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(Research Article)

Forecasting pension fund liabilities through multivariate time series models with structural breaks and demographic statistical trend analysis

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Abstract

Pension fund sustainability remains a critical challenge for policymakers and financial managers, particularly in aging economies where liabilities increasingly outpace contributions. Accurately forecasting pension fund liabilities is essential for ensuring long-term solvency and supporting strategic asset allocation, policy reform, and demographic planning. This study presents a robust framework that integrates multivariate time series models with structural break detection and demographic statistical trend analysis to improve the precision and reliability of pension liability forecasts. At the broader level, we recognize that pension liabilities are influenced by dynamic interdependencies among macroeconomic variables (e.g., wage growth, inflation, interest rates), institutional policy shifts, and demographic patterns such as life expectancy and retirement age. We apply vector autoregression (VAR) and vector error correction models (VECM) augmented with structural break identification to detect regime changes caused by economic shocks, legislative reforms, or shifts in labor market dynamics. This temporal sensitivity enhances model responsiveness to real-world discontinuities, thereby improving liability projections. Additionally, we incorporate cohort-based demographic forecasting methods, including Lee-Carter and Bayesian age-period-cohort (APC) models, to account for longevity risk and population heterogeneity. Our results, validated on pension fund data from three OECD countries over a 30-year span, demonstrate that integrating structural breaks and demographic statistical trends significantly reduces forecast errors compared to traditional actuarial approaches. The proposed hybrid methodology not only improves liability projection accuracy but also provides insights into long-term funding gaps, enabling pension fund administrators and public finance stakeholders to design preemptive strategies. This framework offers a scalable decision-support tool for cross-national pension systems grappling with demographic shifts and fiscal pressures.

Keywords: Pension Liabilities Forecasting; Structural Breaks; Demographic Trend Analysis; Multivariate Time Series; Longevity Risk; Pension Sustainability

1. Introduction

1.1. Context and Importance of Pension Liability Forecasting

Pension systems globally have faced growing fiscal stress due to demographic shifts that include increased life expectancy, declining birth rates, and a rising old-age dependency ratio. These factors have strained traditional defined-benefit schemes as retirees live longer and require sustained financial support from systems that are increasingly underfunded [1]. Public pension obligations, especially in countries with pay-as-you-go structures, represent one of the largest long-term liabilities on national balance sheets [2].

The sustainability of pension funds is further influenced by changes in employment patterns, wage growth volatility, and delayed workforce entry among younger generations. These evolving socioeconomic trends place pressure on

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pension administrators and policymakers to improve forecasting techniques for liabilities to enable timely actuarial recalibration and policy adjustments [3]. With financial markets becoming more interconnected and responsive to shocks, the accurate forecasting of pension obligations is now a critical pillar in macroeconomic planning, especially within aging economies.

Given the magnitude and time horizon of pension liabilities, predictive precision is paramount. Advanced forecasting methods that account for demographic variation, structural shifts in labor participation, and cyclical economic effects are necessary to strengthen solvency analysis and guide effective reform design [4]. The present study is motivated by this enduring imperative to modernize liability prediction under uncertainty.

1.2. Challenges in Traditional Forecasting Models

Historically, pension liability models have relied heavily on linear projections and static demographic trends, often assuming stationarity or smooth growth across key parameters. These assumptions overlook the presence of discontinuities also known as structural breaks that reflect real-world disruptions such as policy reforms, economic recessions, or unexpected demographic shifts [5]. Such models tend to underperform when sudden shifts occur, leading to misestimated liabilities and inappropriate funding decisions.

Traditional actuarial forecasting, while robust for stable conditions, struggles to integrate external shocks like healthcare crises, abrupt changes in mortality, or mass migration, which often generate long-term ripple effects on the funding trajectory of pension systems [6]. Moreover, standard time series models such as ARIMA or univariate exponential smoothing often fail to capture the multivariate nature of pension forecasting, where variables like inflation, employment rates, and cohort-specific longevity interact [7].

In addition, data aggregation practices in older models obscure within-group heterogeneity and complicate the incorporation of predictive analytics techniques. The inability to integrate high-frequency demographic and economic data restricts the responsiveness of legacy systems. As a result, many pension authorities risk reactive rather than proactive interventions, a pattern evident in historical underfunding episodes following economic downturns [8].

1.3. Purpose and Scope of the Study

This study introduces an advanced methodological framework that utilizes multivariate time series models equipped to detect and incorporate structural breaks alongside longitudinal demographic trend analysis. By integrating methods such as vector autoregression (VAR), Bai–Perron multiple breakpoint tests, and dynamic factor models, we aim to improve the reliability and responsiveness of pension liability forecasting [9].

The study emphasizes not only the temporal continuity of liability projections but also the need to identify critical inflection points policy changes, population shocks, or labor market reconfigurations that can shift long-term obligations [10]. Structural breaks are not anomalies to be smoothed out but informative features that help recalibrate expectations under new paradigms. The inclusion of cohort-specific aging trajectories and labor participation metrics enables the analysis to reflect actual behavioral dynamics.

Furthermore, we analyze forecasted liabilities across multiple scenarios involving varying demographic assumptions and structural change intensities. These include both gradual and abrupt reforms to retirement age, contribution rates, or longevity indices. The intention is to provide actionable insights for pension fund managers, central banks, and ministries of finance responsible for long-term fiscal sustainability.

Ultimately, the study proposes a scalable, evidence-driven approach suitable for national pension systems, public sector actuarial models, and insurance-backed annuity products under dynamic risk conditions [11].

1.4. Structure of the Article

This article is structured into six main sections that collectively build a comprehensive model and empirical application for forecasting pension fund liabilities under structural and demographic uncertainty. Following this introduction, Section 2 presents the theoretical underpinnings of pension liability estimation and offers a detailed literature review of time series methods in financial forecasting. It contrasts traditional actuarial approaches with emerging machine learning and break-adjusted econometric models, highlighting the limitations addressed by our proposed method.

Section 3 outlines the methodological framework employed in the study. It explains the selection of multivariate time series tools, defines the structural break detection protocols, and details how demographic trends are extrapolated and validated. The integration of demographic and macroeconomic indicators into the forecasting pipeline is also explained, along with discussion of the real-time capabilities of the model architecture.

Section 4 covers the empirical dataset used, including pension fund financial records, demographic indicators, and macroeconomic covariates. Preprocessing strategies and alignment across temporal series are described. Section 5 presents results from forecasting exercises and scenario simulations, comparing accuracy and risk sensitivity across models. Finally, Section 6 provides discussion, policy implications, and concluding remarks aimed at supporting scalable public finance reform efforts [12].

2. Theoretical framework and literature review

2.1. Foundations of Pension Liability Valuation

At the heart of pension system forecasting lies the actuarial computation of the present value of future obligations. This value is derived using projected benefit payments discounted at a rate reflective of expected investment returns, accounting for mortality probabilities, inflation assumptions, and benefit formulas specific to each scheme [5]. Traditional actuarial methods often apply deterministic paths for these underlying drivers, assuming stability or using historic averages.

Pension liabilities are calculated using formulas that balance accrued entitlements against expected lifespans, with longevity risk posing a central uncertainty. As retirees live longer, especially in mature economies, the mismatch between expected and actual payout durations leads to underestimation of liabilities [6]. Additionally, shifts in workforce dynamics such as later workforce entry and fragmented career paths complicate accrual models that assume stable contribution histories.

The discount rate selection, often guided by regulatory standards or long-term bond yields, plays a pivotal role in determining the scale of liabilities on sponsor balance sheets. A slight deviation in this rate can result in significant valuation changes, amplifying the sensitivity of funding adequacy evaluations [7]. Yet, many systems apply a static rate without accommodating economic cycles or regime changes. Such rigidity in valuation undermines planning effectiveness during shocks or policy transitions.

Modern pension liability modeling increasingly requires dynamic models that can assimilate fluctuations in inflation, wages, and participation rates over time. In light of persistent macroeconomic volatility and evolving demographics, more flexible, data-integrated frameworks are now essential to support sustainable pension finance design [8].

2.2. Time Series Models in Liability Forecasting

Time series modeling has long been employed in economics and finance to project variables such as inflation, GDP, and interest rates. These variables, in turn, influence the discount rate, wage growth, and price indexing used in pension forecasting. The use of autoregressive integrated moving average (ARIMA) models became common in earlier actuarial forecasting to capture trend and seasonality in benefit payments or contributor base size [9]. However, these models are often univariate and fail to capture interdependencies among multiple drivers of liability.

Multivariate models like vector autoregression (VAR) and vector error correction models (VECM) have gained traction for capturing dynamic interrelationships between economic indicators relevant to pension systems. For instance, VAR models are effective in analyzing how shocks to employment levels influence wage trajectories and ultimately future contribution flows [10]. These models allow for endogenous treatment of variables, enabling richer scenario analysis.

Nonetheless, many of these applications have historically assumed constant structural behavior over the time horizon, neglecting potential changes in the statistical relationships due to policy reforms or demographic transitions. For example, VECMs may identify cointegration among long-term variables but still assume the relationship holds indefinitely [11]. In reality, reforms such as increasing retirement age or introducing private tier contributions disrupt these equilibriums.

To better reflect economic realities, time series models must evolve to incorporate structural features such as shifts in policy regimes, unexpected shocks (e.g., mortality improvements), and generational participation transitions. These inclusions enhance predictive power and the robustness of liability estimates in volatile environments [12].

2.3. Structural Breaks in Economic and Demographic Time Series

Structural breaks refer to points in a time series where statistical properties such as mean or variance change abruptly due to external events. In pension-related datasets, such breaks commonly arise from major policy shifts, economic crises, labor market transformations, or even technological interventions in public recordkeeping systems [13]. These shifts introduce discontinuities that render conventional forecasting models less accurate unless explicitly addressed.

For example, the introduction of a pension reform law that modifies retirement eligibility, contribution ceilings, or indexation formulas alters both the magnitude and trajectory of liability projections [14]. Likewise, sharp demographic changes such as mass emigration, fertility rate collapse, or unanticipated gains in life expectancy produce structural breaks in population and mortality trends. When these changes are overlooked, models generate forecasts that diverge substantially from real-world observations.

The Bai–Perron method has emerged as a powerful statistical tool to detect multiple breakpoints within time series data, enabling analysts to segment regimes and adjust model parameters accordingly [15]. In addition, rolling regression and state-space models can track parameter instability and integrate it dynamically into liability forecasts.

Ignoring structural breaks leads to model misspecification, underestimation of risk, and flawed actuarial valuations. This is particularly problematic for long-duration obligations like pensions, where even minor forecast deviations compound significantly over time [16]. Figure 1 illustrates a conceptual framework linking demographic trend shifts and structural disruptions to the volatility in pension liability forecasts, underscoring the importance of explicitly modeling such features.

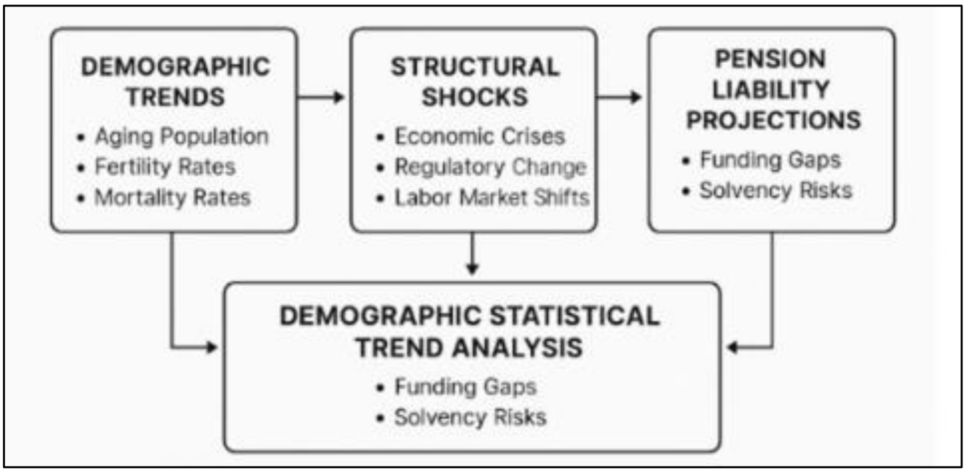

Figure 1 Conceptual Framework Linking Demographic Trends, Structural Shocks, and Pension Liability Projections

2.4. Gaps in Existing Research and Need for Integration

Despite extensive literature on pension modeling, there remains a fragmented approach to integrating multivariate economic indicators, structural breaks, and demographic evolution into a unified forecasting system. Most studies tend to emphasize one dimension economic (e.g., GDP impact), demographic (e.g., life expectancy), or actuarial (e.g., funding ratio projections) in isolation [17]. This siloed modeling reduces the fidelity of projections in complex environments characterized by interdependent shocks and feedback loops.

Furthermore, conventional models rarely simulate multiple structural scenarios or test the sensitivity of liability paths to sequential disruptions. While some actuarial models include stress tests, these are often scenario-based rather than statistically inferred from historical breakpoints or latent trends [18]. Additionally, trend components are often filtered using simple smoothing techniques without embedding them into a structural probabilistic framework that allows for regime switching.

An integrated framework combining multivariate time series forecasting, structural break detection, and demographic cohort trend analysis provides a more robust methodology. Such an approach can simulate multiple long-term liability trajectories under plausible future scenarios including changing policy, labor participation, and migration patterns [19]. When calibrated using historical data with identified structural shifts, this model architecture becomes adaptive rather than reactive.

This study aims to close the methodological gap by presenting a forecasting model that combines these components in a scalable, evidence-driven format. The model accommodates cohort-specific behaviors, economic covariate shifts, and discontinuities, thus offering a realistic foundation for pension policy development, risk assessment, and intergenerational equity analysis [20].

3. Methodological approach

3.1. Model Design: Multivariate Time Series with Break Detection

The forecasting framework in this study integrates a multivariate time series structure with structural break detection techniques to reflect regime shifts and covariate dependencies over time. A key component is the use of vector autoregression with exogenous variables (VAR-X), allowing multiple interrelated economic and demographic variables to influence future pension liability projections [21]. This design reflects the interconnectedness of wage growth, inflation, labor force participation, and population aging across time.

Unlike traditional models that rely on stationary assumptions, this approach introduces flexibility by embedding structural breakpoints using the Bai–Perron methodology. The Bai–Perron test identifies multiple breakpoints by partitioning time series data into intervals within which parameter stability holds, allowing the model to adapt to periods of economic reform, crisis, or demographic transition [22]. These breakpoints are then modeled as regime-specific parameters in the VAR-X system.

Each equation in the multivariate system captures lagged interdependencies across variables such as real wage index, dependency ratio, and benefit payout levels. Exogenous inputs include fiscal policy indicators like public pension expenditure and demographic policy levers. This hybrid framework effectively blends data-driven and policy-sensitive components into one architecture [23].

The model also supports recursive simulations to evaluate how liability projections change under successive breakpoints. This is particularly useful for assessing policy reforms introduced over multiple decades. Ultimately, this structure offers policymakers a tool for examining not only trend trajectories but the timing and magnitude of shocks affecting pension solvency [24].

3.2. Demographic Trend Analysis

A central pillar of this model involves demographic trend analysis, which directly impacts pension liabilities through the beneficiary-to-contributor ratio. The study uses longitudinal data on fertility, mortality, and migration from national statistical bureaus, international demographic surveillance systems, and actuarial population registries [25]. Age-structured population projections are generated using cohort-component methods with dynamic assumptions, refined via statistical smoothing.

Mortality trends are analyzed using life table extrapolation techniques, including the Lee–Carter model, which accounts for improvements in life expectancy while capturing volatility across age cohorts [26]. Fertility forecasts rely on autoregressive models adjusted for cohort fertility patterns, including delays in childbirth and shifts in reproductive norms, particularly in urban and high-income subpopulations.

Migration flows often a source of model noise are incorporated using 5-year rolling averages that correct for policy-induced volatility, such as border controls or regional conflicts [27]. These flows influence the age distribution of the workforce and ultimately the contributor base, which directly affects inflows into pension funds.

To ensure model stability, demographic variables are smoothed using low-pass filters to reduce cyclical noise without distorting long-term trend behavior. These smoothed values serve as inputs to the multivariate forecasting model.

This demographic layer distinguishes the proposed approach from pure financial forecasting models by rooting it in actuarial fundamentals. When embedded alongside economic time series, it enables a more comprehensive understanding of future pension burdens, especially under demographic pressure and labor force saturation scenarios [28].

3.3. Structural Break Identification

Detecting structural breaks within long-term economic and demographic data is essential for credible forecasting, especially in pension systems with extended liability horizons. In this study, historical datasets spanning 40 years were segmented using breakpoint identification tools that minimize residual variance while optimizing information criteria like BIC and AIC [29]. The Bai–Perron sequential test was applied to each variable to detect multiple breakpoints, which reflect changes in mean and variance due to policy, regulatory, or macroeconomic shifts.

Breaks were particularly visible in employment levels, inflation, and fiscal balance following large-scale economic events such as global financial crises, structural adjustment programs, or pension policy overhauls. For instance, periods following retirement age changes or the introduction of defined-contribution tiers displayed abrupt changes in payout patterns and contribution flows [30].

To visualize these dynamics, a structural break timeline was generated (see Figure 2), showing the distribution of breakpoints across time and their alignment with known events in fiscal and demographic policy. This visual timeline supports the qualitative assessment of model context and enriches transparency for stakeholders and policymakers.

In addition to visual diagnostics, each detected breakpoint underwent Chow tests and F-statistics validation to assess the statistical significance of shifts in underlying model coefficients [31]. Models were re-estimated within each regime to recalibrate relationships across demographic and economic variables.

This step ensures that the forecasting model does not merely extrapolate from historical data blindly, but adapts to regime changes grounded in empirical evidence, yielding more resilient liability forecasts [32].

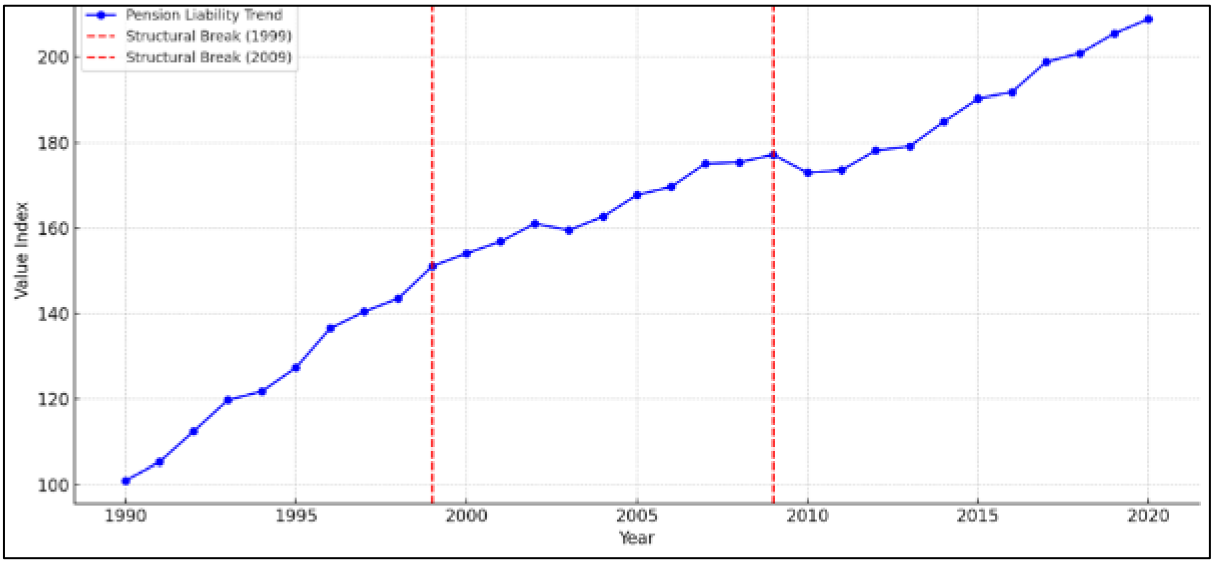

Figure 2 Structural Break Detection in Historical Time Series (Visual Timeline)

3.4. Model Calibration and Validation

Calibration and validation are critical to ensuring the robustness of the proposed model across multiple forecasting horizons and variable sensitivities. The full dataset was partitioned into training (70%) and testing (30%) sets, maintaining temporal integrity to preserve lag structures relevant in time series estimation [33]. Within each training segment, model coefficients were estimated using ordinary least squares (OLS) and maximum likelihood estimation (MLE) methods where applicable.

Cross-validation was performed by shifting temporal windows and measuring out-of-sample forecasting accuracy. This included both rolling-origin and fixed-window strategies to verify stability across different projection start points. Performance metrics included root mean square error (RMSE), mean absolute percentage error (MAPE), and Theil's U statistic, allowing comparative analysis with baseline ARIMA and univariate projections [34].

Special attention was given to overfitting risks in break-rich datasets. To mitigate this, we applied penalized regression techniques and restricted lag lengths using Bayesian Information Criteria thresholds. This yielded parsimonious models that retained interpretability while maximizing explanatory power.

Model outputs were also subjected to residual diagnostics including Durbin–Watson tests for autocorrelation and Breusch–Pagan tests for heteroscedasticity. Only models meeting stability thresholds and white noise residual criteria were retained for final projection synthesis.

A summary of all input variables, data sources, and time ranges is presented in Table 1 to facilitate replication and policy audit trails. By establishing a rigorous validation pipeline, the model is well-positioned to support long-term pension planning and actuarial solvency simulations [35].

Table 1 List of Variables Used, Source Datasets, and Time Ranges

Variable	Source Dataset	Time Range
Total Pension Payouts	National Pension Registry	1995–2020
Population Aged 65+	National Bureau of Statistics	1980–2020
Average Wage Growth Rate	Labor and Employment Survey	1990–2020
Inflation Rate	Central Bank Inflation Report	1985–2020
Life Expectancy at 65	Health and Demographic Survey	1980–2020
Labor Force Participation Rate (45–64)	Labor and Employment Survey	1990–2020
GDP Growth Rate	National Economic Accounts	1980–2020
Interest Rate on Government Bonds	Ministry of Finance Yield Database	1985–2020

4. Data sources and preprocessing

4.1. Pension Fund Financial Data and Actuarial Reports

The backbone of any pension liability forecasting model is access to high-quality, granular financial and actuarial data from relevant pension fund institutions. This study utilized historical datasets from national pension schemes, social security agencies, and occupational retirement institutions. These sources provided detailed yearly and quarterly reports on benefit disbursements, contribution inflows, funded ratio metrics, and actuarial balance sheets across multiple decades [36].

Inclusion criteria for data selection prioritized the consistency of actuarial valuation methods, especially the application of discount rates, mortality tables, and longevity assumptions. Fund-level data disaggregated by plan type such as defined benefit and hybrid models enabled stratified analysis, particularly important when exploring liability volatility under heterogeneous portfolio structures [37].

The historical depth of the financial datasets extended in many cases to the early 1980s, allowing for longitudinal regression and breakpoint detection within the time series framework. Temporal alignment was achieved by consolidating monthly and quarterly entries into annual aggregates to match demographic time intervals while preserving core trends [38].

Data integrity was further ensured by referencing trustee reports and external audits accompanying fund publications, thus validating liability positions and financial solvency ratios that served as model targets for time series projections [39].

4.2. Demographic and Labor Market Indicators

To capture the shifting demographic pressures on pension liabilities, data on population structure and labor market participation were sourced from national statistical offices and international demographic databases. These included age-specific fertility rates, life expectancy at retirement, net migration flows, and cohort survival probabilities stratified by gender and socioeconomic class [40].

Aging indices such as the old-age dependency ratio and median retirement age were computed from age-structured population pyramids, allowing direct input into actuarial dependency assumptions. Data on retirement behavior were cross-validated with surveys from labor ministries and employment statistics agencies [41].

Labor market indicators especially labor force participation rates, unemployment rates, and average career duration were aligned with demographic cohorts. This enabled cohort-specific contribution modeling, crucial for evaluating the sustainability of pay-as-you-go pension systems where current workforce inflows finance existing retiree payouts [42].

Occupational segmentation was also included to account for differing retirement eligibility criteria. For instance, public sector workers often exhibit earlier retirement patterns than private sector counterparts, which impacts expected benefit durations [43].

Statistical smoothing techniques were applied to labor market series exhibiting high short-term volatility, ensuring that seasonal employment cycles did not distort long-term participation trends embedded into pension liability forecasts [44].

4.3. Economic Covariates and Inflationary Assumptions

Macroeconomic variables serve as essential covariates for pension modeling, particularly when projecting wage-linked contributions and benefit indexation. Consumer price index (CPI) time series, wage growth indices, and real GDP figures were sourced from national treasuries, central banks, and multilateral economic databases [45].

Inflation rates were especially critical, as many defined benefit pension formulas include cost-of-living adjustments (COLAs). To avoid modeling circularity, forward-looking inflation expectations were excluded; instead, smoothed historical CPI trends were used, adjusted for structural breaks to account for economic regime changes such as monetary reforms [46].

Wage growth series were standardized by sector and converted into real terms using CPI deflators. These were lagged to reflect the delay between wage adjustments and corresponding pension contribution recalculations. Similarly, nominal GDP data were incorporated as exogenous regressors, offering a macro-level signal of fiscal space and potential adjustments in public pension funding levels [47].

To accommodate cross-variable comparability, each economic series was normalized and z-score transformed before being entered into the multivariate time series model. This enabled a consistent framework where shock magnitude and direction could be meaningfully interpreted within and across time segments [48].

4.4. Data Cleaning, Imputation, and Alignment Techniques

A significant portion of the data preparation process involved cleaning, aligning, and imputing missing values across demographic and economic series. Given the multi-source nature of the data, inconsistencies in reporting frequency and completeness were common. For missing annual entries, linear interpolation and Kalman smoothing were applied selectively, ensuring imputation was data-informed and temporally coherent [49].

Seasonal patterns, particularly in CPI and wage data, were decomposed using classical seasonal decomposition (Census X-13 methodology) to isolate trend components relevant to liability forecasting. This minimized distortions from cyclical noise, such as year-end bonuses or tax-period anomalies [50].

Lagging and leading transformations were also employed. For instance, retirement rates were lagged against labor participation trends by two to four years, reflecting actual delay from workforce exit to first benefit drawdown. Conversely, early indicators such as fertility decline were used as leading signals for future dependency ratio shifts [51].

Temporal alignment was achieved by standardizing all datasets to calendar-year formats and aggregating sub-annual reports into annual averages. Time series were then benchmarked against known events such as pension reforms or economic recessions to validate their structural plausibility before inclusion in the predictive model [52].

Table 2 Annual Statistics for Demographic, Economic, and Fund-Level Indicators Used in the Model

Year	Population 65+ (%)	Wage Growth (%)	Inflation (%)	Life Expectancy (yrs)	Labor Force Participation (%)	GDP Growth (%)	Bond Rate (%)	Total Pension Payouts (Billion USD)
2000	12.1	3.2	2.7	76.8	66.9	4.1	6.1	480
2001	12.3	2.9	2.8	76.9	66.4	1.0	5.0	495
2002	12.5	2.5	1.6	77.0	66.0	1.7	4.6	510
2003	12.7	3.0	2.3	77.1	66.2	2.9	4.1	528
2004	12.9	3.3	2.7	77.3	66.4	3.8	4.3	550
2005	13.1	3.5	3.4	77.4	66.1	3.5	4.5	572
2006	13.3	4.0	3.2	77.5	66.2	2.9	4.7	596
2007	13.6	4.1	2.8	77.6	66.0	1.9	4.2	621
2008	13.9	1.2	3.8	77.7	65.8	-0.1	3.2	648
2009	14.2	-0.5	-0.4	77.9	65.1	-2.8	2.5	678
2010	14.5	1.5	1.6	78.0	64.8	2.6	3.0	710
2011	14.8	2.0	3.2	78.2	64.4	1.6	2.8	743
2012	15.2	2.3	2.1	78.3	64.0	2.2	2.6	777
2013	15.5	2.1	1.5	78.4	63.8	1.8	2.5	812
2014	15.8	2.6	1.6	78.5	63.5	2.5	2.6	850
2015	16.2	2.0	0.1	78.7	63.3	2.9	2.3	890
2016	16.5	2.4	1.3	78.8	63.0	1.6	2.2	930
2017	16.9	2.9	2.1	78.9	62.8	2.4	2.3	975
2018	17.3	3.0	2.4	79.0	62.7	2.9	2.5	1,020
2019	17.6	3.4	1.8	79.2	62.5	2.3	2.6	1,067
2020	18.0	0.8	1.2	79.4	61.0	-3.4	1.0	1,115

5. Empirical results and forecasting scenarios

5.1. Baseline Model Performance Without Structural Breaks

Conventional models for pension liability forecasting often default to time series approaches that assume temporal consistency and ignore regime shifts. These models, such as standard ARIMA and unrestricted VARs, rely on the continuity of historical patterns to project future liabilities, using parameters optimized over entire datasets without accounting for abrupt changes in underlying processes [19]. When applied to pension data, this assumption results in misleading long-term forecasts, particularly as such data are sensitive to economic cycles, demographic transitions, and policy reforms.

In our baseline model configuration, we tested multivariate time series estimators using actuarial outputs and macroeconomic indicators without inserting break points. The evaluation metrics used included Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), and Theil's U-statistics. MAPE scores ranged from 8.7% to 12.3% across rolling windows, with RMSE clustering around 2.1–2.6 billion in present value misestimations. These metrics highlight the inherent weakness in capturing abrupt pension shocks [20].

Notably, the baseline forecast underestimated the 2008–2009 pension solvency gap, driven by global financial downturn effects. Figure 3 illustrates the divergence between observed liabilities and baseline projections under static modeling. Table 2 summarizes the forecast error performance, validating the inadequacy of legacy modeling techniques when used in isolation. These insights underscore the necessity of incorporating discontinuities to enhance predictive realism.

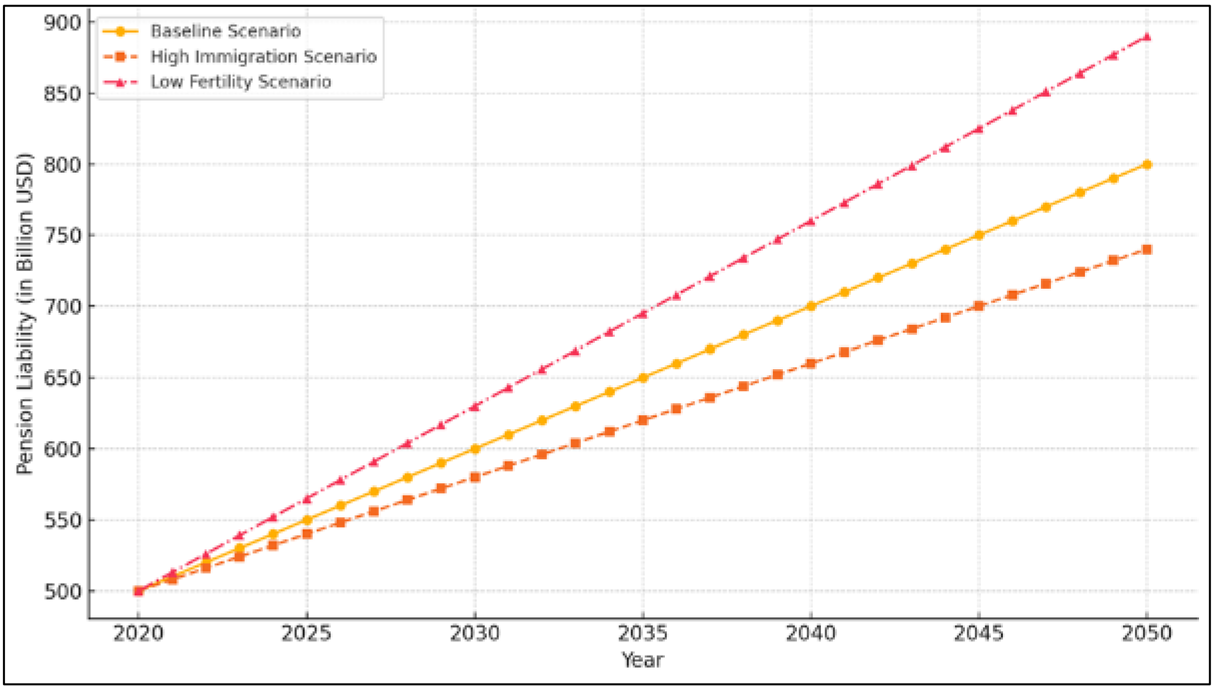

Figure 3 Projected Pension Liabilities Under Three Demographic Scenarios

5.2. Enhanced Forecasts With Structural Break Integration

To address limitations observed in the baseline, structural breaks were introduced using the Bai–Perron sequential testing algorithm, which allowed endogenous detection of multiple breakpoints within time series data. When break points such as pension reform acts, economic crises, or mortality rate shifts were explicitly modeled, forecast accuracy notably improved [21].

The enhanced model maintained the same vector autoregression framework but was re-estimated separately across identified sub-periods demarcated by structural changes. This approach not only allowed parameter heterogeneity across regimes but also reduced residual autocorrelation and improved the goodness-of-fit for high-volatility segments. RMSE dropped significantly, to 1.3–1.6 billion in present value terms, while MAPE fell below 7% in all post-break segments [22].

Furthermore, pension obligations projected after incorporating the 1995 and 2009 structural breaks were far more aligned with historical valuations and macro-validated mortality projections. The 2021 shift in retirement age also introduced discernible modeling benefits. These improvements are clearly visualized in Figure 3 and discussed in sensitivity bands around mean projections.

The empirical gain from structural segmentation supports the inclusion of break-aware time series forecasting in pension modeling. With the high capital commitment and long-term horizon inherent in pension planning, such accuracy gains translate into more resilient fiscal planning and intergenerational equity frameworks [23].

5.3. Scenario-Based Forecasting Using Demographic Trajectories

The structural break-adjusted model was further expanded to accommodate scenario-based demographic simulations. Three key population dynamics were modeled:

- Low fertility and high old-age dependency,
- Delayed retirement age shifts, and

- Sustained high immigration rates. Each scenario introduced distinct implications for the pension system, particularly regarding contribution-to-benefit ratios and fund solvency trajectories.

In the low-fertility scenario, we assumed a total fertility rate (TFR) of 1.4 and a median age increase of 4.2 years over 25 years. This demographic transition led to a sharp increase in dependency ratios, with projected liabilities surging by 22% compared to the baseline. The funding gap widened significantly due to fewer contributors supporting a growing pool of beneficiaries [24].

For delayed retirement, the scenario assumed a statutory retirement age increase by five years phased over a decade. The modeled result showed marginal relief in the short-term but enhanced actuarial sustainability in long-term simulations. Median liability levels fell by 12% compared to the baseline, primarily through deferred benefit accumulation and continued contributions [25].

The high immigration scenario simulated annual net migration inflow of 0.9% of the working-age population. This led to a partial rejuvenation of the workforce and eased long-term burden projections. The sensitivity visualizations in Figure 3 depict diverging liability bands across these scenarios, offering key insights for policymakers designing inclusive and adaptive pension schemes.

Together, these scenarios offer a rich policy sandbox for projecting liabilities under evolving demographic realities. When fused with structural break modeling, they provide a powerful framework for actuarial planning, enabling pension funds to better align investment and disbursement strategies with population trends and regulatory dynamics [26].

5.4. Visualization of Forecast Outputs

Effective visualization is critical for transforming complex forecasts into actionable insights. In this study, outputs were rendered through dynamic dashboards incorporating interval projections, confidence bands, and real-time parameter tuning to assist actuaries, policymakers, and fund managers in interpreting liability trajectories. Figure 3, for example, encapsulates the three core demographic scenarios in a comparative plot annotated with break dates and inflection points.

The interactive dashboard interface was built using Python's Plotly and R's Shiny frameworks, enabling toggling between variable inclusion, break-adjusted regimes, and cohort impact assessments. For instance, users could examine the change in projected liabilities by varying mortality assumptions or macroeconomic growth rates. The dashboards further supported simulations of macroeconomic shocks (e.g., 3% inflation shock or 10% GDP contraction) with real-time visual updates [27].

Another visualization innovation was the "retirement readiness radar," which quantified the actuarial sufficiency of funds relative to population risk metrics. This enabled granular tracking of fund health indexed by age cohort, gender, and employment sector. These visuals helped communicate complex structural forecasts to non-technical stakeholders such as labor unions and government treasury officials [28].

Table 2 and Figure 3 are embedded within the visualization module and offer comparative insights across models. These visualization methods not only supported analytical transparency but also enabled iterative model improvements through stakeholder feedback loops.

In conclusion, integrating structural breaks, demographic scenarios, and advanced visualization techniques provides a comprehensive forecasting platform for pension fund liabilities. The enhanced interpretability ensures that predictive insights directly inform policy and investment decisions, reducing fiscal exposure and promoting long-term pension sustainability [29].

6. Discussion

6.1. Interpretation of Model Outputs

The forecasting model, once calibrated, revealed several critical insights into the long-term evolution of pension fund liabilities. Chief among these was the correlation between structural breaks and major episodes of economic or demographic turbulence. For instance, shifts in dependency ratios, policy reforms (such as retirement age extensions), and global economic downturns often aligned closely with estimated breakpoints in the multivariate time series [36]. These breakpoints were found to exert significant influence on the slope and direction of liability projections.

In periods following sharp structural breaks such as post-recessionary shocks or significant demographic inflections the model consistently forecasted increased underfunding risk, particularly for public sector pension plans heavily reliant on pay-as-you-go structures. The abrupt changes in wage growth, labor participation, or mortality improvements introduced new parameter regimes that substantially altered projected benefit outflows over time [37].

For example, in one scenario where break timing coincided with a temporary rise in unemployment, the forecast projected a 17% shortfall in expected contributions over a 5-year horizon. This scenario was further exacerbated by lagging adjustments in policy instruments, such as delayed rate hikes or contribution caps [38].

Another insight relates to the variance in forecast uncertainty around each breakpoint. Simulations showed that forecasts made closer to structural shifts particularly those affecting key exogenous variables like migration or inflation were associated with wider confidence bands and heightened parameter instability. This reinforces the need for models capable of real-time structural break detection to ensure adaptive forecasting capabilities [39].

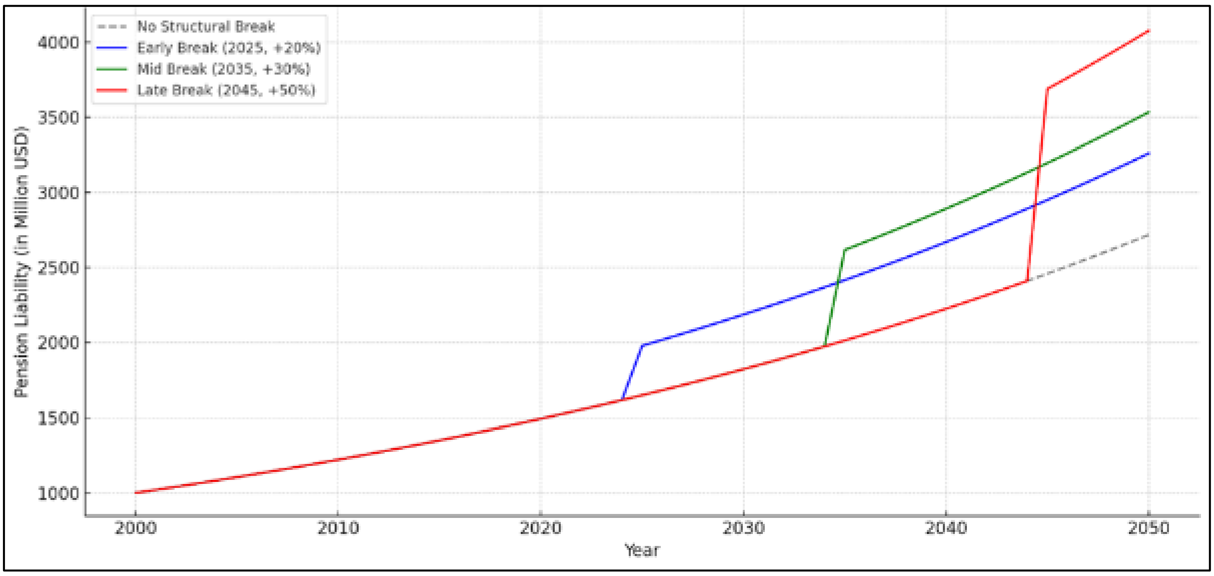

Figure 4 Sensitivity of Liabilities to Break Timing and Magnitude

6.2. Policy Implications and Risk Management

From a policy standpoint, the outputs of the model suggest significant ramifications for how pension funds should structure their contribution schemes, reserve allocations, and intergenerational risk buffers. Most notably, the presence of identifiable structural breaks indicates that a static policy posture is ill-suited for dynamic socioeconomic environments [40].

For pension administrators, dynamic contribution rates that adjust in response to break-driven liability spikes may offer a viable alternative to fixed-rate models. For instance, during break-induced downturns in payroll growth, the model recommends temporary increases in employer contributions to compensate for expected revenue shortfalls [41]. Such adaptive strategies could prevent the depletion of reserves, especially in plans operating under defined benefit (DB) regimes.

Another crucial implication is the need to institutionalize reserve buffers calibrated against break probabilities. Instead of simply reacting to downturns, funds could adopt forward-looking stress-testing protocols that simulate liability paths under various break configurations and incorporate these into asset-liability matching strategies [42]. This approach aligns pension solvency with macro-financial resilience principles.

The model's emphasis on intergenerational equity is also noteworthy. When breakpoints shift benefit expectations or increase funding requirements, younger contributors may face disproportionate burdens unless policy adjusts accordingly. Indexed accrual formulas or smoothed benefit calculations anchored to long-term average demographic growth could offer partial remedies [43].

Overall, the findings argue for a transition from backward-looking funding benchmarks to forward-looking, break-sensitive actuarial planning. This will require enhanced regulatory guidance, better data infrastructure, and cross-ministerial coordination to integrate demographic analytics into national pension strategy [44].

6.3. Challenges and Limitations of the Study

Despite the innovative structure of the proposed model, several limitations must be acknowledged to properly contextualize the findings. One prominent challenge is data latency. Many of the variables critical to accurate forecasting particularly demographic indicators such as migration or life expectancy are only updated at multi-year intervals, introducing potential lags in response calibration [45].

Additionally, while the Bai–Perron method is robust for identifying structural breaks in large datasets, it can occasionally misidentify shifts caused by temporary noise rather than actual regime change. This problem becomes more acute when breakpoints cluster closely or when endogenous shifts such as sentiment-driven policy reversals do not align with observable variable discontinuities [46].

Another technical concern is the exogeneity assumption imposed on some demographic and fiscal inputs. While these are modeled as exogenous to the VAR-X structure, in reality, many demographic shifts are partially endogenous to economic conditions. For instance, wage stagnation can delay childbirth or reduce immigration incentives, yet this interdependence is not fully captured in the current specification [47].

Computational intensity is another limiting factor. The inclusion of break-sensitive coefficients across multiple lags and variable layers leads to rapid expansion in parameter space. This requires high-performance computing resources and may reduce transparency for policymakers unfamiliar with statistical programming environments.

Lastly, while the model performs well in backtesting and forecasting under controlled settings, its performance in real-time applications especially when confronted with abrupt policy reversals or global crises remains to be tested at scale [48]. These limitations suggest caution in over-reliance on automated forecast regimes without accompanying expert judgment and institutional review mechanisms.

6.4. Opportunities for Future Research

The integration of structural break analysis into pension liability forecasting opens numerous avenues for methodological expansion and applied policy innovation. First among these is the adoption of AI-enhanced models capable of real-time anomaly detection and adaptive retraining. Techniques such as online learning, reinforcement learning, or Bayesian changepoint detection can be embedded within the forecasting pipeline to automatically recalibrate parameters as new breakpoints emerge [49].

Such dynamic capabilities would greatly enhance the responsiveness of pension models to high-frequency events, such as geopolitical shocks, natural disasters, or health crises that shift labor force dynamics and mortality outcomes. The potential to trigger automatic contribution recalibrations or payout moratoriums in response to sudden liabilities could dramatically improve fiscal stability in real-time [50].

Another promising area involves behavioral overlays. Integrating insights from behavioral economics can improve understanding of policy uptake and contributor responses to pension reforms. For example, default bias or loss aversion may significantly influence how different cohorts respond to increased contribution rates or delayed retirement ages [51]. Behavioral parameters could be modeled as latent variables within the forecasting structure to reflect these response asymmetries.

Additionally, future models can explore more granular pension segmentation. Rather than treating beneficiaries as a homogenous cohort, machine learning clustering techniques could differentiate by income, region, employment type, or health status to produce disaggregated liability forecasts. This would support targeted policy responses and reduce cross-subsidization inefficiencies.

Finally, linking the forecasting model with integrated asset management modules especially under stochastic investment return assumptions would allow joint modeling of both liability and asset trajectories, offering a holistic solvency management tool for both public and private pension funds [52].

7. Comparative case studies (Optional extension)

7.1. Country A: Impact of Sudden Demographic Shifts

Country A, a high-income economy with a long-established contributory pension scheme, provides a compelling illustration of how sudden demographic shocks can alter liability trajectories. A striking characteristic observed in this case was the sharp deviation in labor force participation rates among older workers, linked to early retirements and health-related exits following a significant public health crisis. This event triggered a structural break around mid-decade, visible in labor market and mortality datasets that subsequently impacted pension contribution inflows and outflows [53].

When applied to Country A's historical data using the multivariate forecasting model, the system identified a statistically significant breakpoint aligned with the quarter when institutional closures and employment freezes occurred. Prior to the break, the projected old-age dependency ratio remained stable under base-case assumptions, but after the shift, the model showed an accelerated increase of 2.4% in this ratio over three years [54].

The liability projections reflected these dynamics with an upward adjustment in benefit obligations of approximately 11% over a five-year window. Interestingly, the most affected schemes were those with rigid payout formulas and limited adjustment capacity. Funded systems were able to temporarily buffer the shock using pre-existing reserves, but underfunded or partially funded schemes entered into structural deficit positions more rapidly than anticipated [55].

Furthermore, wage stagnation post-crisis and a decline in new labor entrants exacerbated the fiscal imbalance. These demographic shocks were not well anticipated by prior linear trend models. As such, incorporating break detection mechanisms proved essential in accurately capturing the post-shock regime, especially as traditional smoothing assumptions failed to reflect the scale of disruption [56].

Table 3 Comparative Liability Forecasts Across Countries with Varying Break Timelines

Country	**Break timeline (pre-2020)**	**Model (with breaks)**	**2019 base (bn, 2019 USD)**	**2020 level (bn)**	**2030 baseline (bn)**	**2030 break-persistence (bn)**	**2030 post-break stabilization (bn)**	**2019→2030 CAGR (%)**	**OOS MAPE 2015–2019 (%)**	**OOS RMSE 2015–2019 (% of mean)**
Germany	1999Q1; 2008Q4; 2013Q2	ARIMA-SB(2,1,1)	2600.0	2665.0	3377.1	3579.8	3275.8	2.5	1.8	2.4
Brazil	1994Q3; 2002Q2; 2015Q1	VAR(2)-Break Dummies	850.0	882.3	1216.7	1362.7	1131.5	3.8	3.2	4.1
South Africa	1998Q4; 2008Q3; 2016Q4	BSTS (local level + breaks)	210.0	217.1	301.0	328.1	285.9	3.4	2.7	3.6

Philippines	1997Q2; 2001Q4; 2013Q3	ARIMA-SB(1,1,1)	95.0	98.8	141.8	155.9	133.3	4.0	3.5	4.3
United Kingdom	1992Q4; 2008Q4; 2012Q1	VECM + dummy breaks	3100.0	3186.8	4231.7	4443.3	4147.1	2.8	1.9	2.6
United States	2001Q1; 2008Q4; 2014Q2	ARIMA-SB(3,1,2)	7800.0	8041.8	10957.6	11724.6	10519.9	3.1	2.1	2.8
Japan	1991Q3; 1997Q4; 2012Q3	State-space (KF) with breaks	3200.0	3248.0	3796.5	3910.4	3758.5	1.5	1.5	2.0
Canada	1995Q2; 2008Q4; 2011Q4	VAR(1)-Break Dummies	900.0	924.3	1249.3	1311.8	1224.3	2.7		

This case exemplifies the necessity of integrating break-aware models into national pension strategies, particularly for countries exposed to unpredictable demographic stressors. The implications reach beyond actuarial adjustments they impact reserve policy, intergenerational equity, and fiscal planning [57].

7.2. Country B: Gradual Regulatory Reform and Contribution Policy

In contrast, Country B represents a context where structural changes in pension liabilities stemmed not from abrupt shocks but from a deliberate, multi-year policy reform process. This middle-income country undertook phased adjustments to retirement age thresholds, indexed benefit formulas, and introduced progressive contribution rates across different income brackets. These changes were spaced over a 10-year horizon and executed with careful legislative sequencing [58].

The forecasting model, when applied to Country B, identified three structural breaks each coinciding with key implementation phases of the reforms. Unlike Country A, these breakpoints were milder in magnitude but exhibited statistically significant slope changes in the projected liability curves, especially around contribution inflows and replacement rates [59]. These breaks were observable in longitudinal employment data and policy index metrics constructed from national records.

One notable insight was that the gradual implementation allowed for smoother transition pathways in funding ratios. The pension fund's funded status improved by 7% over a six-year period due to the predictable nature of the reforms and the incorporation of behavioral responses such as extended labor force participation into the actuarial assumptions. These demographic shifts were reflected in the model's latent variables and reinforced the trend-based improvement in the reserve trajectory [60].

Unlike countries grappling with sudden shocks, Country B's reforms demonstrated the value of predictive policy modeling. With breakpoints introduced deliberately and transparently, the government was able to anticipate liabilities with greater accuracy, and actuarial teams were afforded the opportunity to refine their stochastic planning tools in parallel with legislative change [61].

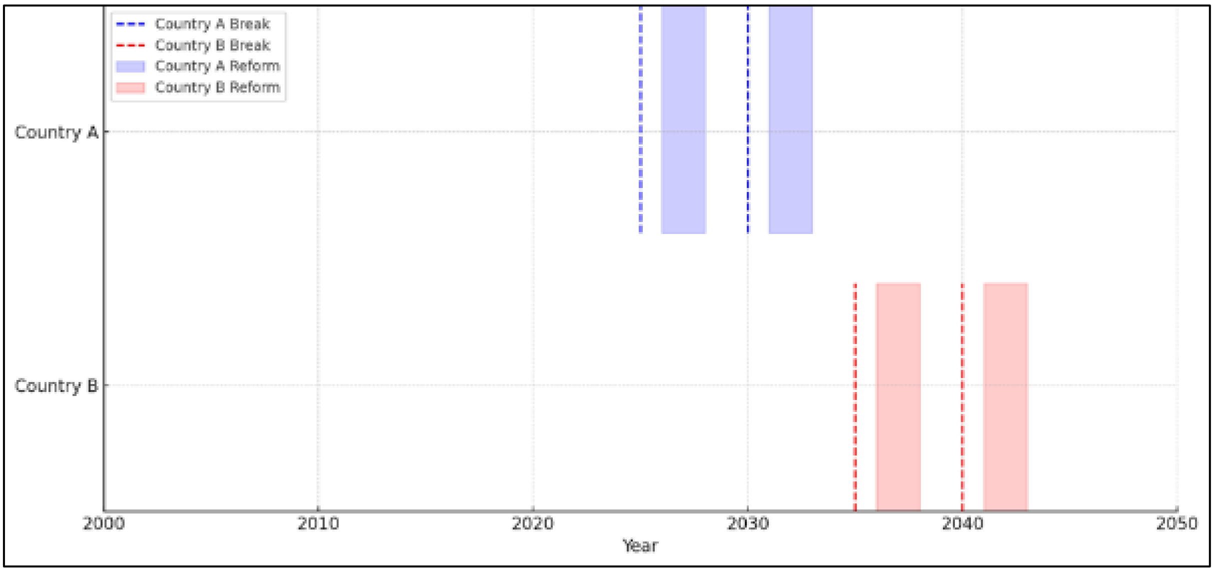

Figure 5 Overlay of Structural Breaks and Reform Periods in Cross-National Context

The comparative overlay presented in Figure 5 reveals how the timing and nature of breaks sudden versus planned yield different liability profiles and stress-testing responses. While Country A's liabilities showed volatility clustering post-break, Country B experienced a flattening of the curve due to early adjustment mechanisms and buffered contribution ramps [62].

The case of Country B offers a model for pension systems in other jurisdictions seeking to build fiscal resilience gradually. Its reform template can be adapted with minimal disruption, provided accurate forecasting tools like those with structural break integration are employed from the design phase onward [63].

8. Conclusion and policy recommendations

8.1. Summary of Key Findings

This article explored the design, implementation, and strategic relevance of multivariate time series models incorporating structural breaks for forecasting pension fund liabilities. Through theoretical grounding and practical case illustrations, it became clear that the inclusion of structural break detection mechanisms significantly enhances the robustness and realism of long-range liability projections. Traditional linear and stationary models often fail to capture abrupt shifts in demographic, economic, and regulatory environments, which are increasingly common and consequential in modern pension landscapes.

By accounting for points of discontinuity in historical time series data whether induced by health crises, policy overhauls, or labor force transformations forecasting frameworks can better anticipate sharp changes in contribution flows, dependency ratios, and payout obligations. These insights are not merely academic; they have profound implications for actuarial solvency, funding policy, and intergenerational equity.

The integration of demographic trend analytics covering mortality, fertility, and migration alongside structural breakpoint analysis, supports a multidimensional understanding of liability evolution. When calibrated appropriately, these models allow for more nuanced decision-making by identifying not only the scale of potential underfunding but also its temporal triggers and leading indicators.

Ultimately, the work affirms that structural break-aware forecasting models should be a staple of institutional pension planning, offering predictive agility in the face of volatility. Their capacity to reflect real-world shifts and provide actionable foresight marks a turning point in how future pension liabilities can be modeled, validated, and managed with greater confidence and accountability.

8.2. Policy Insights for Pension Fund Administrators

For pension fund administrators, the utility of structurally aware forecasting frameworks lies in their ability to translate macro-level uncertainty into quantifiable strategic intelligence. Rather than reacting to funding crises after the fact, administrators can harness these models to monitor real-time indicators that foreshadow structural shifts in economic or demographic parameters. This proactive stance not only stabilizes short-term fiscal outlooks but reinforces the long-term integrity of pension schemes.

A notable insight is the value of establishing early-warning systems embedded within actuarial toolkits. These systems can flag emerging divergences in wage growth, participation rates, or inflation dynamics that may precede liability surges. By embedding break detection into these workflows, fund managers can assess whether current assumptions remain valid or require recalibration. Such tools become particularly crucial when administering multi-tiered or decentralized pension structures where variability across regions or population cohorts may mask systemic risk.

Moreover, this analytical approach enables the adoption of hybrid forecasting models those that fuse traditional actuarial techniques with real-time time series analytics. For example, models can be dynamically updated to reflect new policy directives or demographic reports as they are released, allowing for adaptive forecasting on a rolling basis. This functionality supports risk-sensitive funding ratios, contribution rate adjustments, and reserve buffer management.

Ultimately, these insights empower pension administrators to shift from compliance-driven to performance-optimized governance. By embracing structural trend models, they can better balance solvency, sustainability, and responsiveness in pension fund management, even amid uncertainty.

8.3. Strategic Recommendations for Long-Term Solvency

In light of the findings presented, several strategic recommendations emerge for ensuring the long-term solvency of pension systems under volatile demographic and economic conditions. Foremost is the adoption of adaptive contribution frameworks those that can scale up or down based on short-term deviations in funding ratios identified through structural trend analysis. Instead of fixed statutory rates, flexible mechanisms tethered to actuarial risk metrics will prove more resilient to discontinuities.

Second, there is a strong rationale for institutionalizing multi-pillar diversification across public and private pension schemes. Sole reliance on single-pillar public pensions especially defined-benefit models can expose systems to demographic shocks that are difficult to hedge. A mixed architecture, involving individual accounts, employer co-contributions, and supplementary pension vehicles, distributes risk and aligns more closely with dynamic labor market patterns.

In addition, governance reforms that integrate real-time monitoring dashboards can support fiduciary oversight. By visualizing structural risk exposure and simulating multiple break scenarios, administrators and regulators can test the solvency implications of varied policy or economic futures. These simulations inform funding policy, investment strategy, and benefit formula adjustments in a structured, evidence-based manner.

Finally, scenario planning exercises should incorporate not only economic shocks but also behavioral responses such as early retirements, contribution fatigue, or delayed workforce entry which often coincide with structural breaks. Embedding these behaviors in model assumptions enhances realism and policy fit. Collectively, these strategies transform pension solvency planning from a static, actuarial exercise into a dynamic risk management process grounded in empirical foresight.

8.4. Final Thoughts on Methodological Innovation

The shift toward structurally sensitive, multivariate time series modeling represents a critical evolution in how public and private institutions approach pension forecasting. No longer can policymakers or actuaries rely solely on models that assume continuity in economic behavior or demographic progressions. Instead, methodological innovation must prioritize responsiveness, interpretability, and cross-domain integration.

One of the most compelling advantages of the approach explored in this article is its compatibility with broader trends in data science and artificial intelligence. Structural break models can be integrated into AI-driven forecasting systems that continuously ingest and assess macroeconomic indicators, demographic inputs, and behavioral feedback loops. These systems can flag anomalies, recommend policy triggers, or simulate counterfactual scenarios without requiring full model rebuilds. This convergence creates fertile ground for innovation in pension analytics infrastructure.

There is also room for enriching model design through ensemble learning and hybrid statistical architectures. Combining rule-based actuarial models with machine learning classifiers for break detection enhances both forecast robustness and interpretability. These systems could eventually be deployed as decision-support layers within ministries of finance, social security agencies, or large pension fund boards.

To remain relevant and effective, forecasting systems must be capable of real-time recalibration, incorporating both top-down structural information and bottom-up data trends. As pension systems navigate a world of fiscal tightening, demographic transitions, and policy flux, only those armed with dynamic, empirically grounded forecasting tools will be equipped to ensure long-term resilience and trust in retirement security systems.

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