling. The Bicubic geoid model as given in Kirici and Sisman [16] and Oduyebo et al. [15] is

$$N = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 XY + a_5 Y^2 + a_6 X^3 + a_7 X^2 Y + a_8 XY^2 + a_9 Y^3$$
(2)

Also, the Multiquadratic model given by Sanlioglu et al. [17] is

$$N = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 Y^2 + a_5 XY + a_6 X^2 Y + a_7 XY^2 + a_8 X^2 Y^2$$
(3)

Where,

Y = Northing, coordinate of observed station

X = Easting, coordinate of observed station

 $a_0, a_1, a_2, \dots a_n$ = Model coefficient/parameters

N = Geoid height

1.3. Observation Equation Method of Least Squares Adjustment

The fitting of the polynomial interpolation surface to a set of geoid heights requires the model parameters (variable coefficients) to be computed. The computation of these coefficients is done by observation equation method of least squares adjustment technique. The functional relationship between adjusted observations and the adjusted parameters as given by Eteje and Oduyebo [18] is

$$L_a = F(X_a) \tag{4}$$

Where, L_a = adjusted observations and X_a = adjusted parameters. Equation (4) is a linear function from which the general observation equation model was obtained. The system of observation equations is presented by matrix notation as [19]

$$V = AX - L \tag{5}$$

Where,

A = Design Matrix,

X = Vector of Unknowns

L = Observation Matrix.

V = Residual Matrix

The residual (V) is the difference between the estimate and the observed value. It is useful when applying the least squares adjustment technique in the computation of geoid model parameters. Since it is equal to the difference between the model geoid heights and the computed geoid heights from GNSS observations and spirit levelling, it is useful as a check. The model used for the computation of the unknown parameter as given in Oluyori et al. [20] is

$$X = (A^T A)^{-1} A^T L \tag{6}$$

Where,

 $(A^{T}A)^{-1}$ = Inverse of the normal matrix

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The step by step procedures for the computation of the polynomial interpolation model Coefficients (a_{mn}) of the variables (x, y) are detailed in Eteje and Oduyebo [18].

1.4. Accuracy of the Model

The accuracy of geometric geoid model is obtained using the Root Mean Square Error, RMSE index. To compute the accuracy of the model, the model geoid heights of the test points are compared with their corresponding computed geoid heights to obtain the geoid height residuals. The computed geoid heights are obtained by finding the differences between the known/existing orthometric heights and the GNSS ellipsoidal heights. The geoid height residuals and the total number of the test points are used to compute the RMSE, as well as the accuracy of the model. Here, the accuracy of the model varies inversely to the computed value of the RMSE [21]. The Root Mean Square Error, RMSE index used for the computation of the geometric geoid model accuracy as given by Yilmaz and Kozlu [22] is

$$RMSE = \pm \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\delta N_{\text{Residual}} \right)^2}$$
(7)

Where,

 $\partial N_{\text{Residual}} = (N_{\text{Computed}} - N_{\text{Model}})$ $N_{\text{Computed}} =$ Geoid height from existing orthometric and GNSS ellipsoidal heights $N_{\text{Model}} =$ Geometric Geoid Height n = Number of Points

2. Methodology

2.1. Data Acquisition

A total of 26 points were used in the study, 20 points for modelling the local geoid and 6 test points. The orthometric heights have previously determined by gravimetric means [23]. GNSS observation was carried out with Trimble GNSS dual-frequency Receivers in static mode to obtain the coordinates and ellipsoidal heights of the points. The occupation period of each point during the observation was 3 hours. It was carried out from 27th February, 2019 to 22nd April, 2019. Figure 3 shows the GNSS receiver at station U2092.



Figure 3 GNSS Receiver at Station U2092

2.2. Data Processing

The GNSS observations were processed using Bernese (V5.2) and Spectra Precision Survey Office (v4.1) software in ITRF2005 (Epoch 2010.0) datum (UTM zone 36N). The geoid heights of the points were computed by subtracting the existing orthometric heights from their corresponding ellipsoidal heights obtained from the processed GNSS

observations using equation (1). Table 1 shows the GNSS ellipsoidal heights, existing orthometric heights and the computed geoid heights.

Station	GNSS Ellipsoidal Height (m)	Existing Orthometric Height (m)	Computed Geoid Height (m)
U2022	1158.850	1172.250	-13.400
U1068	1044.100	1058.240	-14.140
U1057	1086.010	1100.350	-14.340
U1033	1181.940	1197.050	-15.110
U2113	1086.570	1101.140	-14.570
U2112	1097.770	1112.350	-14.580
U2110	1125.030	1139.270	-14.240
U2109	1104.950	1119.250	-14.300
U2108	1092.070	1106.750	-14.680
U2107	1123.340	1138.090	-14.750
U2094	1190.660	1205.870	-15.210
U2092	1167.760	1182.900	-15.140
U2027	1064.240	1077.550	-13.310
U2025	1186.120	1199.240	-13.120
U2024	1179.020	1192.240	-13.220
U2023	1173.210	1186.640	-13.430
U1060	1093.550	1107.100	-13.550
U1023	1133.060	1148.360	-15.300
U1031	1136.210	1150.940	-14.730
U1039	1168.900	1183.260	-14.360

Table 1 Ellipsoidal, Existing Orthometric and Computed Geoid Heights

The Bicubic and Multiquadratic models' (equations (2) and (3) respectively) parameters were computed with the coordinates and geoid heights of the points using the least squares technique, as well as equation (6). The computations were done with Microsoft Excel programs. The fit, as well as the models' parameters, is necessary to enable the interpolation of geoid heights of new points in the study area using their (the points') coordinates. The parameters were substituted accordingly into their respective equations, and Microsoft Excel programs developed for the implementation of the models in the study area. The computed Bicubic and Multiquadratic models' parameters are respectively.

Bicubic Model Parameters



Multiquadratic Model Parameters



The accuracy of the models were computed with the 6 test points' existing orthometric heights and the GNSS ellipsoidal heights. The geoid heights of the points were computed by finding the differences between the ellipsoidal heights and the orthometric heights. The computed geoid heights were compared with their respective geoid heights from the two models to obtain the residuals. The accuracy computation was done using the RMSE index, as well as equation (7).

3. Results

Table 2 presents the computed geoid heights, Bicubic and Multiquadratic models' geoid heights and the averages of the absolute differences between the computed and the two models' geoid heights. It was done to present the minimum and maximum geoid heights of the two models and the averages of their absolute differences between the computed geoid heights.

Station	Computed Geoid Height (m) (A)	Bicubic Geoid (m) (B)	Model Height	Multiquadratic Model Geoid Height (m) (C)	I(A) - (B)I (m)	I(A) - (C)I (m)
U2022	-13.400	-13.407		-13.403	0.007	0.003
U1068	-14.140	-14.170		-14.179	0.030	0.039
U1057	-14.340	-14.331		-14.316	0.009	0.024
U1033	-15.110	-15.072		-15.063	0.038	0.047
U2113	-14.570	-14.516		-14.520	0.054	0.050
U2112	-14.580	-14.531		-14.534	0.049	0.046
U2110	-14.240	-14.241		-14.242	0.001	0.002
U2109	-14.300	-14.310		-14.309	0.010	0.009
U2108	-14.680	-14.749		-14.754	0.069	0.074
U2107	-14.750	-14.860		-14.860	0.110	0.110
U2094	-15.210	-15.128		-15.120	0.082	0.090
U2092	-15.140	-15.111		-15.128	0.029	0.012
U2027	-13.310	-13.386		-13.373	0.076	0.063
U2025	-13.120	-13.064		-13.077	0.056	0.043
U2024	-13.220	-13.224		-13.230	0.004	0.010
U2023	-13.430	-13.466		-13.464	0.036	0.034
U1060	-13.550	-13.517		-13.512	0.033	0.038
U1023	-15.300	-15.386		-15.388	0.086	0.088
U1031	-14.730	-14.666		-14.652	0.064	0.078
U1039	-14.360	-14.343		-14.356	0.017	0.004
				Average =	0.043	0.043

Table 2 Computed, Bicubic, Multiquadratic Geoid Heights and their Absolute Differences

Figures 4 to 6 and Figure 7 respectively present the contour and linear plots of the computed, Bicubic and Multiquadratic geoid heights of the points. They were plotted to show graphically, the shapes and agreements of the Bicubic and the Multiquadratic models' geoid heights with the computed geoid heights.





Figure 7 Plots of Computed, Bicubic and Multiquadratic Geoid Heights

Table 3 presents the Root Mean Square Errors (RMSEs) of the Bicubic and Multiquadratic geoid models using the 6 test points. The RMSEs were computed to show and compare the accuracy of the two models to determine which of them (the models) is more suitable for use in the study area.

Table 3 Bicubic and Multiquadratic Models' RMSE Computation Using the Test Points

Computed N (m) (P)	Bicubic N (m) (Q)	Multiquadratic N (m) (S)	((P) -(Q)) ² Bicubic	((P) -(S))² Multiquadratic
-14.200	-14.212	-14.207	0.000133	0.000054
-14.660	-14.768	-14.775	0.011688	0.013194
-14.520	-14.495	-14.495	0.000618	0.000629
-15.170	-15.107	-15.108	0.004011	0.003825
-13.250	-13.269	-13.266	0.000376	0.000264
-13.740	-13.871	-13.833	0.017162	0.008668
		RMSE (m) =	0.075264	0.066625

4. Discussion

In Table 2, the minimum and maximum geoid heights of the Bicubic and Multiquadratic geoid models are respectively 15.386m and -13.064m, and -15.388m and -13.077m. It shows that geoid heights can be respectively interpolated in the study area with the Bicubic and Multiquadratic models within the ranges of -15.386m to -13.064m, and -15.388m to - 13.077m. Also, in Table 2, the averages of the absolute differences between the computed geoid heights and the two models geoid heights are 0.043m. It implies a good fit of the two geoid surfaces to the computed geoid heights.

In Figures 4 to 6, the shapes of the two geoid models' contour maps are identical with that of the computed geoid heights. It shows the agreement of the two geoid models' geoid heights with the computed geoid heights of the study area. Also, in Figure 7, the linear plots of the Bicubic and Multiquadratic models' geoid heights are identical with that of the computed geoid heights. It also indicates the agreement, as well as a good fit of the two geoid surfaces to the computed geoid heights.

It can again be seen in Table 3 that the RMSEs of the Bicubic and Multiquadratic geoid models are respectively 0.075m and 0.067m. According to Oluyori and Eteje (2020), the accuracy of the model varies inversely to the computed value of the RMSE. It implies that the Multiquadratic geoid model is more suitable for application in the study area. It also shows that geoid heights can be interpolated in the study area with an accuracy of 0.067m with the Multiquadratic geoid model. The accuracy achieved in this study is within the ones obtained in previous studies [4-10]. With an accuracy of less than 10cm for the recommended model, it is fit for applications like engineering feasibility studies, GIS topographic mapping, remote sensing land-cover mapping projects and oceanographic applications [24].

5. Conclusion

In conclusion, the study has established the local geoid model of Busoga, Uganda, using the geometric method. It has shown that the Multiquadratic geoid model is more suitable for application in the study area. It has also revealed that geoid heights can be interpolated in the study area with an accuracy of 0.067m using the recommended geoid model. The study has developed a Microsoft Excel program for implementation of the determined geoid model in the study area.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declare that there is no conflict of interest in this study

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